

# Building Privacy-Aware Database Systems

CS848 Winter 2021

Module 2



UNIVERSITY OF  
**WATERLOO**

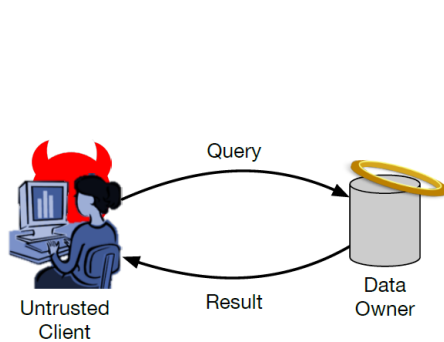


Data  
Systems  
Group

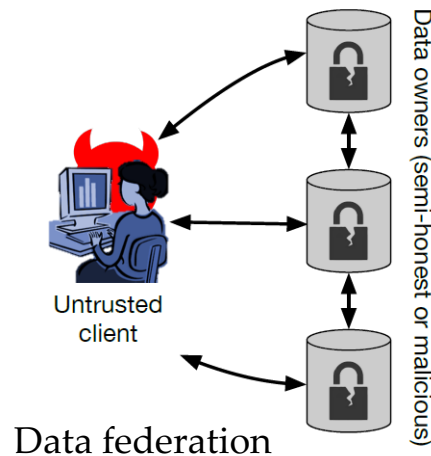
# This course will explore ...

- How to define a good privacy promise?
- How to design a privacy-preserving algorithms?
- How to build a privacy-aware database systems?

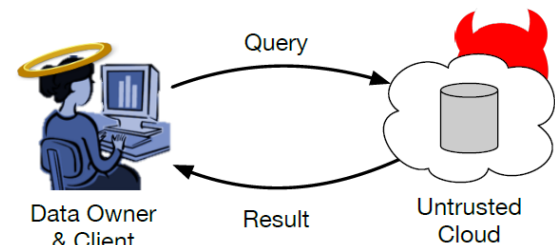
Greatly depend on  
the architecture setup and trust assumptions



Client-server with  
trusted data curator



Data federation



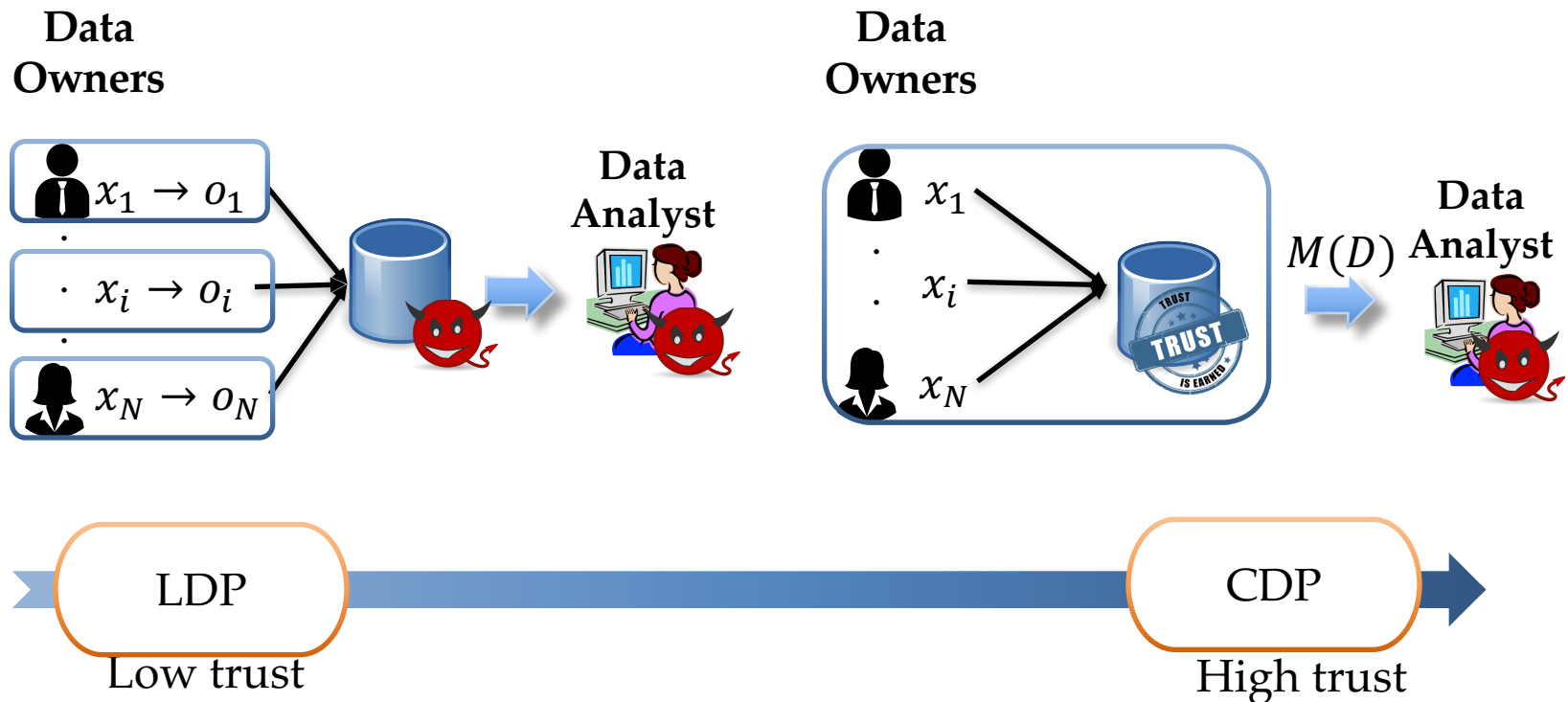
Cloud service provider

# Outline

- Part I: Local Differential Privacy (LDP)
- Part II: Marrying DP with Crypto
- Upcoming Papers and Announcements

# No Trusted Data Curator

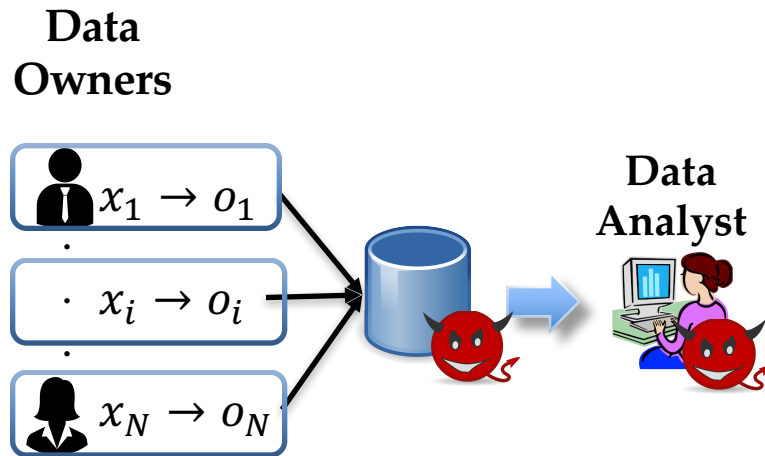
- Local DP
  - No trusted data curator
- Centralized DP
  - Trusted data curator



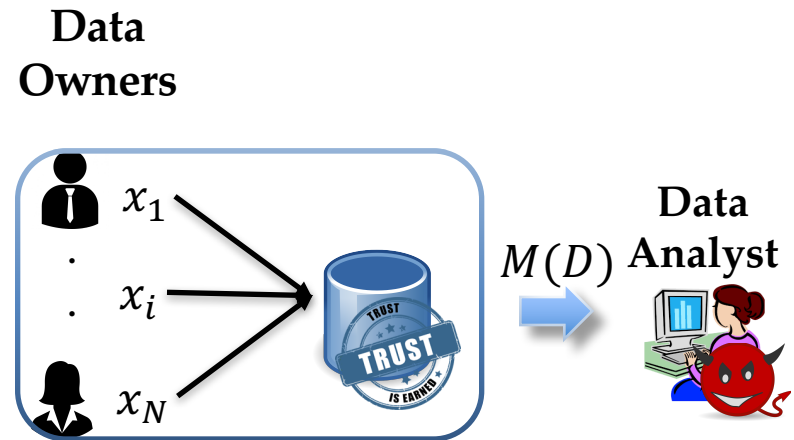


# No Trusted Data Curator

- Local DP
  - No trusted data curator
- Centralized DP
  - Trusted data curator

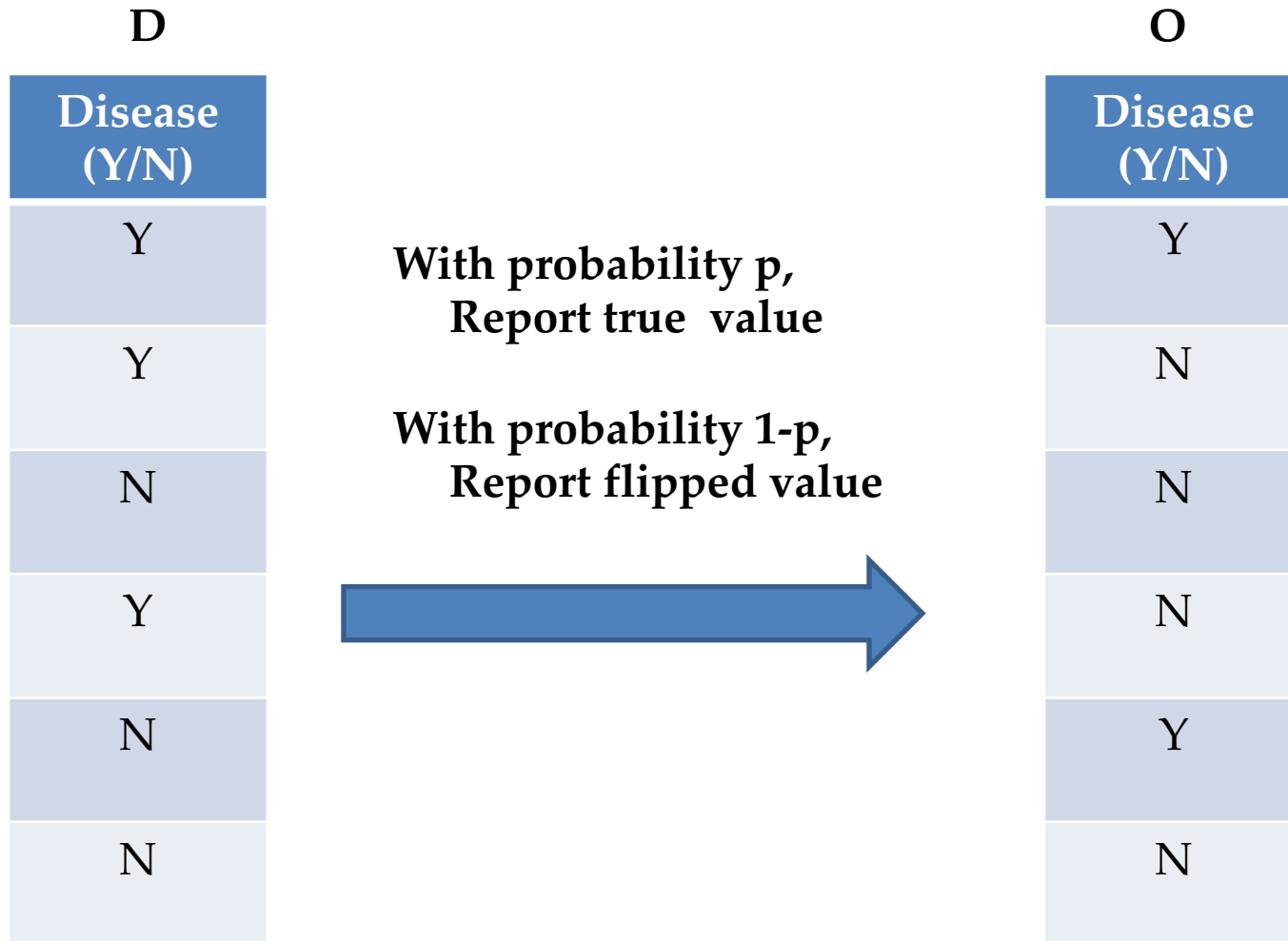


$$\ln \left( \frac{\Pr[A(x_i) = o]}{\Pr[A(x_i') = o]} \right) \leq \varepsilon$$



$$\ln \left( \frac{\Pr[M(D) = o]}{\Pr[M(D') = o]} \right) \leq \varepsilon$$

# Randomized Response (a.k.a. local randomization)



# Privacy Analysis of RR

- Considering a record taking values  $(x, x')$
- Consider some output/response  $O$

		Output	
		Yes	No
Input	Y	$p$	$1-p$
	N	$1-p$	$p$

$$e^{-\epsilon} \leq \frac{\Pr[A(N) = No]}{\Pr[A(Y) = No]} \leq e^{\epsilon}$$

$$e^{-\epsilon} \leq \frac{\Pr[A(Y) = Yes]}{\Pr[A(N) = Yes]} \leq e^{\epsilon}$$

$$\frac{1}{1 + e^{\epsilon}} \leq p \leq \frac{e^{\epsilon}}{1 + e^{\epsilon}}$$



$$e^{-\epsilon} \leq \frac{p}{1 - p} \leq e^{\epsilon}$$

# Utility Analysis of RR

- Suppose  $y$  out of  $N$  people replied “yes”, and rest said “no”; what is the best estimate for  $\pi$  = fraction of people with disease =  $Y$ ?

- Expected number of “yes” responses:

$$E[y] = \pi N \cdot p + (1 - \pi)N \cdot (1 - p)$$

- Unbiased estimator for  $\pi$ :  $\hat{\pi} = \frac{\frac{y}{N} - (1-p)}{2p-1}$

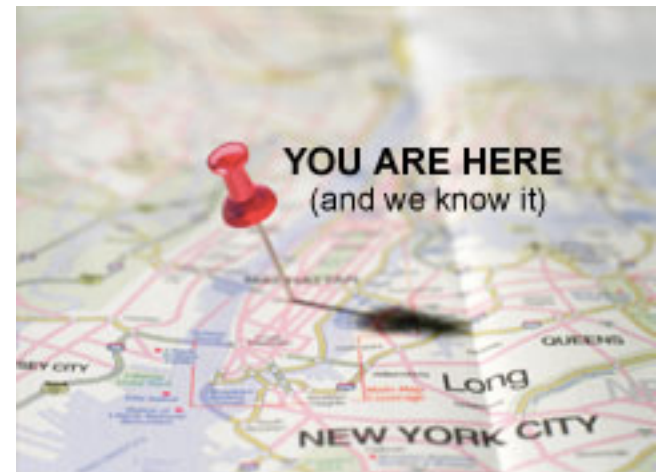
$$- E(\hat{\pi}) = E\left[\frac{\frac{y}{N} - (1-p)}{2p-1}\right] = \pi$$

$$- Var(\hat{\pi}) = \frac{\pi(1-\pi)}{N} + \frac{1}{N\left(16\left(p-\frac{1}{2}\right)^2 - \frac{1}{4}\right)}$$

**Sampling    Variance due to coin flips**

# RR for Larger Domains (mini-assignment 2)

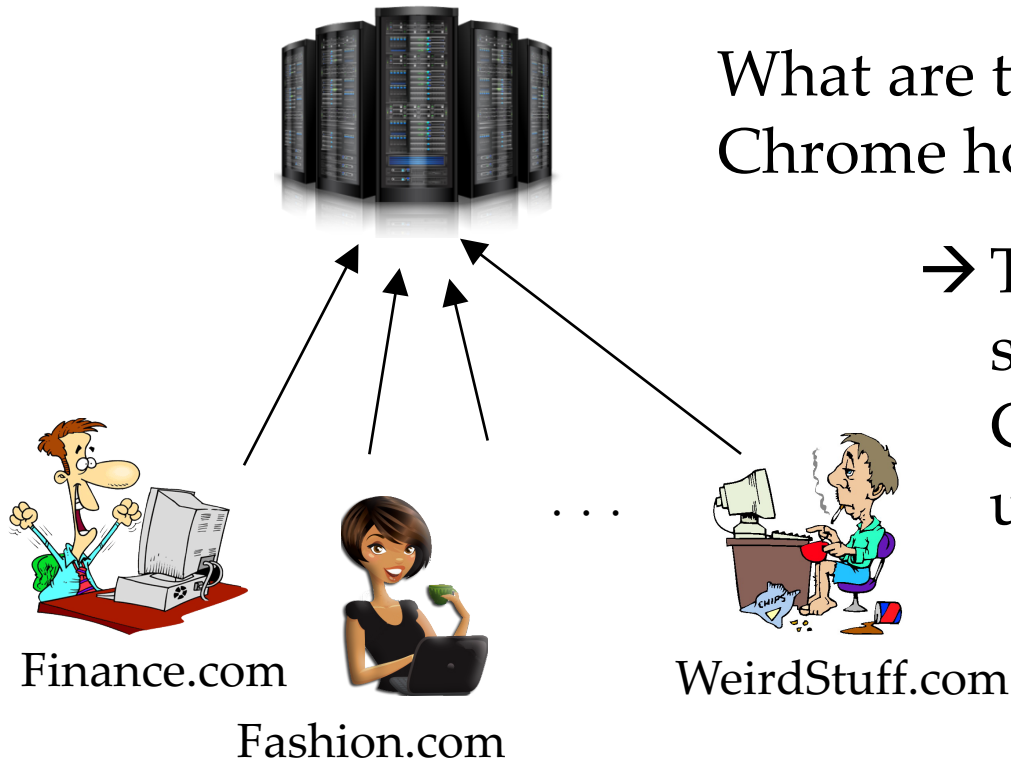
- Suppose area is divided into  $k \times k$  uniform grid.
- How to achieve LDP?
  - What is the probability of reporting the true location?
  - What is the probability of reporting a false location?





# Problem

[Erlingsson et al CCS'14]



What are the *frequent* unexpected Chrome homepage domains?

→ To learn malicious software that change Chrome setting without users' consent

# How to ensure privacy?

Can use Randomized Response ...

On a binary domain:

With probability  $p$  report true value

With probability  $1-p$  report false value

... but the domain of all urls is very large ...  
... original value is reported with very low prob.

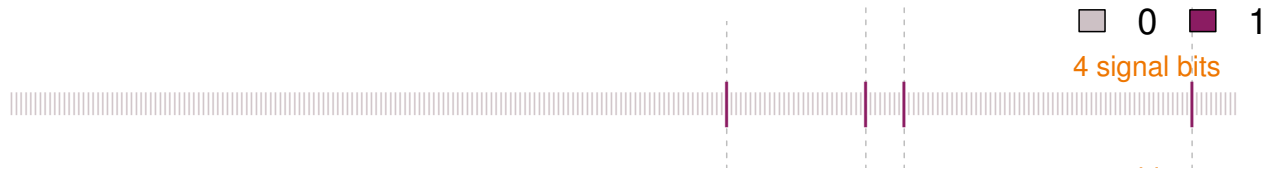


# RAPPOR Solution

- Idea 1: Use bloom filters to reduce the domain size



Bloom filter (B):

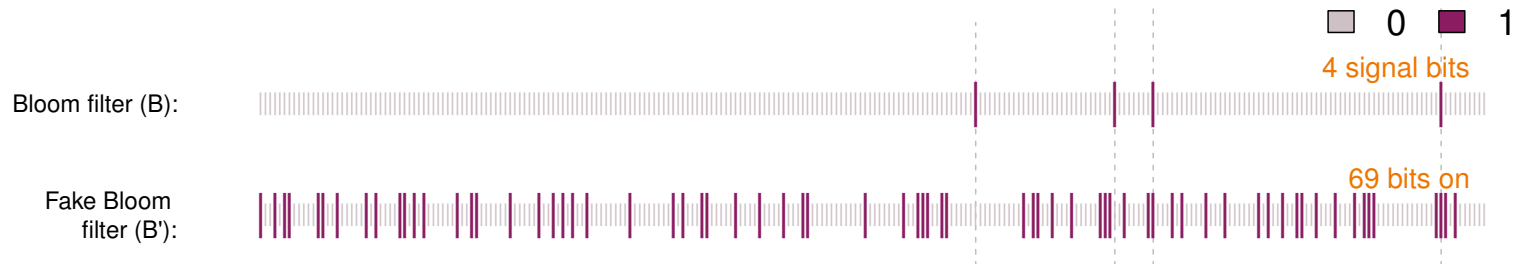


# RAPPOR Solution

- Idea 2: Use RR on bloom filter bits



Finance.com



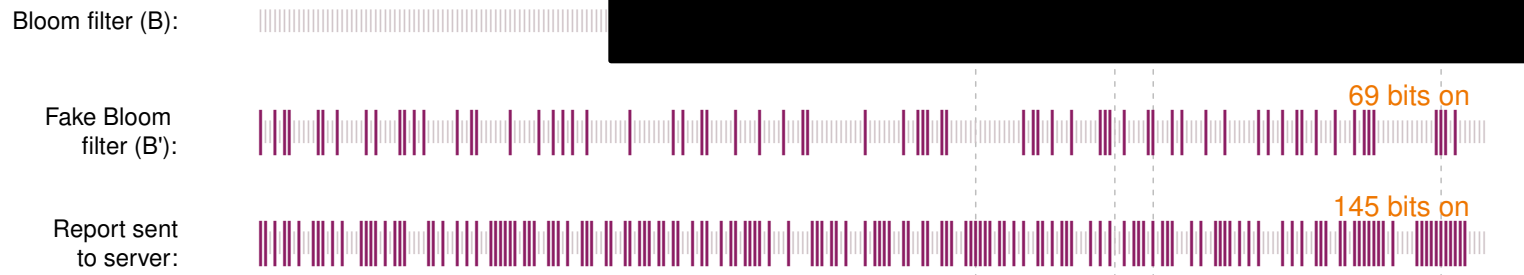
# RAPPOR Solution

- Idea 3: Again use RR on the Fake bloom filter



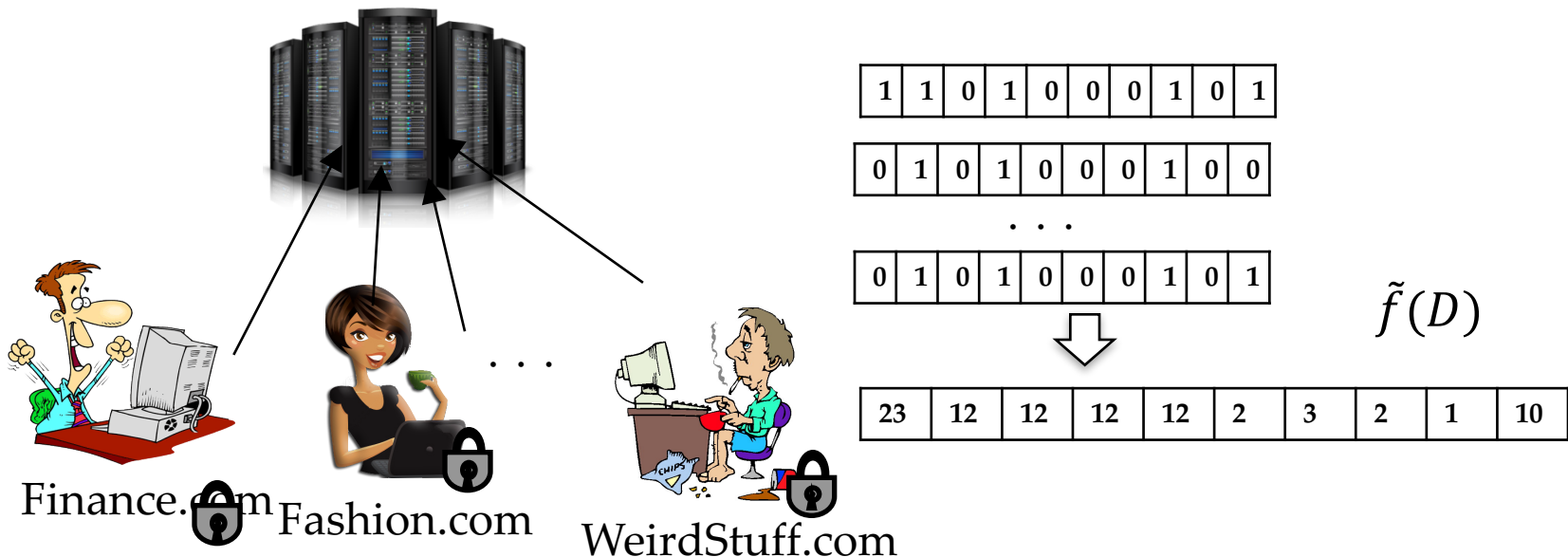
Why randomize two times?

- Chrome collects information each day
- Want perturbed values to look different on different days to avoid linking



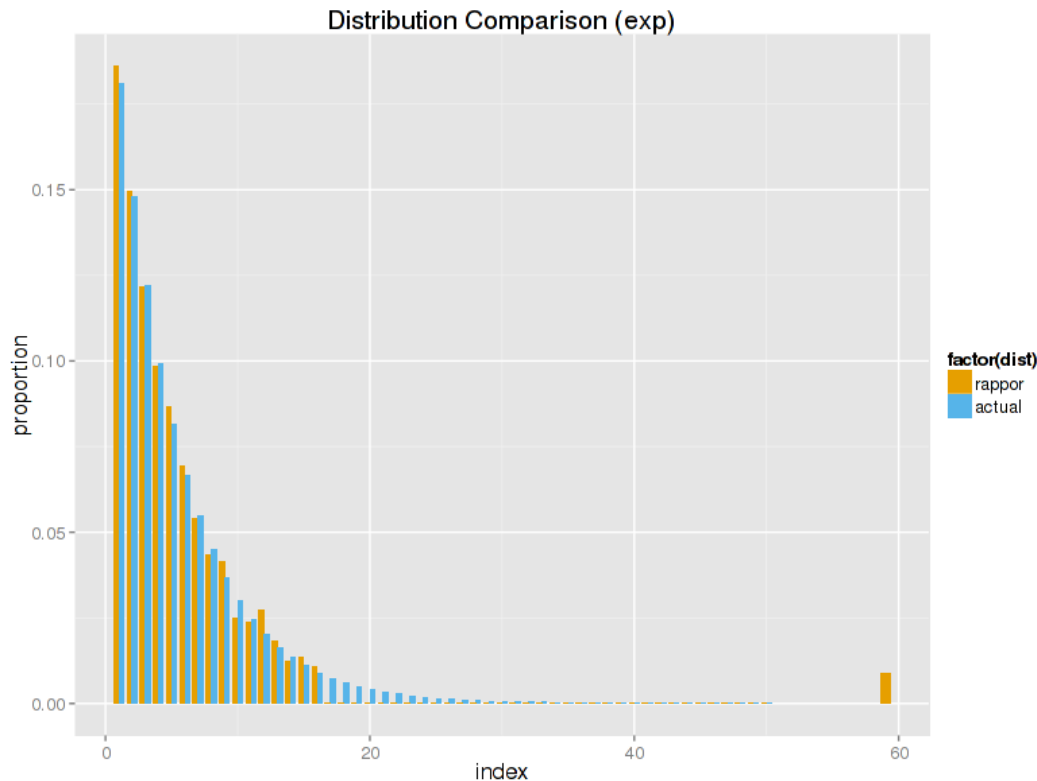
# Server Report Decoding

- Step 4: estimates bit frequency from reports  $\tilde{f}(D)$
- Step 5: estimate frequency of candidate strings with regression from  $\tilde{f}(D)$



# Evaluation

<http://google.github.io/rappor/examples/report.html>



## Simulation Input

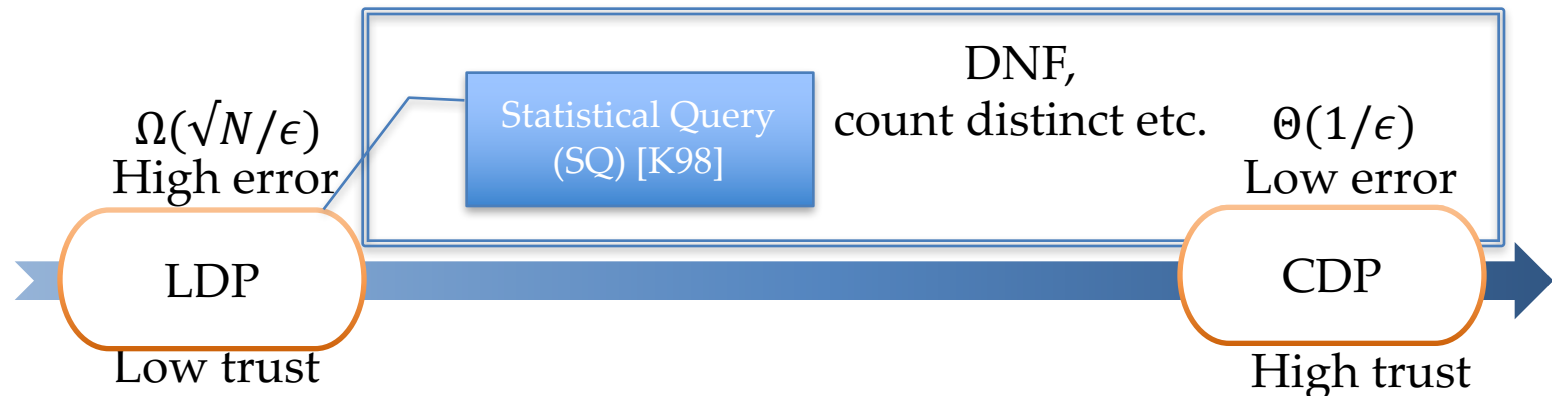
Number of clients	100,000
Total values reported / obfuscated	700,000
Unique values reported / obfuscated	50

## RAPPOR Parameters

<b>k</b>	Size of Bloom filter in bits	16
<b>h</b>	Hash functions in Bloom filter	2
<b>m</b>	Number of Cohorts	64
<b>p</b>	Probability p	0.5
<b>q</b>	Probability q	0.75
<b>f</b>	Probability f	0.5

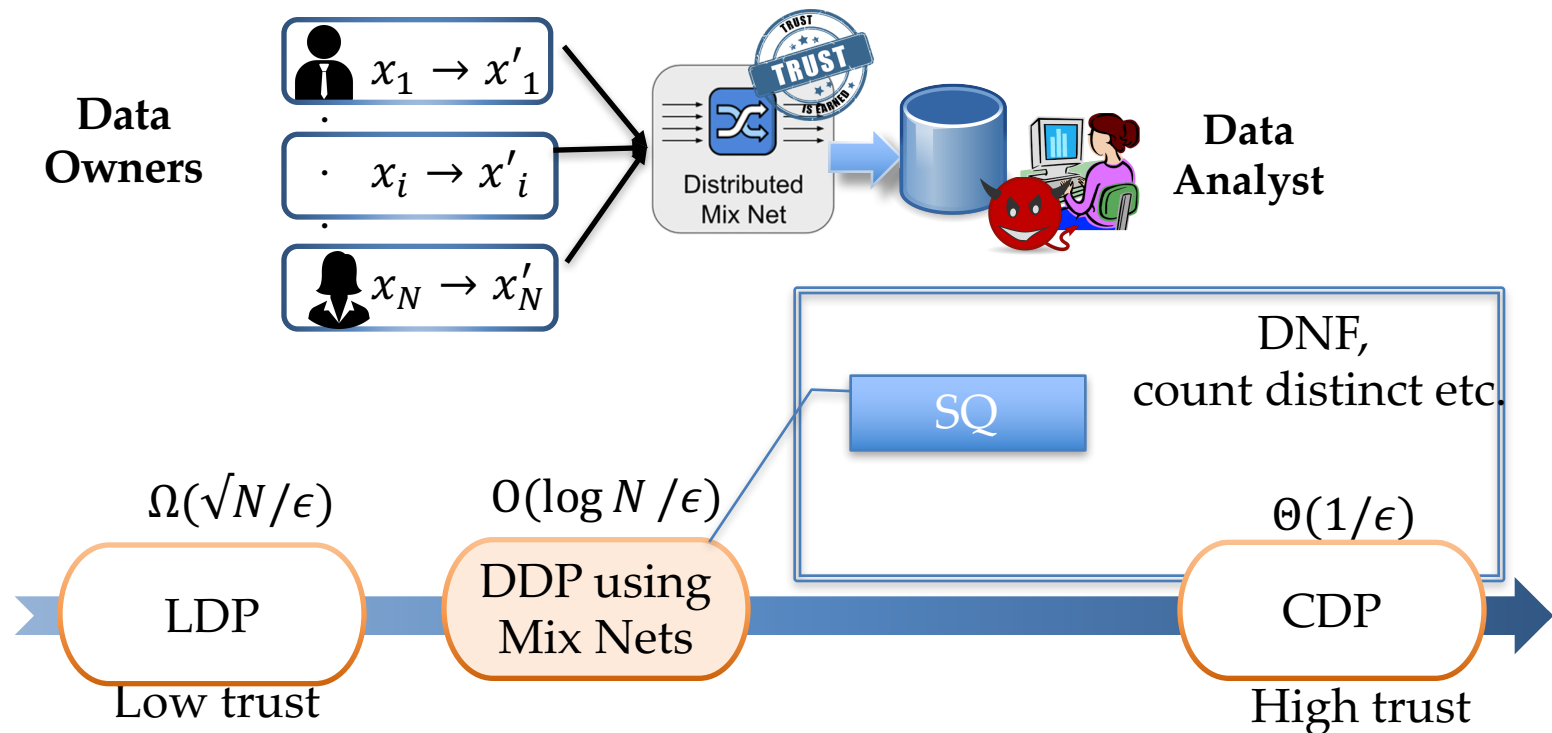
# Limitations of Local DP

- Local DP: Less accurate/expressive
  - $\Omega(\sqrt{N}/\epsilon)$  for statistical counting queries, where  $N$  is datasize
  - Separation results between the accuracy and sample complexity of LDP and CDP [KLNRS08]
    - E.g. disjunctive normal form (DNF) queries



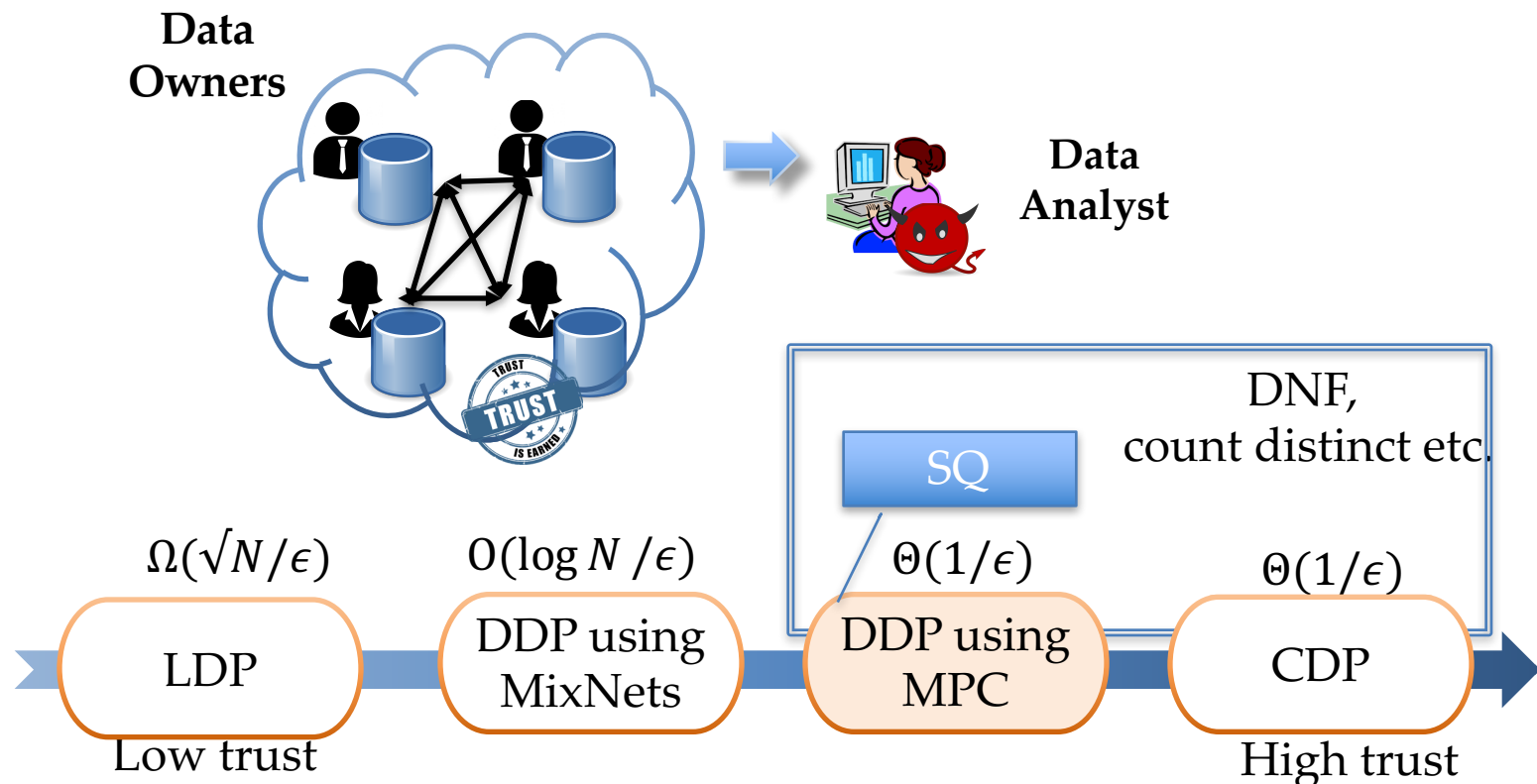
# Shifting Trust Assumptions

- Trusted anonymous communication channels  
[BEMMRLRKTS17, CSUZZ18, EFMRTT19, BBGN19]



# Shifting Trust Assumptions

- Trusted multi-party secure computation (MPC)  
[NH12, BEEGKR17, AHKM18]



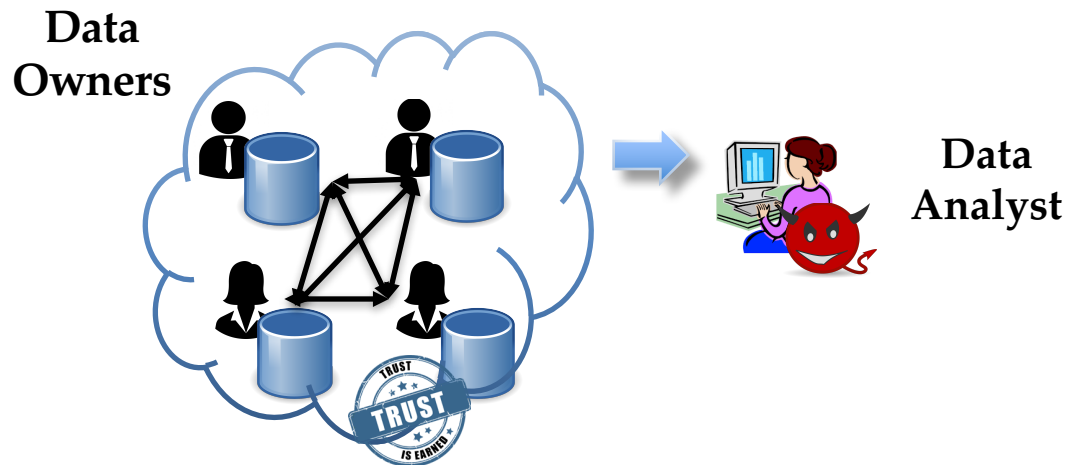


# Outline

- Part I: Local DP
  - Randomized Response
  - RAPPOR
  - Limitations of Local DP
- Part II: Marrying DP with Crypto
  - Secure Multi-party computation
  - Crypte
- Upcoming Papers and Annoucements

# No Trusted Data Curator

- Trusted multi-party secure computation (MPC)  
[NH12, BEEGKR17, AHKM18 ]



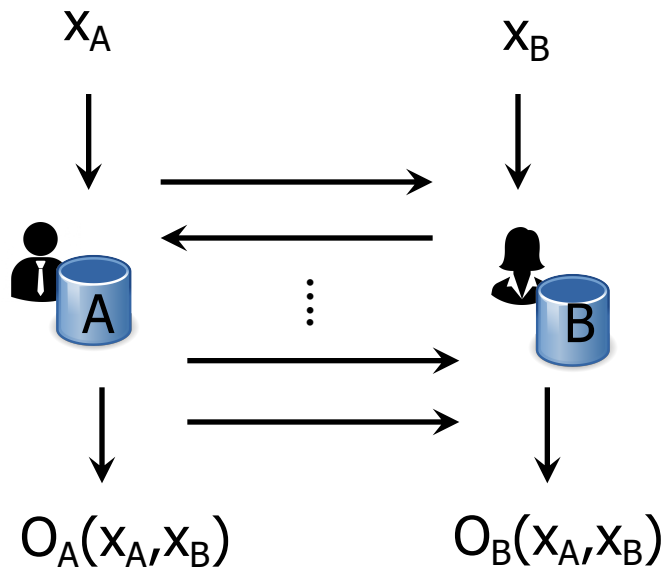
- MPC: (informally) to compute a **function** of **private inputs** without revealing information about the inputs beyond what is revealed by the function

# Multi-party Secure Computation

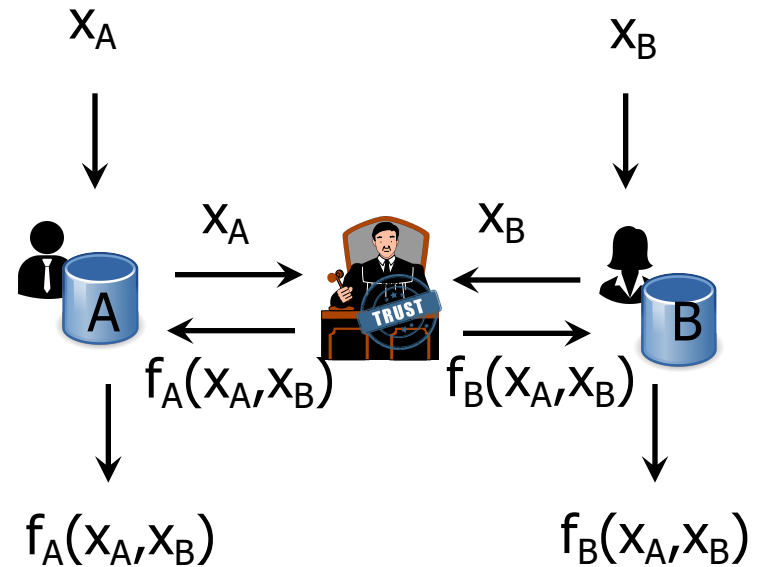
- Motivated use cases:
  - Can we have an auction without auctioneer?
  - Hospitals which cannot share their patient records with anyone want to mine on the combined data.
- Emulate a source of **trusted** computation
  - It will not “leak” a party’s information to others
  - And it will not cheat in the computation

# Simulation-based MPC

- 2-party example



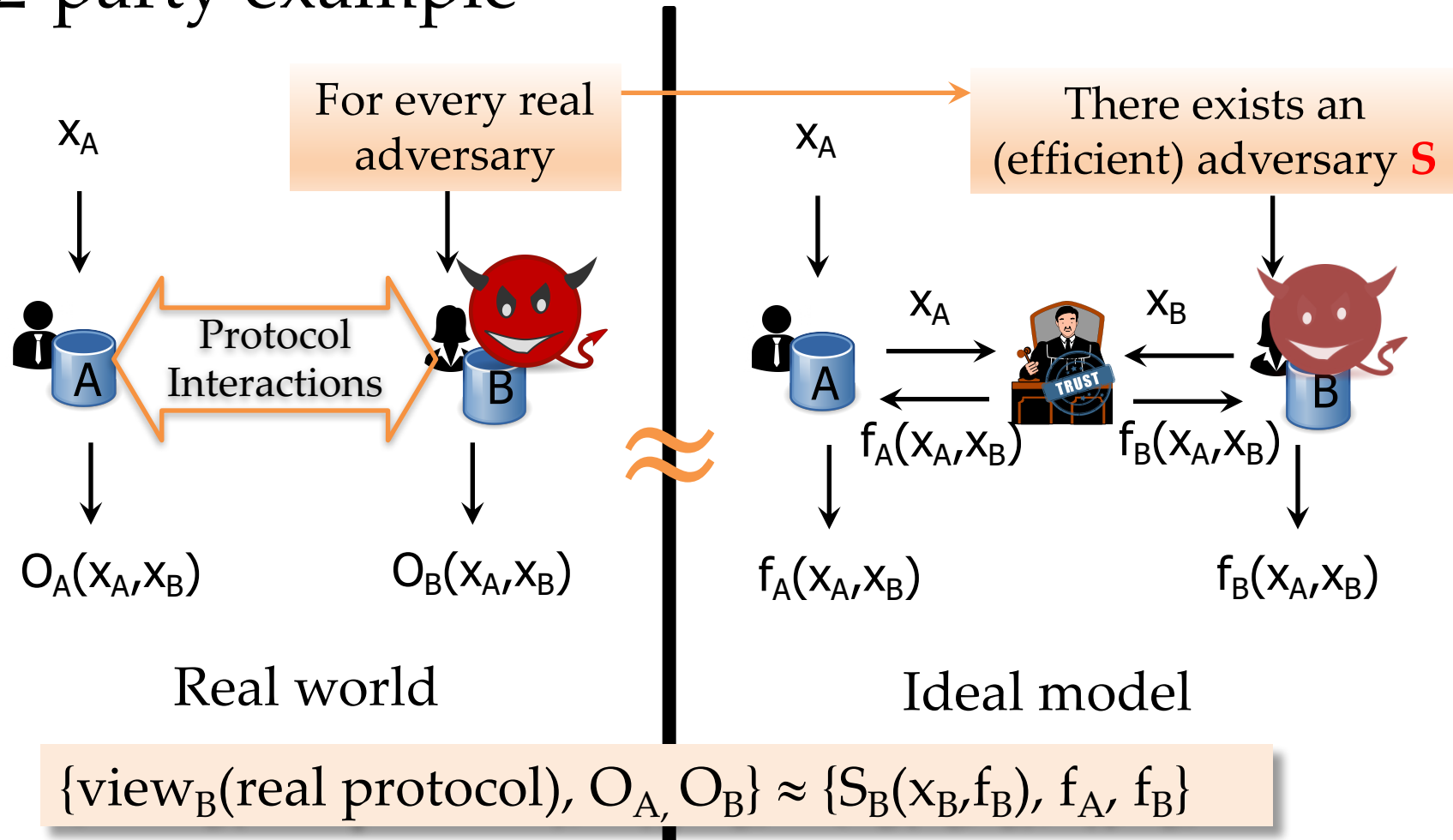
Real world



Ideal model

# Simulation-based MPC

- 2-party example



# Simulation-based MPC

- Protocol for computing  $f(X_A, X_B)$  betw. A and B is **secure** if there exist efficient simulator algorithms  $S_A$  and  $S_B$  such that for all input pairs  $(x_A, x_B)$ 

$$\{\text{view}_A(\text{real protocol}), O_A, O_B\} \approx \{S_A(x_A, f_A), f_A, f_B\}$$

$$\{\text{view}_B(\text{real protocol}), O_A, O_B\} \approx \{S_B(x_B, f_B), f_A, f_B\}$$
- Correctness:  $(O_A, O_B) \approx f(x_A, x_B)$ 
  - In the ideal model, the function is always computed correctly
  - Thus, the same is true in the real-model

# Adversary Model

- Computation power:
  - Prob. polynomial time v.s. all-powerful
- Adversarial behavior:
  - Semi-honest: follows the protocol, but tries to learn more (aka passive; honest-but curious)
  - Malicious: deviates from the protocol in arbitrary ways
- Corruption behavior:
  - Static: set of corrupted parties fixed at onset
  - Adaptive: can choose to corrupt parties at time during computation
- Number of corruptions:
  - Honest majority v.s. unlimited corruptions

# Feasibility

- Any multiparty functionality can be securely computed
  - For any number of corrupted parties: security with abort is achieved, assuming enhanced trapdoor permutations [Yao,GMW]
  - With an honest majority: full security is achieved, assume private channels only [BGW,CCD]



# Public-key Encryption

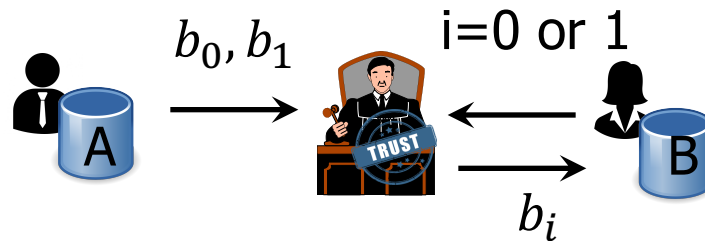
- Let  $(G, E, D)$  be a public-key encryption scheme
  - $G$  : a key-generation algorithm  $(pk, sk) \leftarrow G$ 
    - $pk$  : public key;  $sk$  : secret key
    - Terms:  $m$  denotes plaintext;  $c$  denotes ciphertext
  - Encryption:  $c = E_{pk}(m)$
  - Decryption:  $m = D_{sk}(c)$
  - Concept of one-way function: knowing  $c, pk, E_{pk}()$ , it is still computationally intractable to find  $m$

# Construction Paradigms

- Passively-secure computation for two-parties
  - Use **oblivious transfer** to securely select a value
- Passively-secure computation with shares
  - Use **secret sharing scheme** such that data can be reconstructed from some shares
- From passively-secure protocols to actively secure protocols
  - Use **zero-knowledge proofs** to force parties to behave in a way consistent with the passively secure protocol

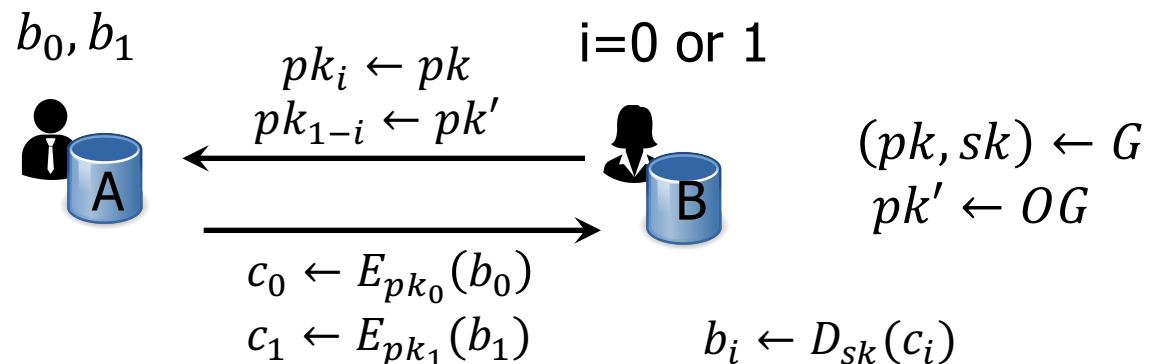
# 1-out-of-2 Oblivious Transfer (OT)

- A inputs two bits
- B inputs the index of one of A's bits
- B learns his chosen bit, A learns nothing
  - A does not learn which bit B has chosen; B does not learn the value of the bit that he did not choose



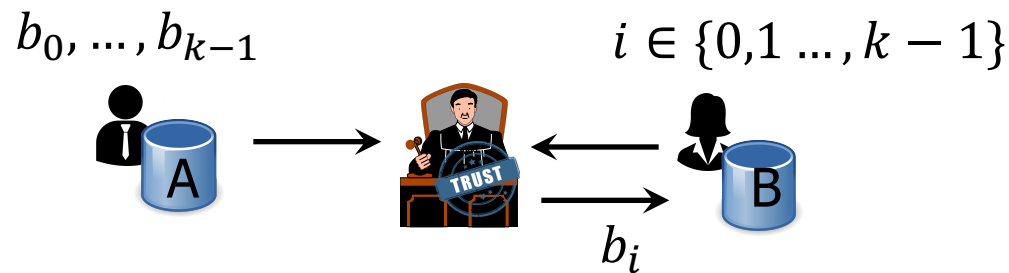
# Semi-Honest OT

- Let  $(G, E, D)$  be a public-key encryption scheme
  - $G$  : a key-generation algorithm  $(pk, sk) \leftarrow G$ 
    - Assume that a  $pk$  can be sampled without knowledge of its  $sk$  [oblivious key generation, e.g., El-Gamal encryption]:  $pk \leftarrow OG$
  - Encryption:  $c = E_{pk}(m)$
  - Decryption:  $m = D_{sk}(c)$



# Generalization [min-assignment 2]

- Can define 1-out-of-k oblivious transfer
- How?



# General GMW Construction

[Goldreich-Micali-Wigderson]

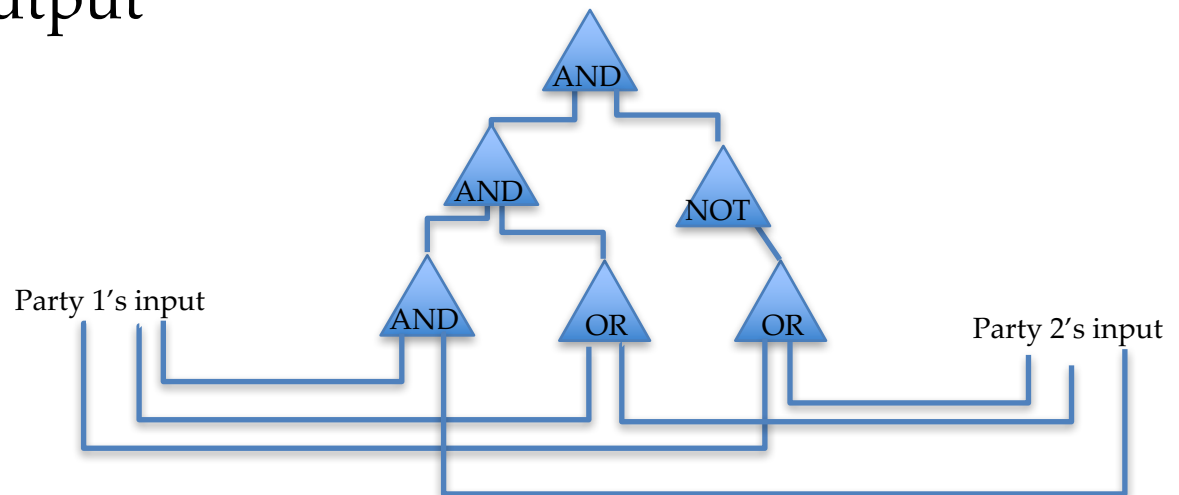
- For simplicity – consider 2-party
  - Let  $f$  be the function that the two parties wish to compute over their inputs  $(a,b)$ :  $f(a,b)$
- Idea:
  - Represent  $f$  as an arithmetic circuit with addition and multiplication gates
  - Aim to compute gate-by-gate, revealing only random shares each time

# Random Shares Paradigm

- Let  $a$  be some value:
  - Party 1 holds a random value  $a_1$
  - Party 2 holds  $a + a_1$
  - Note that without knowing  $a_1$ ,  $a + a_1$  is just a random value revealing nothing of  $a$
  - We say that parties hold random shares of  $a$
- The computation will be such that all the intermediate values are random shares (and so they reveal nothing)

# Circuit Computation

- Stage 1: each party randomly shares its input with the other party
- Stage 2: compute gates of circuits as follows
  - Given random shares to the input wires, compute random shares of the output wires
- Stage 3: combine shares of the output wires in order to obtain actual output





# Addition Gates

- Input wires to gate have values  $a$  and  $b$ :
  - Party 1 has shares  $a_1$  and  $b_1$
  - Party 2 has shares  $a_2$  and  $b_2$
  - Note:  $a_1+a_2=a$ ,  $b_1+b_2=b$
- To compute random shares of output  $c=a+b$ 
  - Party 1 locally computes  $c_1=a_1+b_1$
  - Party 2 locally computes  $c_2=a_2+b_2$
  - Note:  $c_1+c_2 = a_1+b_1+a_2+b_2 = a+b=c$

# Multiplication Gates

- Input wires to gate have values  $a$  and  $b$ :
  - Party 1 has shares  $a_1$  and  $b_1$
  - Party 2 has shares  $a_2$  and  $b_2$
  - Wish to compute  $c = ab = (a_1+a_2)(b_1+b_2)$
- Party 1 knows  $(a_1, b_1)$
- Party 2's values  $(a_2, b_2)$  are unknown to Party 1, but there are only 4 possibilities
  - (depending on correspondence to 00,01,10,11)

# Multiplication Gates (cont.)

- Let  $r$  be a random bit chosen by Party 1
- Party 1 prepares a table as follows:
  - Row 1:  $ab+r$  when  $a_2=0, b_2=0$
  - Row 2:  $ab+r$  when  $a_2=0, b_2=1$
  - Row 3:  $ab+r$  when  $a_2=1, b_2=0$
  - Row 4:  $ab+r$  when  $a_2=1, b_2=1$

- For example

- Assume  $a_1=0, b_1=1$
- Assume  $r=1$

Row	Party 2's shares	Output Value
1	$a_2=0, b_2=0$	$(0+0)(1+0)+1=1$
2	$a_2=0, b_2=1$	$(0+0)(1+1)+1=1$
3	$a_2=1, b_2=0$	$(0+1)(1+0)+1=0$
4	$a_2=1, b_2=1$	$(0+1)(1+1)+1=1$

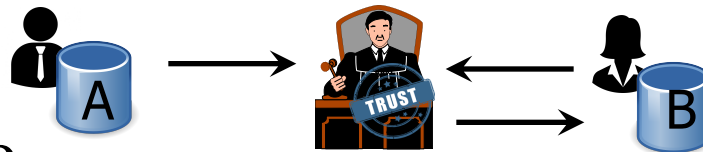
# The Gate Protocol

- The parties run a 1-out-of-4 OT

Row	$ab+r$ value
1	$(0+0)(1+0)+1=1$
2	$(0+0)(1+1)+1=1$
3	$(0+1)(1+0)+1=0$
4	$(0+1)(1+1)+1=1$

Row	Party 2's shares
1	$a_2=0, b_2=0$
2	$a_2=0, b_2=1$
3	$a_2=1, b_2=0$
4	$a_2=1, b_2=1$

$i = 1, \text{or } 2, \text{ or } \mathbf{3}, \text{ or } 4$



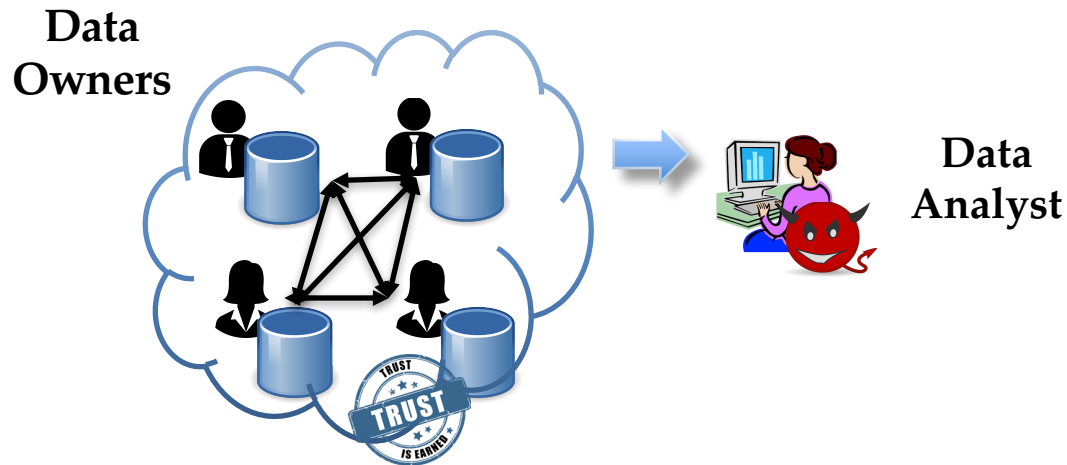
- For example

- Assume  $a_1=0, b_1=1$
- Assume  $r=1$

$$row_3 = ab+r$$

# Challenges in Practice

- $(m+1)$ -party MPC protocols are usually expensive



- Data owners need to be online

# Challenges in Practice

- Security/privacy proofs can be tricky
  - Even for stand-alone crypto/DP mechanisms [BR06, LSL17]
  - Hybrid approach is vulnerable to faulty proofs [HMFS17]

**//NoisyMax [DR14]:**

For  $i = 1, \dots, L$ :  $[c'_i] \leftarrow [c_i] + [\eta_i]$ , where  $\eta_i \sim \text{Lap}(\frac{1}{\epsilon})$

Release  $\text{argmax}_{i \in [1, L]} c'_i$

**//Release counts for  $i \leq 10$ :**

~~For  $i = 1, \dots, 10$ :  $[c'_i] \leftarrow [c_i] + [\eta_i]$ , where  $\eta_i \sim \text{Lap}(\frac{\Delta}{\epsilon})$~~

Release  $(c'_1, c'_2, \dots, c'_{10})$

$2\epsilon$ -DP

$[xxx]$  denotes cipher text

# Challenges in Practice

- Performance optimization is non-trivial

**//Accurate Histogram Publication (AHP) [ZCXM14]:**

For  $i \in [1, L]$ :  $[c'_i] \leftarrow [c_i] + [\eta_i]$ , where  $\eta_i \sim \text{Lap}(\frac{1}{\epsilon_1})$

$([c'_{i_1}], \dots, [c'_{i_L}]) \leftarrow \text{Sort}(\text{Threshold}([c'_1], \dots, [c'_L]))$

$C \leftarrow \text{Cluster}([c'_{i_1}], \dots, [c'_{i_L}])$

For  $C_i \in C$ :  $[C_i] \leftarrow \sum_{c_j \in C_i} [c'_j] / |C_i|$

For  $i \in [1, L]$ :  $[c''_i] \leftarrow [C_i] + [\eta_i] / |C_i|$ , where  $\eta_i \sim \text{Lap}(\frac{1}{\epsilon_2})$

Release  $(c''_1, \dots, c''_L)$

EMP toolkit  
800s for a dataset  
of size 33k  
→ 3.4x less time  
with optimization!!

# Outline

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  - Limitations of Local DP
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  - Crypte
- Upcoming Papers and Annoucements



# Crypte

- ~~(m+1)-party MPC protocols are usually expensive~~

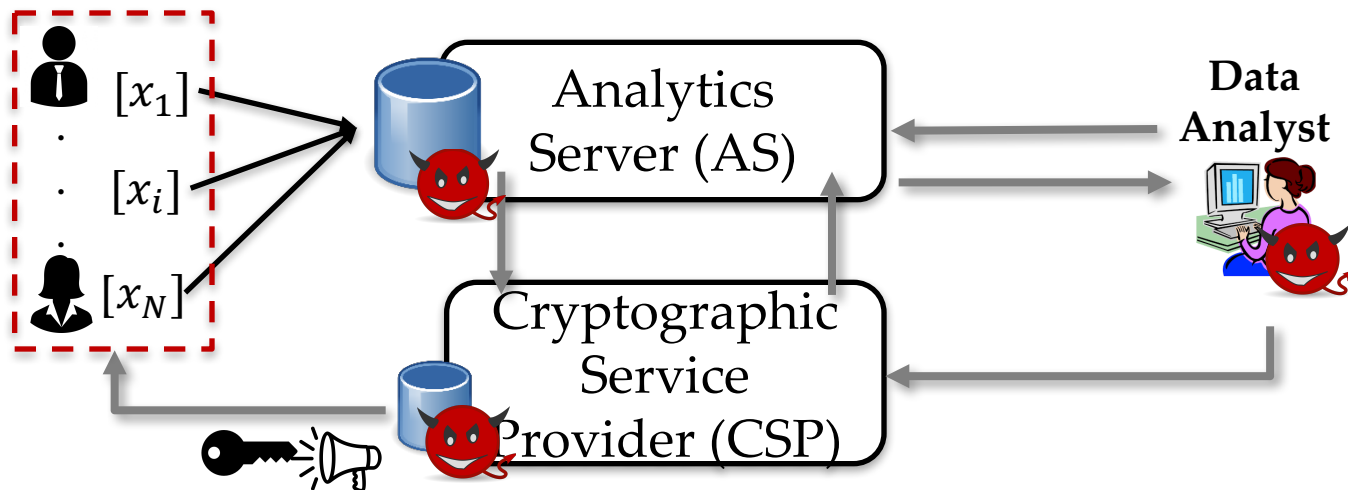
- Data owners need to be online

- Security

- Performance

## 2-Server Model with minimal trust assumption

- Computationally bounded adversary
- Semi-honest behavior and non-collusion

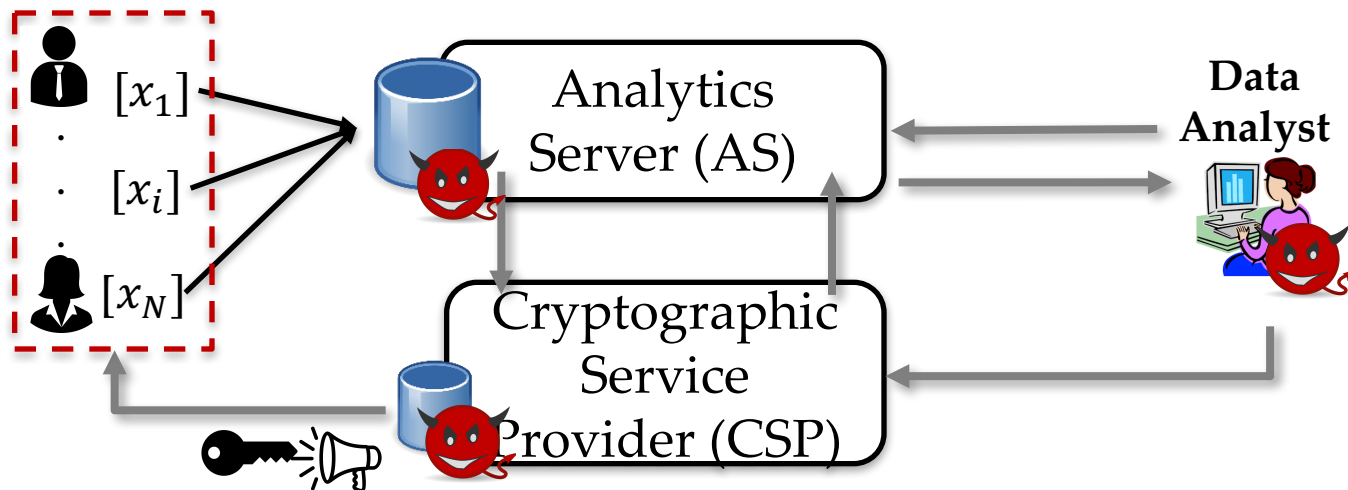


# Crypte

- (m+1)-party MPC protocols are usually expensive
- ~~Data owners need to be online~~

## Data owners are offline

- CSP acts on behalf of data owners
- CSP is minimally involved in the protocols

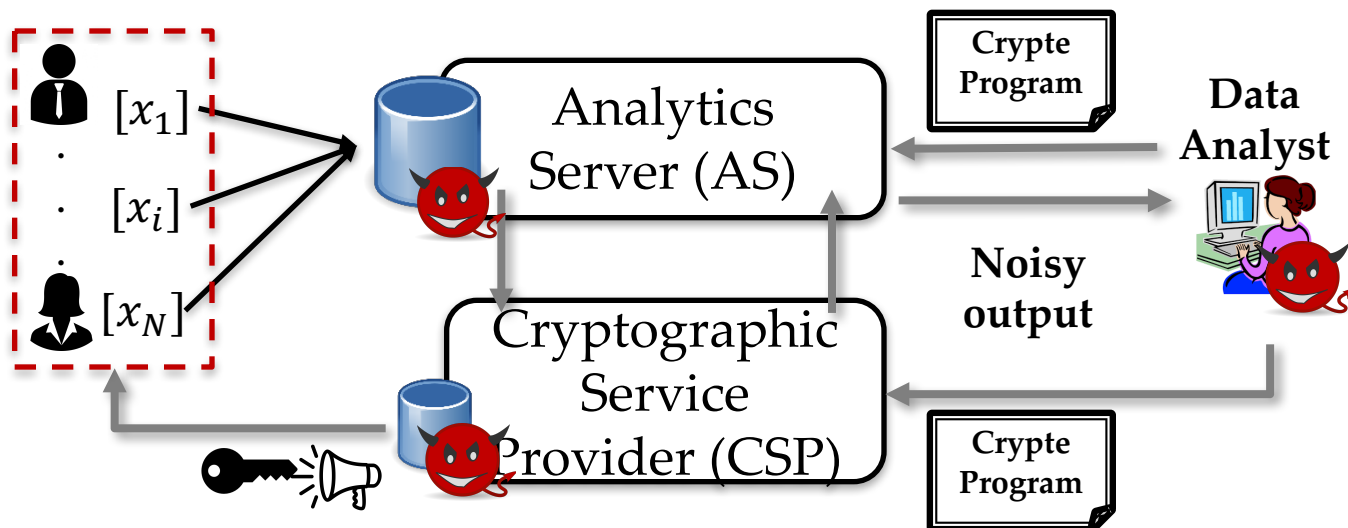


# Homomorphic Encryption

- Allows computations on the ciphertext **without knowing the secret key**, meanwhile ensures that the decryption of the resulting ciphertext is the same **as** the computations over the plaintext.
- Development
  - Idea about privacy homomorphism was proposed [RAD78]
  - Partially Homomorphic Encryption Schemes [RSA78] [Paillier99]
  - 1st generation FHE based on ideal lattice (bootstrapping) [Gen09b]
  - 2nd generation FHE based on RLWE (key/modulus switch) [BV11b][BGV12]
  - 3rd generation FHE based on LWE (approximate eigenvector) [GSW13]

# Crypte

- (m+1) Programming Framework (logical)
  - Compile high level operators down to black box
  - Prove security/privacy in modular fashion
- Data
- ~~Security/privacy proofs can be tricky~~
- Performance optimization is non-trivial

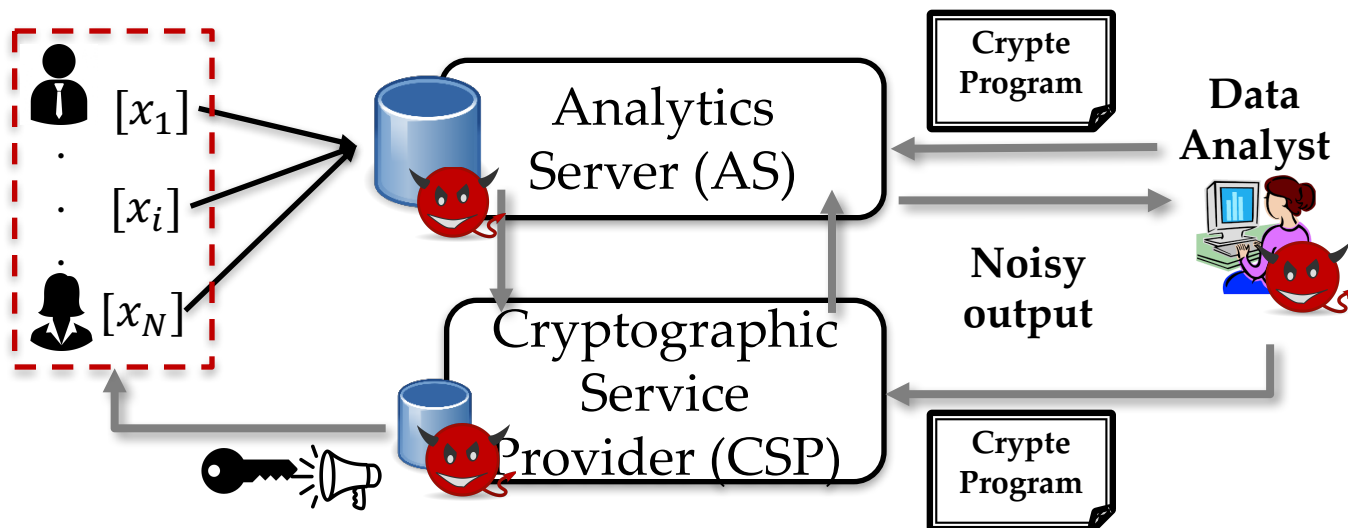


# Crypte

- (m+1)-party MPC protocols are usually expensive
- Data confidentiality
- Security
- ~~Performance optimization is non-trivial~~

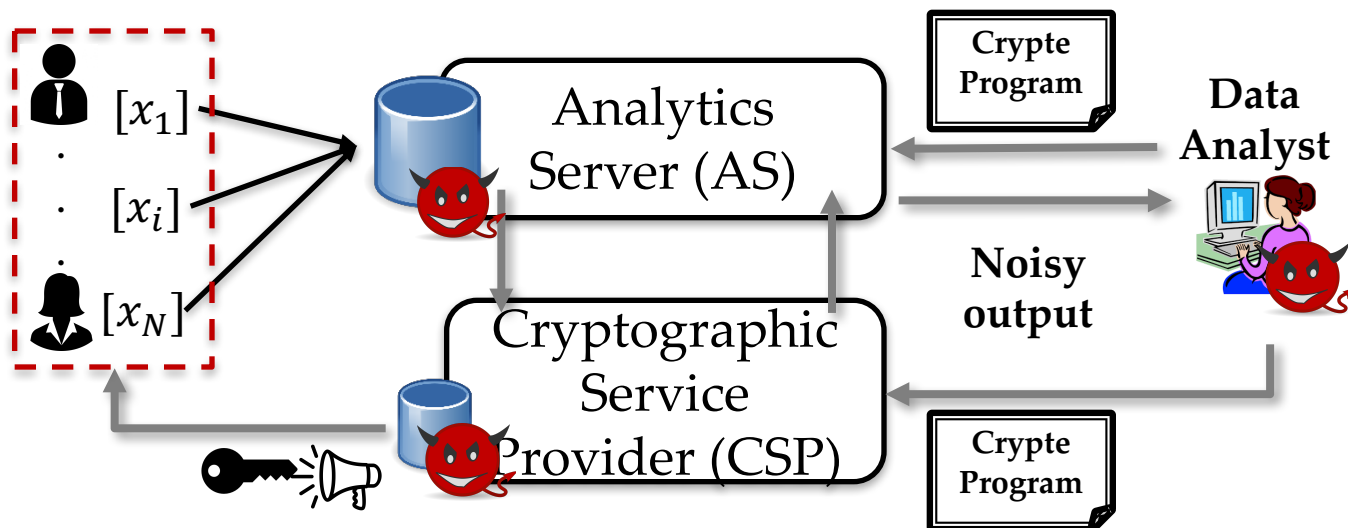
## Built-in Performance Optimization

- DP-index optimization
- Crypto-engineering optimization

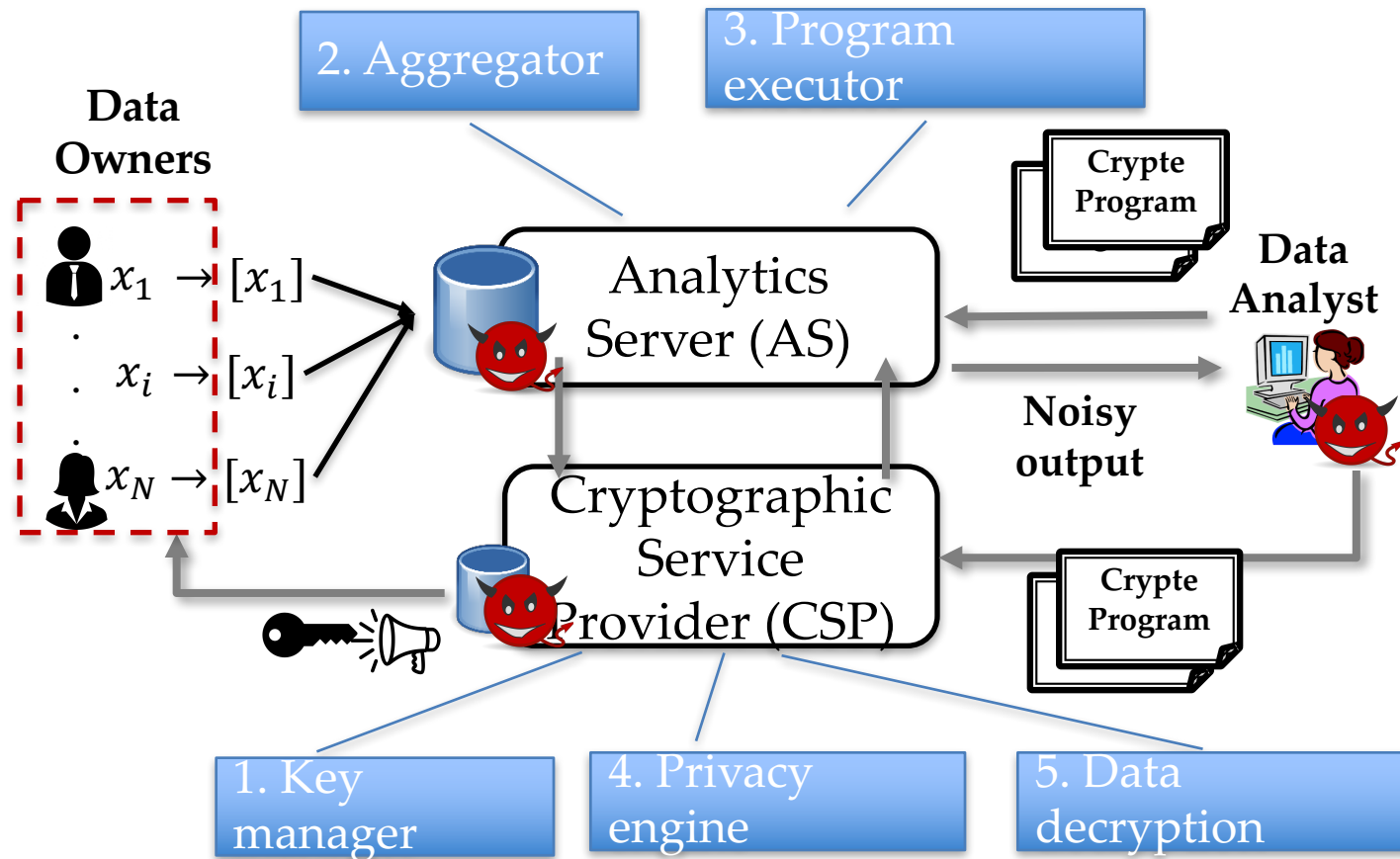


# Design Principles in Crypte

- 2-server model with minimal trust assumptions
- Data owners are offline
- Programming framework
- Built-in performance optimization

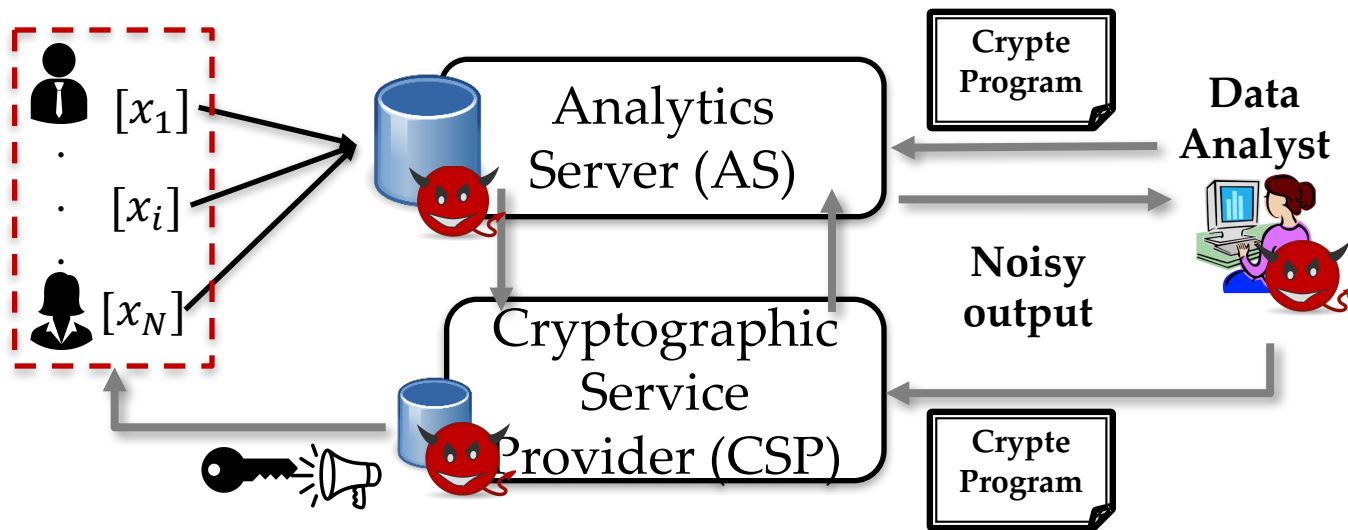


# Crypte Overview



# Design Principles

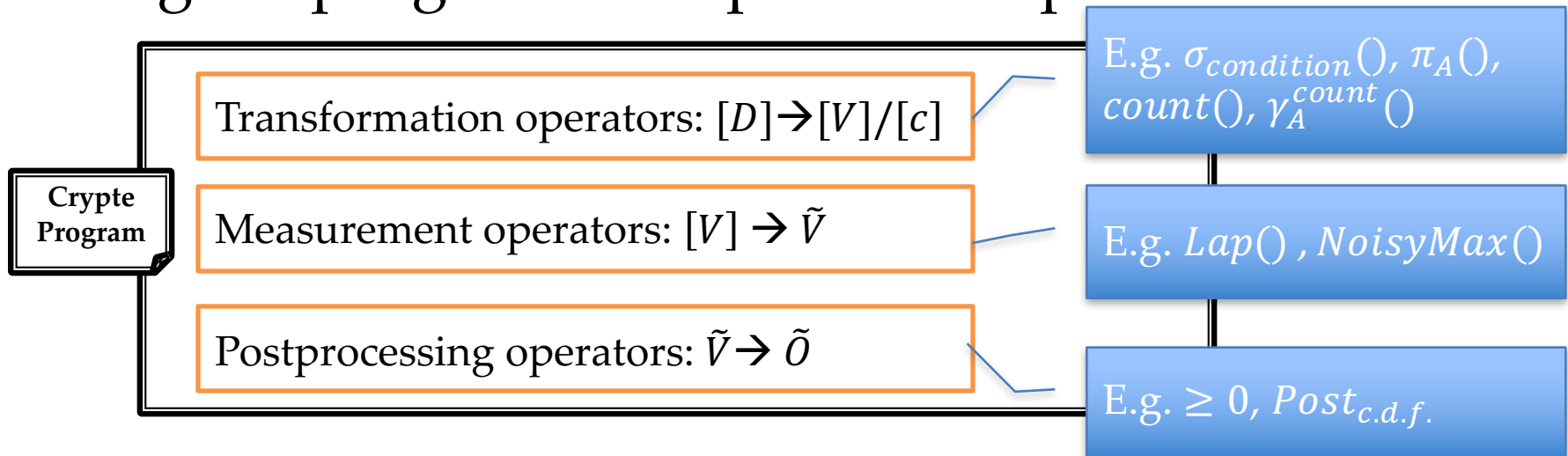
- 2-server model with minimal trust assumptions
- Data owners are offline
- Programming framework
- Built-in performance optimization





# Programming Framework

- Logical program: a sequence of operators



- Views of AS and CSP
  - AS sees either encrypted values or noised values in the clear
  - CSP always sees noised values (encrypted/in the clear)

# Program Example

- Database schema:  $Age(A)$ ,  $Gender(G)$ ,  $NativeCountry(N)$ ,  $Race(R)$
- P1: Compute the cumulative distribution of  $Age$  ranged  $[1,100]$

For  $i \in [1,100]$ :

$$\hat{c}_i \leftarrow Lap_{\epsilon_i, \Delta=1} \left( count \left( \sigma_{Age \in (0, i]} \left( \pi_{Age}(\tilde{D}) \right) \right) \right);$$

$output \leftarrow post_{c.d.f.}([\hat{c}_1, \dots, \hat{c}_{100}])$

DP noised

Encrypted

# Program Example

- Database schema:  $Age(A)$ ,  $Gender(G)$ ,  $NativeCountry(N)$ ,  $Race(R)$
- P5: Count the no. of male employees of Mexico having age 30

$$\hat{c} \leftarrow Lap_{\epsilon, \Delta=1} \left( count \left( \sigma_{A=30 \wedge G=M \wedge N=Mexico} \left( \pi_{A,G,N}(\tilde{D}) \right) \right) \right)$$

- P7: Count the no. of age values having at least 100 records

$$\hat{c} \leftarrow Lap_{\epsilon, \Delta=2} \left( count \left( \sigma_{Count \in [100, m]} \left( \gamma_A^{count} \left( \pi_A(\tilde{D}) \right) \right) \right) \right);$$

DP noised

Encrypted

# Implementation Details

$$\hat{c}_{30} \leftarrow \text{Lap}_{\epsilon_{30}, \Delta=1} \left( \text{count} \left( \sigma_{\text{Age} \in (0, 30]} \left( \pi_{\text{Age}}(\tilde{D}) \right) \right) \right);$$

A	G	N	R	Bit
[18]				[1]
[35]				[1]
...				...
[42]				[1]
[23]				[1]

A	Bit
[18]	[1]
[35]	[1]
...	...
[42]	[1]
[23]	[1]

A	Bit
[18]	[0]
[35]	[1]
...	...
[42]	[1]
[23]	[0]

[0] [0] ... [1] ... [0] [0] [0]

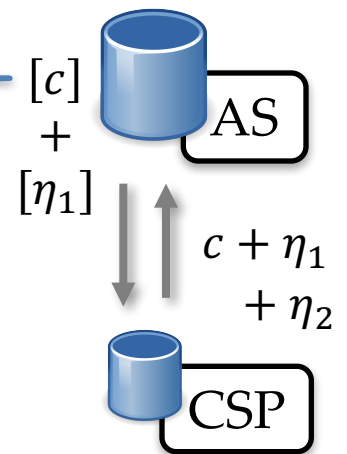
18<sup>th</sup> position

Linear Homomorphic Encryption:  $\text{Dec}([x] + [y]) = x + y$

Data Analyst



$$c + \eta_1 + \eta_2$$



# Alternative Implementations

- Secret share based MPC protocol
  - 2 servers do the similar amount of work
- Joint noise generation [DKMMN06, NH12]
  - More expensive MPC protocol
- Other improvements:
  - Data representation: multi-attribute one-hot-encoding
  - Optimized HE scheme or GC

All are possible, due to the separation of logical programming framework and underlying physical implementation!

# Security Sketch (Semi-honest)

For  $i \in [1, 100]$ :

$$\hat{c}_i \leftarrow \text{Lap}_{\epsilon_i, \Delta=1} \left( \text{count} \left( \sigma_{\text{Age} \in (0, i]} \left( \pi_{\text{Age}}(\tilde{D}) \right) \right) \right);$$

Program  $P$

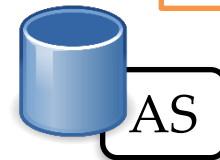
$$\text{output} \leftarrow \text{post}_{c.d.f.}([\hat{c}_1, \dots, \hat{c}_{100}])$$

Satisfy SIM-CDP [MVRV09]

Exists PPT  $\text{Sim}_{AS}, \text{Sim}_{CSP}$ , such that

$$\text{Sim}_{AS}(P^{CDP}(D, \epsilon)) =_c \left( \text{View}_{AS}^{\Pi}(P, D, \epsilon), \text{Output}^{\Pi}(P, D, \epsilon) \right)$$

$$\text{Sim}_{CSP}(P^{CDP}(D, \epsilon)) =_c \left( \text{View}_{CSP}^{\Pi}(P, D, \epsilon), \text{Output}^{\Pi}(P, D, \epsilon) \right)$$



Protocol  $\Pi$

r.v.: the output of  
running  $P$  in CDP  
model

# Security Sketch (Semi-honest)

For  $i \in [1, 100]$ :

$$\hat{c}_i \leftarrow \text{Lap}_{\epsilon_i, \Delta=1} \left( \text{count} \left( \sigma_{\text{Age} \in (0, i]} \left( \pi_{\text{Age}}(\tilde{D}) \right) \right) \right);$$

$\text{output} \leftarrow \text{post}_{c.d.f.}([\hat{c}_1, \dots, \hat{c}_{100}])$

Program  $P$

Satisfy SIM-CDP [MVRV09]

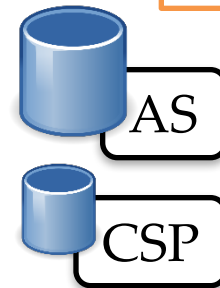
Encryption

...

Garbled circuit

Laplace operator

Protocol  $\Pi$



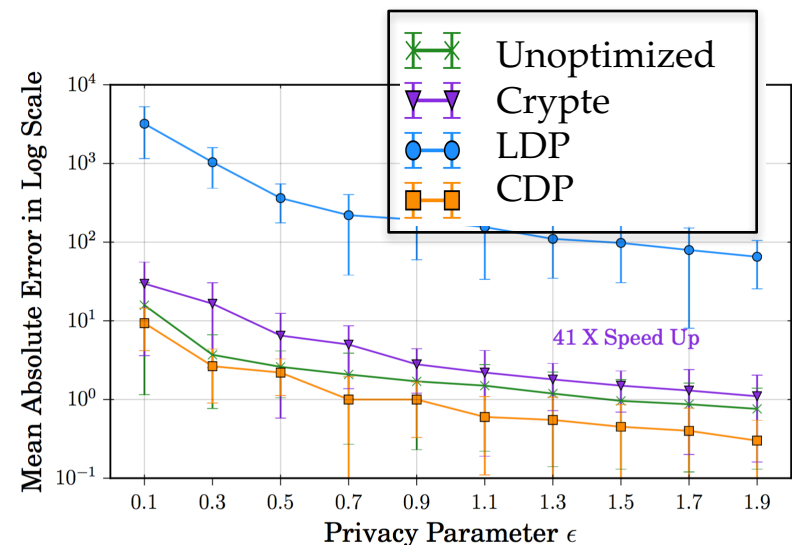
**Composition theorem**

[G. Oded. Foundations of  
Cryptography]

# Design Principles

- 2-server model with minimal trust assumptions
- Data owners are offline
- Programming framework
- Built-in performance optimization
  - DP-index Opt
    - E.g. index on *NativeCountry*
  - Crypto-engineering Opt
    - DP range tree
    - Pre-computation
    - Offline processing

P5: 1201.12s → 29.21s





# Evaluation

- Dataset of 32,651 rows and 7 Crypte programs
- Accuracy:
  - **The same order** as that of CDP implementation
  - 2-orders of smaller error than that of LDP implementation
- Performance:
  - Optimizations improve the performance by up to **5667×**
  - A large class of programs execute within **5 mins** and scale linearly with the data size
  - AS performs majority of the work for most programs

# Take-away and Future work

Key: Separation of logical programming framework and underlying physical implementation!

- User-specified query in a high-level language
- A larger class of programs
- Malicious setting

Key: Leaky crypto for performance-privacy trade-off!

- DP for Crypto to improve performance
  - Computation [BHEMR17]
  - Access pattern [TDG16, WCM18]
  - Communication [HLZZ15, TGLZZ17, LGZ18]

# Summary

- Part I: Local DP
  - Randomized Response
  - RAPPOR
  - Limitations of Local DP
- Part II: Marrying DP with Crypto
  - Secure Multi-party Computation
  - Crypte

# Discussion Time



# Paper Readings

- Week 7
  - 2a. Prochlo
  - 2b. Orchard
  - 2c. EncryptedDB
- Week 8
  - 2d. Collecting data jointly under LDP
  - 2e. DJoin
  - 2f. Shrinkwrap

# Announcement

- Assignment 1
  - Release after Wed session
  - Latex file will be available
  - Submit pdf on Learn, by Feb 22, 11pm
- Feedback on projects will be emailed before/during the reading week

# Slides Credits

- <http://sigmod2017.org/wp-content/uploads/2017/03/04-Differential-Privacy-in-the-wild-2.pdf>
- [https://www.cs.utexas.edu/~shmat/courses/cs380s\\_fall09/15smc.ppt](https://www.cs.utexas.edu/~shmat/courses/cs380s_fall09/15smc.ppt)
- [http://www.mathcs.emory.edu/~lxiong/cs573\\_s12/share/slides/12crypt.pdf](http://www.mathcs.emory.edu/~lxiong/cs573_s12/share/slides/12crypt.pdf)