CS480/680: Introduction to Machine Learning

Lec 09: Convolutional Neural Network

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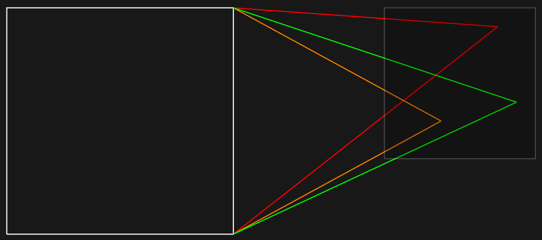
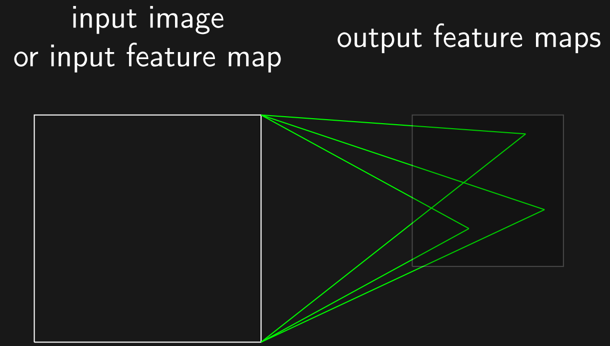
MLP Rearranged



- Fully connected; say input/output dim $m \times n \times d$ / $p \times q$
  - how many weights are there?

Weight Sharing



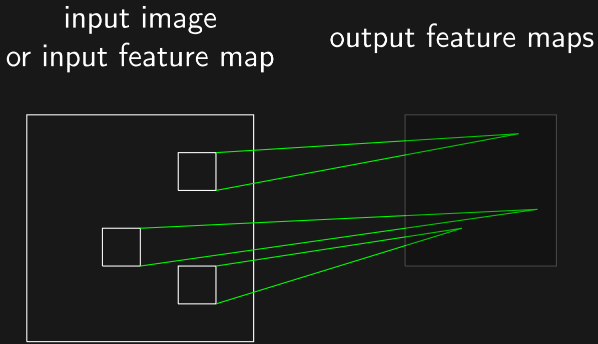
- Share weights; say input/output dim $m \times n \times d$ / $p \times q$
  - how many weights are there?
  - is there any new issue?

Enumerating All Possibilities

$$\mathbf{h} = \sigma(\langle \mathbf{w}, \mathbf{x} \rangle + b)$$

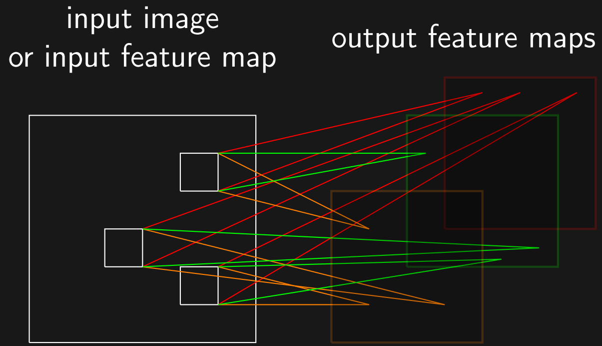
- Same $\mathbf{x}$, different $\mathbf{w}$: MLP
- Same $\mathbf{x}$, same $\mathbf{w}$
- Different $\mathbf{x}$, same $\mathbf{w}$
- Different $\mathbf{x}$, different $\mathbf{w}$

Weight Sharing with A Restricted Field



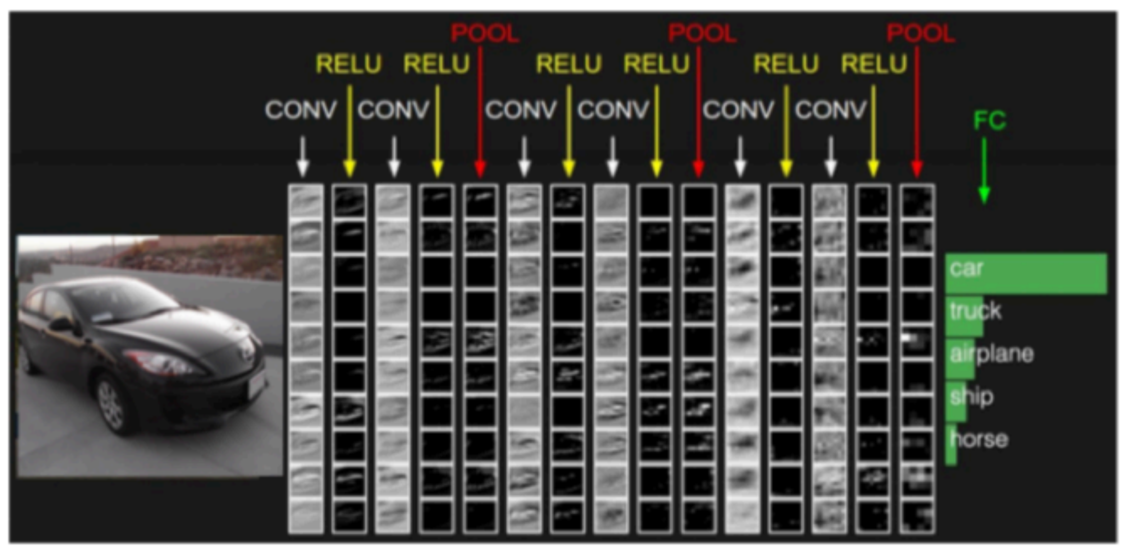
- Share weights; say input/output dim $m \times n \times d$ / $p \times q$, filter size $a \times b$
  - how many weights are there?
  - is there any new issue?

Weight Sharing and Convolution

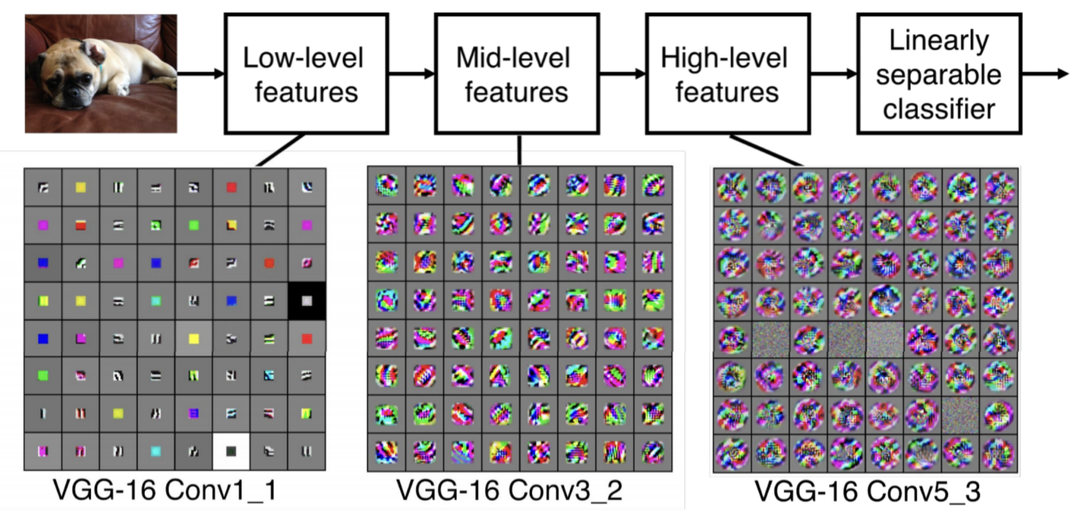


- Share weights; say input/output dim $m \times n \times d$ / $p \times q \times c$, filter size $a \times b$
  - how many weights are there?

Layers in Convolutional Neural Networks (CNN)



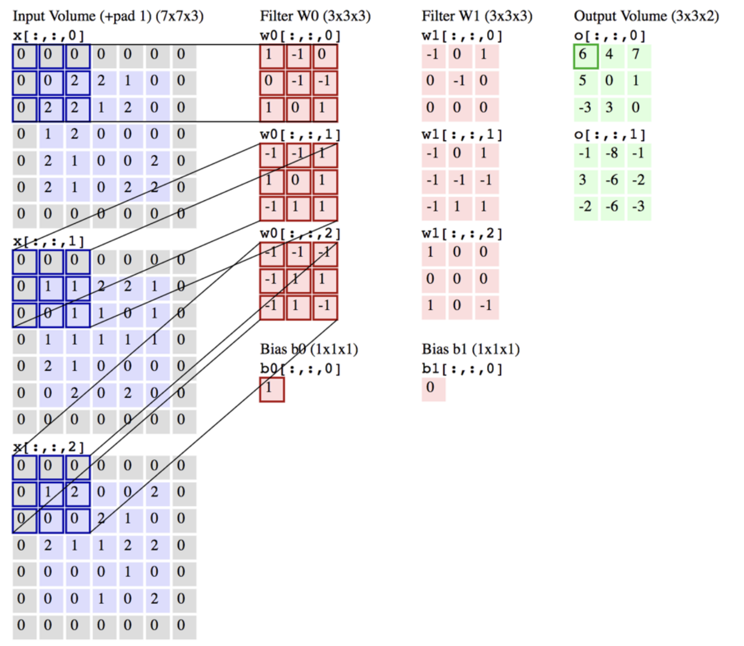
Hierarchical Feature Representation



M. D. Zeiler and R. Fergus. "Visualizing and Understanding Convolutional Networks". In: *European Conference on Computer Vision (ECCV)*. 2014, pp. 818–833.

Controling the Convolution

- Filter size: width x height, e.g. 3 x 3 or 5 x 5; by default, depth of each filter is the same as that of the input
- Number of filters: weights are not shared between filters; determine depth (channel) of output
- Stride: how many pixels to move the filter each time
  - typically stride $\leq$ filter size so as to leave no "gap"
  - larger stride makes neighboring outputs less similar due to less overlap in the input window
- Padding: add zeros (or any other value) around boundary of input
  - make the output size more standard (e.g. same as input, or 2^k for some k)



Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 2 1 0 0
0 2 2 1 2 0 0
0 1 2 0 0 0 0
0 2 1 0 0 2 0
0 2 1 0 2 2 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 1 2 2 1 0
0 0 1 1 0 1 0
0 1 1 1 1 1 0
0 2 1 0 0 0 0
0 0 2 0 2 0 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 0 0 2 0
0 0 0 2 1 0 0
0 2 1 1 2 2 0
0 0 0 0 1 0 0
0 0 0 1 0 2 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 0
0 -1 -1
1 0 1
w0[:,:,1]
-1 -1 1
1 0 1
-1 1 1
w0[:,:,2]
-1 -1 -1
-1 1 1
-1 1 -1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
-1 0 1
0 -1 0
0 0 0
w1[:,:,1]
-1 0 1
-1 -1 -1
-1 1 1
w1[:,:,2]
1 0 0
0 0 0
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 4 7
5 0 1
-3 3 0
o[:,:,1]
-1 -8 -1
3 -6 -2
-2 -6 -3

Size Calculation

Input size: $m \times n \times c$, filter size: $a \times b$, stride: $s \times t$, padding: $p \times q$

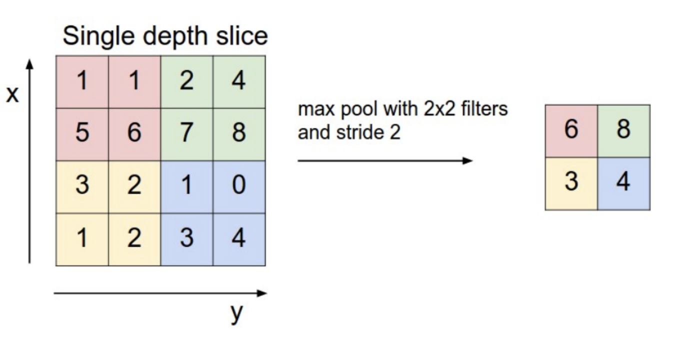
- Pad p pixels on left/right and q pixels on top/bottom (typically $p = q$)
- Filter size is $a \times b \times c$ but we omit the last dimension
- Move s pixels horizontally and t pixels vertically
- Output size: $\left\lfloor 1 + \frac{m+2p-a}{s} \right\rfloor \times \left\lfloor 1 + \frac{n+2q-b}{t} \right\rfloor$
- With $p = \left\lceil \frac{m(s-1)+a-s}{2} \right\rceil$ and $q = \left\lceil \frac{n(t-1)+b-t}{2} \right\rceil$, output size $=$ input size

Receptive Field

Input size: $m \times n \times c$, filter size: $a \times b$, stride: $s \times t$, padding: $p \times q$

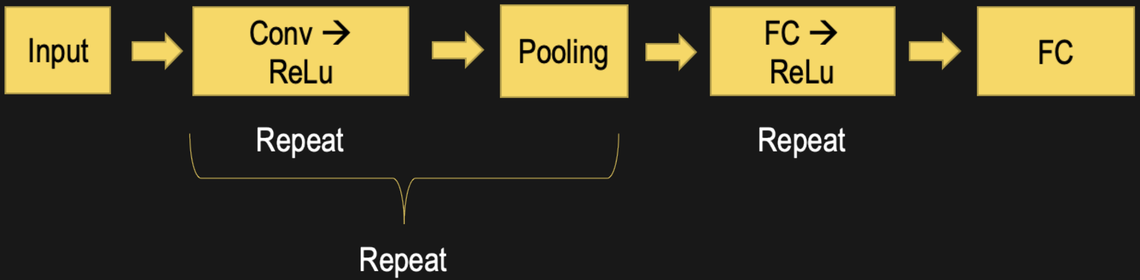
- How many pixels in the input can each output pixel "see" (i.e. depend on)?
  - obviously $a \times b$ (filter size); again, omitting channels by default and ignoring boundary
- How many pixels in the input can a $i \times j$ window in output "see"?
  - $[(i-1)s + a] \times [(j-1)t + b]$, assuming stride $\leq$ filter size and ignoring boundary
- How many pixels in layer k can a $i \times j$ window in layer $l \geq k$ "see"?
  - $i \cdot \prod_{r=k+1}^{l} s_r + \sum_{u=k+1}^{l} (a_u - s_u) \prod_{v=k+1}^{u-1} s_v$, assuming stride $\leq$ filter size and ignoring boundary
  - $i \cdot s^{l-k} + (a-s)\frac{s^{l-k}-1}{s-1}$, and $i + (a-1)(l-k)$ if $s \equiv 1$, i.e. increase by $a-1$ for every layer

Pooling



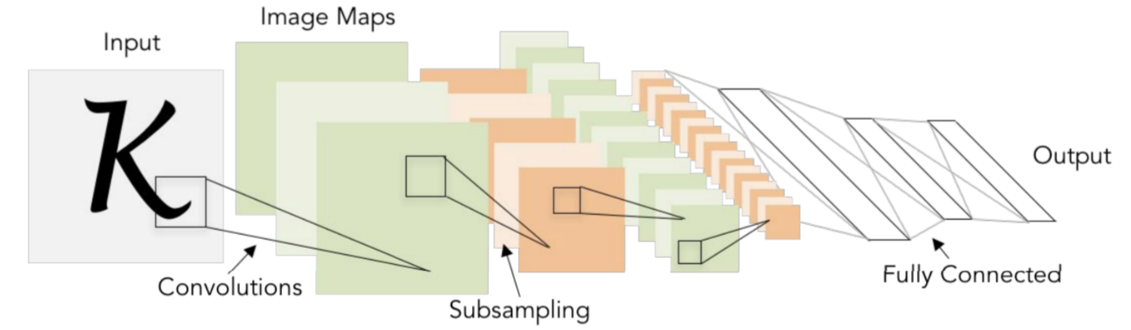
- Down-sample input size to reduce computation and memory
- Pooling by default is performed on each slice separately
  - hence output depth = input depth
  - max-pool, average-pool, (averaged) ℓ_p-norm pool
- Size and stride as in convolution; no parameter; typically no padding
- Global pooling: pooling size = input size

CNN Architecture



- Several standard architectures to choose (examples to follow)
- Try and tweak to fit your problem

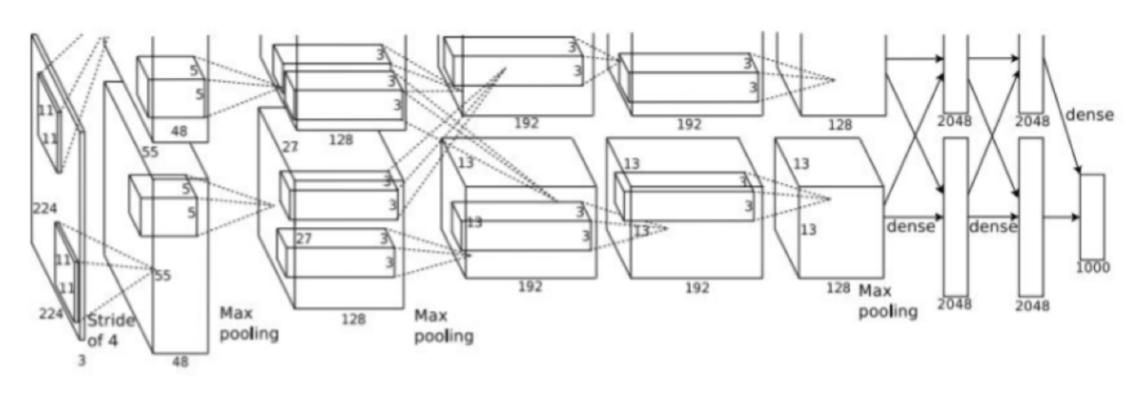
LeNet



Conv filters were 5x5, applied at stride 1
Subsampling (Pooling) layers were 2x2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-FC-FC]

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner. "Gradient-based learning applied to document recognition". *Proceedings of the IEEE*, vol. 86, no. 11 (1998), pp. 2278–2324.

AlexNet



A. Krizhevsky, I. Sutskever, and G. E. Hinton. "ImageNet Classification with Deep Convolutional Neural Networks". In: *Advances in Neural Information Processing Systems 25*. Ed. by F. Pereira, C. J. C. Burges, L. Bottou, and K. Q. Weinberger. 2012, pp. 1097–1105.

Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Details/Retrospectives:
- first use of ReLU
- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

VGGNet

Small filters, Deeper networks

8 layers (AlexNet)
-> 16 - 19 layers (VGG16Net)

Only 3x3 CONV stride 1, pad 1
and 2x2 MAX POOL stride 2

11.7% top 5 error in ILSVRC'13
(ZFNet)
-> 7.3% top 5 error in ILSVRC'14

- 7x7 filter vs. 3x3 filter x 3 (stride 1)
  - both have receptive field 7x7
  - params: $O(7 \times 7)$ vs. $O(3 \times 3 \times 3)$
- nxn vs. nx1 followed by 1xn ?

K. Simonyan and A. Zisserman. "Very Deep Convolutional Networks for Large-scale Image Recognition". In: *International Conference on Learning Representations*. 2015.

INPUT: [224x224x3] memory: 224*224*3=150K params: 0 (not counting biases)
CONV3-64: [224x224x64] memory: **224*224*64=3.2M** params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: **224*224*64=3.2M** params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = **102,760,448**
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

Note:

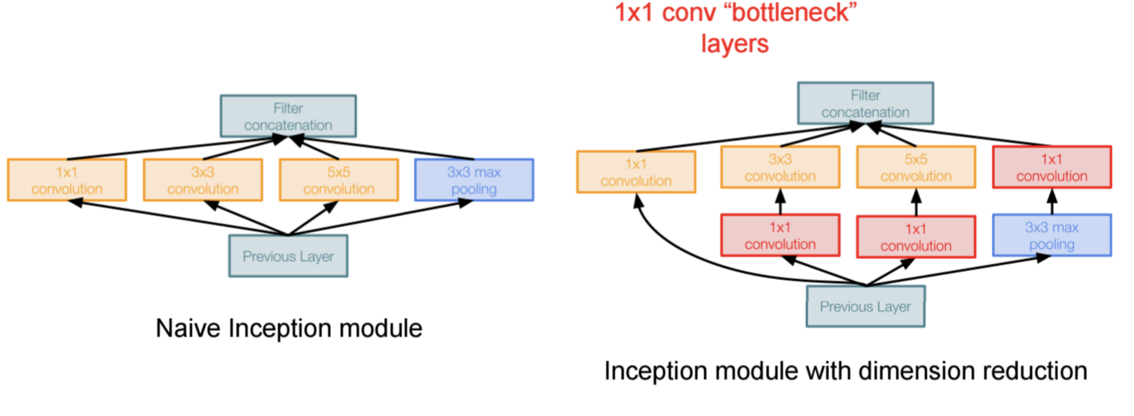
Most memory is in early CONV

Most params are in late FC

TOTAL memory: 24M * 4 bytes ~= 96MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters


THAT'S NOT ENOUGH
WE HAVE TO GO DEEPER

1x1 Convolution



C. Szegedy et al. "Going deeper with convolutions". In: *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 2015, pp. 1–9.

naïve 3x3 conv

m x n x c
3 x 3 x c x k --> m x n x k

O(9mnck)

3x3 conv first, 1x1 conv second

m x n x c
3 x 3 x c x d --> m x n x d
1 x 1 x d x k

O(9mncd + mndk)

1x1 conv first, 3x3 conv second

m x n x c
1 x 1 x c x d --> m x n x d
3 x 3 x d x k

O(mncd + 9mndk)

output dim k > intermediate dim d   input dim c > output dim k

1x1 conv first, 3x3 pool second

m x n x c
1 x 1 x c x k --> m x n x k
3 x 3 x k

O(mnck + mnk)

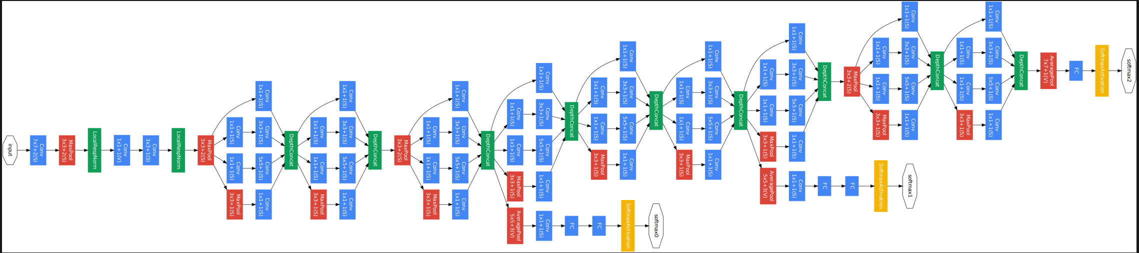
3x3 pool first, 1x1 conv second

m x n x c
3 x 3 x c --> m/3 x n/3 x c
1 x 1 x c x k

O(mnc + mnck/9)

- 1x1 conv allows easy control of the depth (channels)
- Can save computation without affecting output size

GoogLeNet



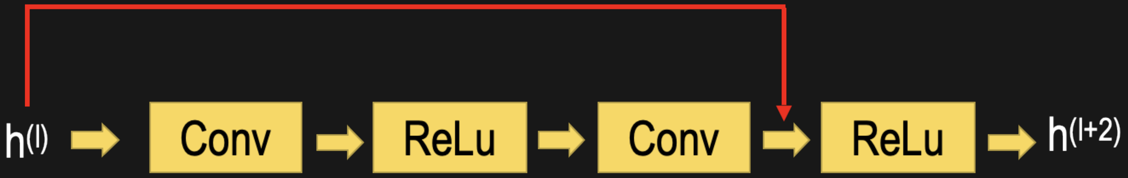
- No fully connected (FC) layers
- Deeper but more efficient and better performance

The Deeper, the Better, but More Difficult to Train

- Deeper models are harder to train due to vanishing / exploding gradient
- Can be worse than shallower networks if not properly trained!

K. He, X. Zhang, S. Ren, and J. Sun. "Deep Residual Learning for Image Recognition". In: *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 2016, pp. 770–778.

Residual Block



a residual block

- Add a shortcut connection that allows "skipping" one or more layers
- Effectively turning the block into learning residual: output - input
- Allows more direct backpropogation of the gradient through the "shortcut"
- Can also concatenate or add a linear layer if dimensions mismatch

Residual Network (ResNet)

Full ResNet architecture:
- Stack residual blocks
- Every residual block has two 3x3 conv layers
- Periodically, double # of filters and downsample spatially using stride 2 (/2 in each dimension)
- Additional conv layer at the beginning
- No FC layers at the end (only FC 1000 to output classes)