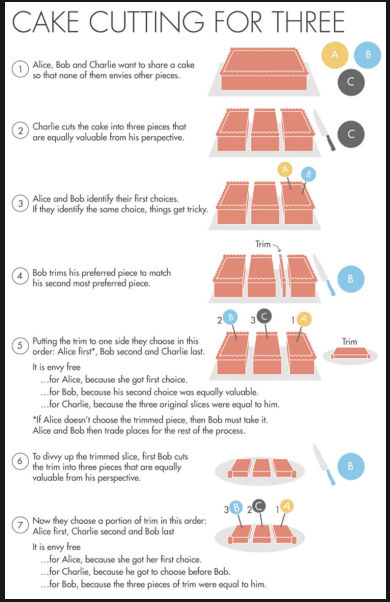
CS480/680: Introduction to Machine Learning

Lec 21: Algorithmic Fairness

Yaoliang Yu

UNIVERSITY OF WATERLOO
FACULTY OF MATHEMATICS
DAVID R. CHERITON SCHOOL OF COMPUTER SCIENCE

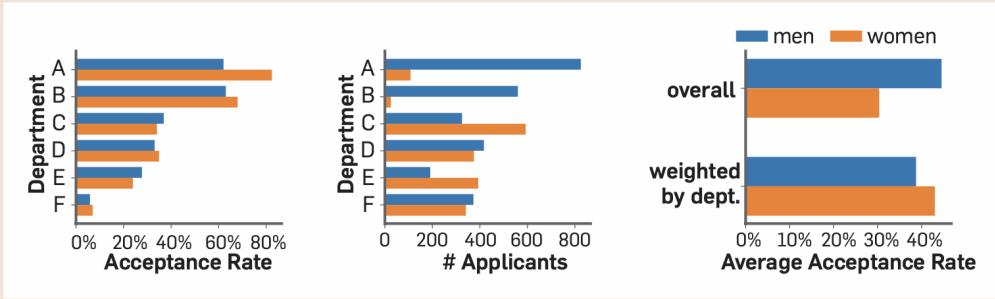
March 27, 2025



H. Aziz and S. Mackenzie. "A bounded and envy-free cake cutting algorithm". *Communications of the ACM*, vol. 63, no. 4 (2020), pp. 119–126.

Simpson's Paradox: Berkeley Admission Statistics (1973 fall)

Figure 2. UC Berkeley admissions statistics for men and women. Left: Acceptance rates. Middle: Number of applicants. Right: Average acceptance rate, either overall or weighted by the total number of applicants (of both groups) for each department.



- Overall acceptance rate for men was higher (44%) than for women (35%)
- For almost all departments, women enjoyed a higher acceptance rate than men

M. Buyl and T. D. Bie. "Inherent Limitations of AI Fairness". *Communications of the ACM*, vol. 67, no. 2 (2024), pp. 48–55.

COMPAS: Correctional Offender Management Profiling for Alternative Sanctions

- Developed by Northpointe in 1998, sold to Toronto-based Constellation Software in 2011
- Used in some US criminal justice systems
- Predicts a defendant's risk of committing a misdemeanor or felony *within 2 years*
  - proxy for lack of groundtruth (committing a crime)
- 137 features about an individual and the individual's past criminal record

Example Features in COMPAS

- Prior arrests and convictions
- Address of the defendant
- Whether the defendant a suspected gang member
- Whether the defendant ever violated parole
- If the defendant's parents separated
- If friends/acquaintances of the defendant were ever arrested
- Whether drugs are available in the defendants neighborhood
- How often the defendant has moved residences
- The defendants high school GPA
- How much money the defendant has
- How often the defendant feels bored or sad
- Age at the time of current offense
- Age at the time of first offense

One variable that doesn't appear is the defendant's race

White				Black			
		Actual				Actual	
		NR	R			NR	R
Predicted	NR	999	408	Predicted	NR	873	473
	R	282	414		R	641	1188
FN	0.50			FN	0.28		
FP	0.22			FP	0.42		

- Unequal base rates: $\frac{408+414}{408+414+282+999} \approx 39\%$ vs. $\frac{473+1188}{473+1188+873+641} \approx 52\%$
- Unequal odds: White higher False Negatives while Black higher False Positives
  - positive prediction (i.e., Recidivism) may be used by the judge against the defendent

A. W. Flores, K. Bechtel, and C. T. Lowenkamp. "False Positives, False Negatives, and False Analyses: A Rejoinder". *Federal Probation*, vol. 80, no. 2 (2016), pp. 38–46.

J. Angwin, J. Larson, S. Mattu, and L. Kirchner. "Machine bias". 2016.

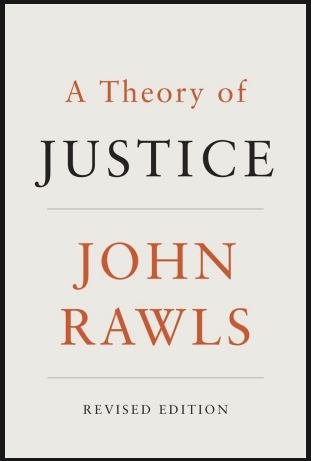
	All	White	Black
Low	32	29	35
Medium	55	53	56
High	75	73	75
Base Rate*	47	39	52
AUC	0.71	0.69	0.70

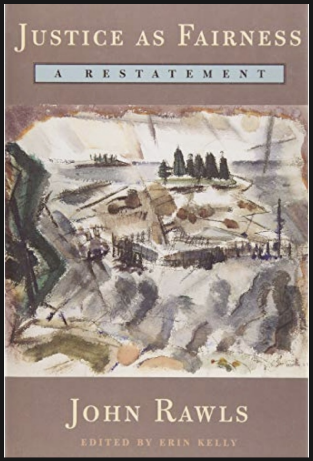
	All	White	Black
Low	11	9	13
Medium	26	22	27
High	45	38	47
Base Rate*	17	12	21
AUC	0.71	0.68	0.70

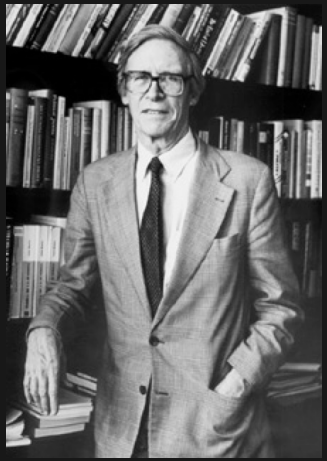
- Pr(Recidivism | race, risk score) roughly calibrated
  - left: any crime; right: violent crime only
- Accuracy parity: $\frac{414+999}{408+414+282+999} \approx 67\%$ vs. $\frac{873+1188}{473+1188+873+641} \approx 65\%$
- No demographic parity: $\frac{282+414}{408+414+282+999} \approx 33\%$ vs. $\frac{641+1188}{473+1188+873+641} \approx 58\%$

A. W. Flores, K. Bechtel, and C. T. Lowenkamp. "False Positives, False Negatives, and False Analyses: A Rejoinder". *Federal Probation*, vol. 80, no. 2 (2016), pp. 38–46.

Each person possesses an inviolability founded on justice that even the welfare of society as a whole cannot override.







original position: *people select what kind of society they would choose to live under if they did not know which social position they would personally occupy.*

Setting

- Features for each individual: $\mathsf{X} \in \mathbb{R}^d$
- Binary labels: $\mathsf{Y} \in \{0, 1\}$
  - $\mathsf{Y} = 1$ being the preferred label, e.g., admission
- Sensitive attributes: $\mathsf{A} \in \{a, b\}$
  - partition individuals into groups
- (Probabilistic) prediction (e.g., by an algorithm or human): $\hat{\mathsf{Y}} = \hat{\mathsf{Y}}(\mathsf{X}) \in [0, 1]$
- Disparate Treatment: prediction $\hat{\mathsf{Y}}$ depends on sensitive attribute A
  - often by law or moral: $\mathsf{A} \notin \mathsf{X}$ (Rawls' original position)
  - proxy: may still be able to predict A based on other features in X

Affirmative Action (AA)

- First introduced in US by President JFK in 1961: government contractors "take affirmative action to ensure that applicants are employed, and employees are treated during employment, without regard to their race, creed, color, or national origin."

- By President LBJ in 1965: government employers to take "affirmative action" to "hire without regard to race, religion and national origin."

- In 1965: gender was added to the list

AA in Action

- Canada: the Canadian Charter of Rights and Freedoms explicitly permits affirmative action but does not require preferential treatment
  - The Canadian Employment Equity Act requires employers in federally-regulated industries to give preferential treatment to Women, persons with disabilities, aboriginal peoples, and visible minorities
- UK: quotas are illegal
- China: lower requirement for minorities in national university entrance exam; quota; dedicated financial aid/scholarship
- India: reservation system for majority (60% college admission or government jobs reserved for 90% majority)

- Regents of the University of California v. Bakke, 438 U.S. 265 (1978)
  - the court upheld affirmative action, allowing race to be one of several factors in college admission policy. However, the court ruled that specific racial quotas were impermissible.
- Grutter v. Bollinger, 539 U.S. 306 (2003)
  - the court held that a student admissions process that favors "underrepresented minority groups" did not violate the Fourteenth Amendment's Equal Protection Clause so long as it took into account other factors evaluated on an individual basis for every applicant
  - the court struck down a points-based admissions system that awarded an automatic bonus to the admissions scores of minority applicants
- Fisher v. University of Texas, 570 U.S. 297 (2013)
- Students for Fair Admissions v. Harvard, 600 U.S. 181 (2023)
  - the court held that race-based affirmative action programs in college admissions processes (excepting military academies) violate the Equal Protection Clause of the Fourteenth Amendment.

More than half a century earlier, the members of the Lawrence Tract met and debated whether to sell their vacant lot to a white or a black family. The Tract had to "weigh the value of the experiment against the need and welfare of the prospective buyer." Deciding between those two goals was far from easy. But if you want to use tipping points in the service of engineering a social outcome, that is what you have to do. You have to decide how far you will go to defend a number. And you have to be honest about what you are doing.

Except that instead of admitting underprivileged students with lower academic credentials, athletic affirmative action admits privileged students with lower academic credentials.

Revenge of the Tipping Point, by Malcom Gladwell, Chapter 5, page 170

Harvard Admissions (Percentage of Admitted Students by Race/Ethnicity)									
	2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
African American	10.5%	10.7%	11.0%	10.8%	11.3%	11.8%	10.2%	11.5%	11.9%
Hispanic	9.8%	10.1%	9.7%	10.9%	10.3%	12.1%	11.2%	11.5%	13.0%
Asian American	17.7%	19.6%	18.5%	17.6%	18.2%	17.8%	20.7%	19.9%	19.7%
Native American	1.4%	1.5%	1.3%	1.3%	2.7%	1.9%	1.7%	2.2%	1.9%
White and Other	60.6%	58.1%	59.5%	59.4%	57.5%	56.4%	56.2%	54.9%	53.5%

Revenge of the Tipping Point, by Malcom Gladwell, Chapter 5, page 153

Fairness Definition 1: Statistical/Demographic Parity

$$\mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = a) = \mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = b) = \mathbb{E}(\hat{\mathsf{Y}})$$

- For deterministic classifiers, i.e., $\hat{\mathsf{Y}} \in \{0, 1\}$, demographic parity means $\hat{\mathsf{Y}} \perp\!\!\!\perp \mathsf{A}$
- But, consider the following two scenarios:
  - scenario 1: For $\mathsf{A} = a$, accept top 10%; for $\mathsf{A} = b$, accept random 10%
  - scenario 2: $\mathsf{Y} = [\![\mathsf{A} = a]\!]$; may disallow (almost) perfect classifier...

Estimated Canadian breast cancer statistics (2024)

Category	Women	Men
New cases	30,500	290
Deaths	5,500	60
5-year net survival (estimates for 2015 to 2017)	89%	76%

https://cancer.ca/en/cancer-information/cancer-types/breast/statistics

Disparate Impact

- Griggs v. Duke Power Co. (1971, US Supremum Court)
  - 1950s: Duke Power held policy restricting black employees to its "Labor" dept.
  - 1955: Added requirement of high school diploma for employment in any dept. but Labor, and offered 2/3 training tuition for employee w/o diploma
  - 1965: Added 2 employment tests (mechanical & IQ) to allow employees w/o diploma to transfer to any dept.
  - Blacks were 10 times less likely to pass
- Supremum court ruling: if such tests disparately impact minority groups, businesses must demonstrate that such tests are "reasonably related" to the job for which the test is required

The Fox and the Stork

80% Rule

$$\frac{\mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = a) \ \wedge \ \mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = b)}{\mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = a) \ \vee \ \mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = b)} \geq \tau = 80\%$$

- Recall that $\mathsf{Y} = 1$ is the preferred label, e.g., hire
- Selection rate for the disadvantageous group (min) is at least 80% of that for the advantageous group (max)
- Advocated by the US Equal Employment Opportunity Commission (1979)
- Completely ignores the true label Y (qualification); quota or preferential treatment

M. Feldman et al. "Certifying and Removing Disparate Impact". In: *Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. 2015, pp. 259–268.

Fairness Definition 2: Equal Odds

$$\mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = a, \mathsf{Y} = y) = \mathbb{E}(\hat{\mathsf{Y}} \mid A = b, \mathsf{Y} = y), \quad \forall y \in \{0, 1\}$$

- For a deterministic classifier, i.e., $\hat{\mathsf{Y}} \in \{0, 1\}$, equal odds means $\hat{\mathsf{Y}} \perp\!\!\!\perp \mathsf{A} \mid \mathsf{Y}$
- If true label $\mathsf{Y} = 1$: (generalization of) equal true positives
- If true lable $\mathsf{Y} = 0$: (generalization of) equal false positives

$$\mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A}) = \int \mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A}, \mathsf{Y} = y) \Pr(\mathsf{Y} = y \mid \mathsf{A}) \, \mathrm{d}y$$

- Equal odds implies demographic parity under equal base rates $\Pr(\mathsf{Y} = y \mid \mathsf{A})$

M. Hardt, E. Price, and N. Srebro. "Equality of Opportunity in Supervised Learning". In: *Advances in Neural Information Processing Systems 29*. 2016, pp. 3315–3323.

Fairness Definition 3: Equal Opportunity

$$\mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = a, \mathsf{Y} = 1) = \mathbb{E}(\hat{\mathsf{Y}} \mid \mathsf{A} = b, \mathsf{Y} = 1)$$

- Recall $\mathsf{Y} = 1$ is the preferred label, e.g., loan approval
- $\mathsf{Y} = 1$: qualified applicants
- Among qualified applicants, equal true positives for different groups
- No requirement on unqualified applicants: maximal utility
- Post-processing: does not have access to Y in test time

M. Hardt, E. Price, and N. Srebro. "Equality of Opportunity in Supervised Learning". In: *Advances in Neural Information Processing Systems 29*. 2016, pp. 3315–3323.

Fairness Definition 4: Group Calibration

$$\mathbb{E}(\mathsf{Y} \mid \hat{\mathsf{Y}}, \mathsf{A} = a) = \hat{\mathsf{Y}} \in [0, 1], \quad \forall a$$

- For a deterministic classifier, i.e., $\hat{\mathsf{Y}} \in \{0, 1\}$, calibrated = perfect
- Among all instances that we predict positive with $\hat{\mathsf{Y}} = 80\%$ probability, indeed $\hat{\mathsf{Y}} = 80\%$ of them have true label 1
- Calibration is often desirable, but it may have little to do with accuracy
  - consider the constant predictor $\hat{\mathsf{Y}} = \mathbb{E}(\mathsf{Y})$: is it calibrated?
- True meaning: $f(\hat{\mathsf{Y}})$ is not more accurate than $\hat{\mathsf{Y}}$ for any post-processing f

---

G. W. Brier. "Verification of Forecasts Expressed in Terms of Probability". *Monthly Weather Review*, vol. 78, no. 1 (1950), pp. 1–3,
M. H. DeGroot and S. E. Fienberg. "The comparison and evaluation of forecasters". *Journal of the Royal Statistical Society: Series D (The Statistician)*, vol. 32, no. 1-2 (1983), pp. 12–22.

Inherent Tradeoff

Theorem: You can't have everything!

If a probabilistic classifier $\hat{Y} = \hat{Y}(X)$ satisfies

$$\mathbb{E}(Y \mid \hat{Y}, A) = \hat{Y} = \mathbb{E}(Y \mid \hat{Y}) \qquad \text{(group calibration)}$$

$$\mathbb{E}(\hat{Y} \mid Y, A) = \mathbb{E}(\hat{Y} \mid Y), \qquad \text{(equal odds)}$$

then either $\hat{Y}$ is a perfect classifier or the base rates match, i.e.,

$$\mathbb{E}(Y \mid A) = \mathbb{E}(Y).$$

- Applies to any probabilistic classifier, algorithmic or human
- When base rates differ, demographic parity contradicts calibration

J. Kleinberg, S. Mullainathan, and M. Raghavan. "Inherent trade-offs in the fair determination of risk scores". In: *ITCS*. 2017, 43:1–43:23.

A. Chouldechova. "Fair prediction with disparate impact: A study of bias in recidivism prediction instruments". In: *Fairness, Accountability, and Transparency in Machine Learning*. 2016.

$$
\begin{aligned}
\mathbb{E}[\hat{Y} \mid Y = 0] &= \mathbb{E}[\hat{Y} \mid Y = 0, A] && \text{(from equal odds)} \\
&= \frac{\mathbb{E}\big[\hat{Y} \cdot [\![Y = 0]\!] \mid A\big]}{\Pr[Y = 0 \mid A]} && \text{(definition of conditional expectation)} \\
&= \frac{\mathbb{E}\big[\hat{Y}(1 - [\![Y = 1]\!]) \mid A\big]}{1 - \mathbb{E}[Y \mid A]} && \text{(Y is binary)} \\
&= \frac{\mathbb{E}[\hat{Y} \mid A] - \mathbb{E}[\hat{Y} \mid Y = 1, A] \cdot \Pr[Y = 1 \mid A]}{1 - \mathbb{E}[Y \mid A]} && \text{(conditional expectation)} \\
&= \frac{\mathbb{E}[Y \mid A] - \mathbb{E}[\hat{Y} \mid Y = 1, A] \cdot \mathbb{E}[Y \mid A]}{1 - \mathbb{E}[Y \mid A]} && \text{(from calibration)} \\
&= \tfrac{\mathbb{E}[Y|A]}{1-\mathbb{E}[Y|A]} \cdot \big(1 - \mathbb{E}\big[\hat{Y} \mid Y = 1, A\big]\big) \\
&= \tfrac{\mathbb{E}[Y|A]}{1-\mathbb{E}[Y|A]} \cdot \big(1 - \mathbb{E}\big[\hat{Y} \mid Y = 1\big]\big) && \text{(equal odds)}
\end{aligned}
$$

- Either base rates $\mathbb{E}[Y|A] = \mathbb{E}[Y]$ do not depend on sensitive attributes A
- Or $\mathbb{E}[\hat{Y} \mid Y = 1] = 1$ and hence $\mathbb{E}[\hat{Y} \mid Y = 0] = 0$, i.e., $\hat{Y} = Y$ is perfect

Estimated Canadian breast cancer statistics (2024)

Category	Women	Men
New cases	30,500	290
Deaths	5,500	60
5-year net survival (estimates for 2015 to 2017)	89%	76%

https://cancer.ca/en/cancer-information/cancer-types/breast/statistics

- Base rates clearly differ
- So far, no classifier is perfectly accurate
- Thus, any existing classifier (algorithmic or not) can meet at most one of calibration and equal odds!

Things Are Easy under Matching Base Rates

- Consider predicting the mean, i.e., $\bar{Y} := \mathbb{E}(\mathrm{Y})$
- Trivially satisfies demographic parity and equal odds
  - more generally, any constant predictor satisfies demographic parity and equal odds
- When base rates match, $\bar{Y}$ also satisfies (group) calibration
- When base rates match, any constant predictor also satisfies accuracy parity

Fairness Definition 5: Individual Fairness

- Similar individuals should be treated similarly
- Transitivity can easily kill us: if a is similar to b, b is similar to c, ..., then we are forced to call a similar to z, even when they are very different

$$\text{dist}\left(\hat{\mathsf{Y}}(\mathsf{X}), \hat{\mathsf{Y}}(\mathsf{Z})\right) \leq \text{dist}\left(\mathsf{X}, \mathsf{Z}\right)$$

- In other words, our predictor $\hat{\mathsf{Y}}$ needs to be Lipschitz continuous
- But, finding an agreeable distance function is difficult

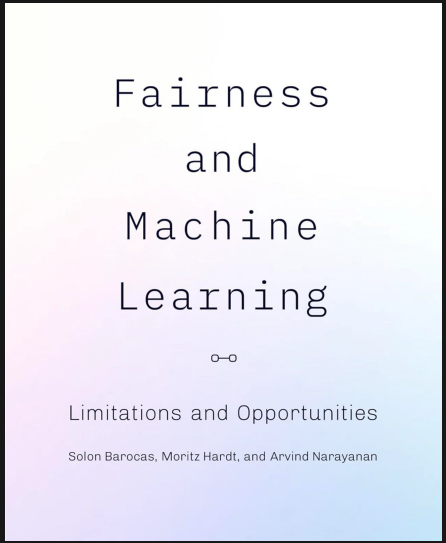
C. Dwork et al. "Fairness Through Awareness". In: *Proceedings of the 3rd Innovations in Theoretical Computer Science Conference*. 2012, pp. 214–226.

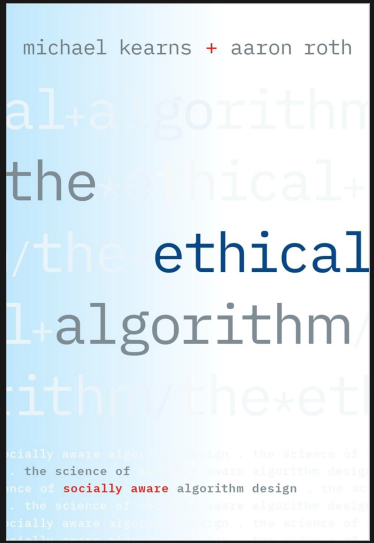
Some Perils of Algorithmic Fairness

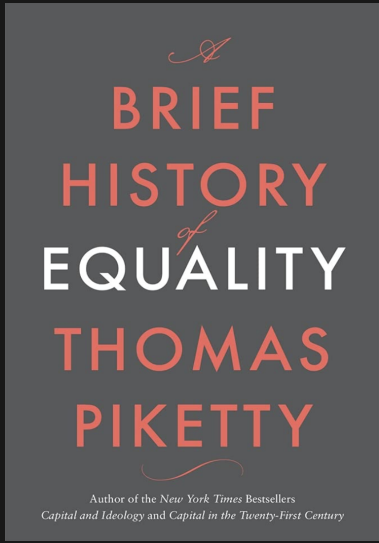
- Limited access to ground-truth label; often resort to questionable proxies
  - commit a crime $\approx$ arrested by police; neither one implies the other
- Need to collect sensitive attributes, something explicitly banned by AA
  - Proposed European AI Act allows processing sensitive data for bias monitoring, detection and correction
- No universally agreed definition (probably never will)
- Limited power over the entire decision pipeline
  - one would be naive to think algorithmic fairness can solve social issues all by itself
- Open to abuse

Goodhart's law: "When a measure becomes a target, it ceases to be a good measure"

M. Buyl and T. D. Bie. "Inherent Limitations of AI Fairness". *Communications of the ACM*, vol. 67, no. 2 (2024), pp. 48–55.


Fairness
and
Machine
Learning
Limitations and Opportunities
Solon Barocas, Moritz Hardt, and Arvind Narayanan


michael kearns + aaron roth
the
ethical
algorithm
the science of
socially aware algorithm design


A
BRIEF
HISTORY
of
EQUALITY
THOMAS
PIKETTY
Author of the New York Times Bestsellers
Capital and Ideology and Capital in the Twenty-First Century

Other Fairness Definitions

- Accuracy parity (for a deterministic predictor):

$$\Pr(\hat{\mathsf{Y}} = \mathsf{Y} \mid \mathsf{A}) = \Pr(\hat{\mathsf{Y}} = \mathsf{Y}),$$

  or more generally for a probabilistic classifier:

$$\mathbb{E}\left[\hat{\mathsf{Y}} \cdot \mathsf{Y} + (1 - \hat{\mathsf{Y}})(1 - \mathsf{Y}) \mid \mathsf{A}\right] = \mathbb{E}\left[\hat{\mathsf{Y}} \cdot \mathsf{Y} + (1 - \hat{\mathsf{Y}})(1 - \mathsf{Y})\right]$$

- Even more generally, we can compare the conditional distributions induced by different groups using any risk measure or divergence
- Causality/Counterfactual based

R. Williamson and A. Menon. "Fairness risk measures". In: *Proceedings of the 36th International Conference on Machine Learning*. 2019, pp. 6786–6797.

N. Kilbertus et al. "Avoiding Discrimination through Causal Reasoning". In: *Advances in Neural Information Processing Systems 30*. 2017, pp. 656–666, M. J. Kusner, J. Loftus, C. Russell, and R. Silva. "Counterfactual Fairness". In: *Advances in Neural Information Processing Systems 30*. 2017, pp. 4066–4076.