CS848 Fall 2025: Algorithmic Aspects of Query Processing

Traditional Query Processing

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Sep 10, 2025

Agenda

- First class: Introduction
- Last and This class: Traditional query processing
  - Relational Algebra
  - Pairwise Framework
  - Yannakakis algorithm
  - Extensions

Semi-Join $R(A,B) \ltimes S(B,C)$

- $R \ltimes S = \pi_{A,B}(R \bowtie S)$: all tuples in R that can be joined with at least one tuple in S
- Law of semi-joins: $R \bowtie S = (R \ltimes S) \bowtie S$
- $O(|R| + |S|)$ ignoring log-factors
- How to use semi-join to remove dangling tuples those won't participate in any join result?

R

A	B
a_1	b_1
a_2	b_1
~~a_1~~	~~b_2~~
a_2	b_3
~~a_1~~	~~b_4~~

S

B	C
b_1	c_3
b_1	c_5
b_3	c_3
b_5	c_5
b_6	c_6

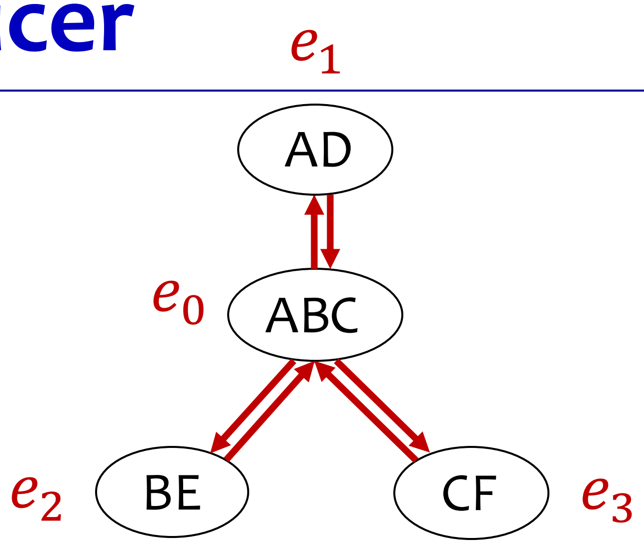
Yannakakis Algorithm: Semi-Join Reducer

Take an arbitrary join tree

- In a bottom-up phase:
  - pick a non-visited node e (with its parent e')
  - update $R_{e'}$ with $R_{e'} \ltimes R_e$

- In a top-down phase:
  - pick a node e
  - For each child e' of e, update $R_{e'}$ with $R_{e'} \ltimes R_e$

- Data Complexity: $O(N)$ ignoring log-factors



$$R_{e_0} \coloneqq R_{e_0} \ltimes R_{e_2}$$
$$R_{e_0} \coloneqq R_{e_0} \ltimes R_{e_3}$$
$$R_{e_1} \coloneqq R_{e_1} \ltimes R_{e_0}$$

$$R_{e_0} \coloneqq R_{e_0} \ltimes R_{e_1}$$
$$R_{e_2} \coloneqq R_{e_2} \ltimes R_{e_0}$$
$$R_{e_3} \coloneqq R_{e_3} \ltimes R_{e_0}$$

Example of Semi-join Reducer

R

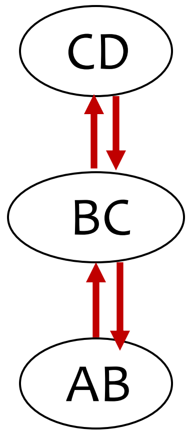
A	B
a_1	b_1
a_2	b_1
a_1	b_2
a_2	b_3
a_1	b_4

S

B	C
b_1	c_3
b_1	c_5
b_3	c_3
b_5	c_5
b_6	c_6

T

C	D
c_1	d_1
c_2	d_2
c_5	d_1
c_6	d_2
c_6	d_3



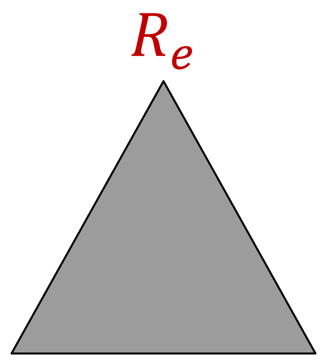
$$Q := R(A,B) \bowtie S(B,C) \bowtie T(C,D)$$

No Dangling tuples

- Every remaining tuple participates in at least one join result.

- In the bottom-up phase:
  - For every node e, once e is reduced, every tuple in R_e participate in at least one result of the sub-join induced by T_e

- In the top-down phase:
  - For every node e, once e is reduced, every tuple in R_e participate in at least one result of the whole join query



T_e: the subtree rooted at node e

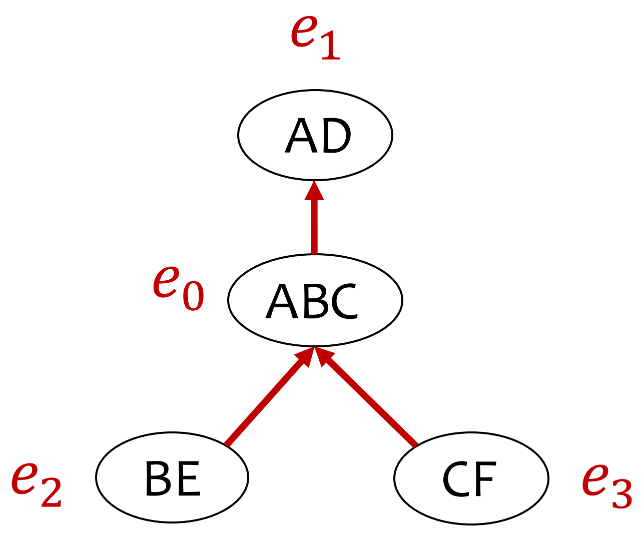
Yannakakis Algorithm: Pairwise Framework

Take an arbitrary join tree

- In a bottom-up phase:
  - pick a non-visited node e (with its parent e')
  - update $R_{e'}$ with $R_{e'} \bowtie R_e$
- Output R_r for the root node r

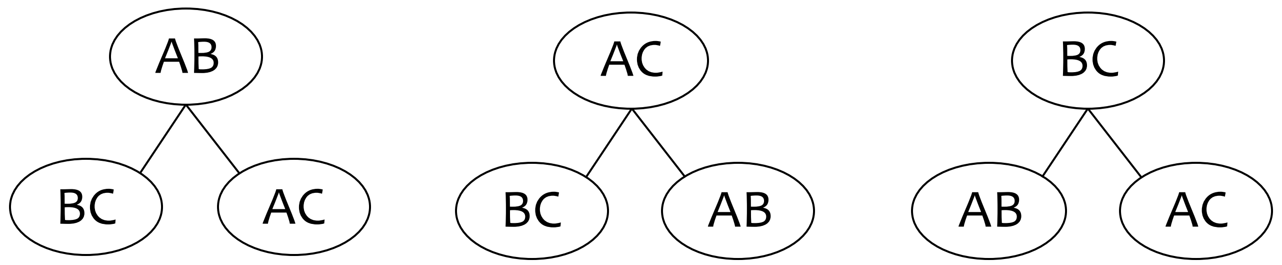
- The intermediate join size is bounded by $O(OUT)$

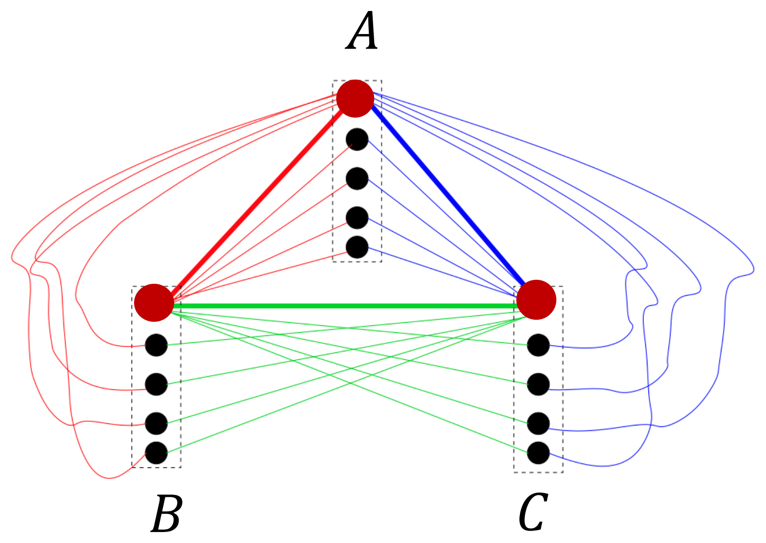
- Data complexity: $O(OUT)$



$$R_{e_0} \coloneqq R_{e_0} \bowtie R_{e_2}$$
$$R_{e_0} \coloneqq R_{e_0} \bowtie R_{e_3}$$
$$R_{e_1} \coloneqq R_{e_1} \bowtie R_{e_0}$$

Discussion: How about Semi-join Reducer on Cyclic Joins?





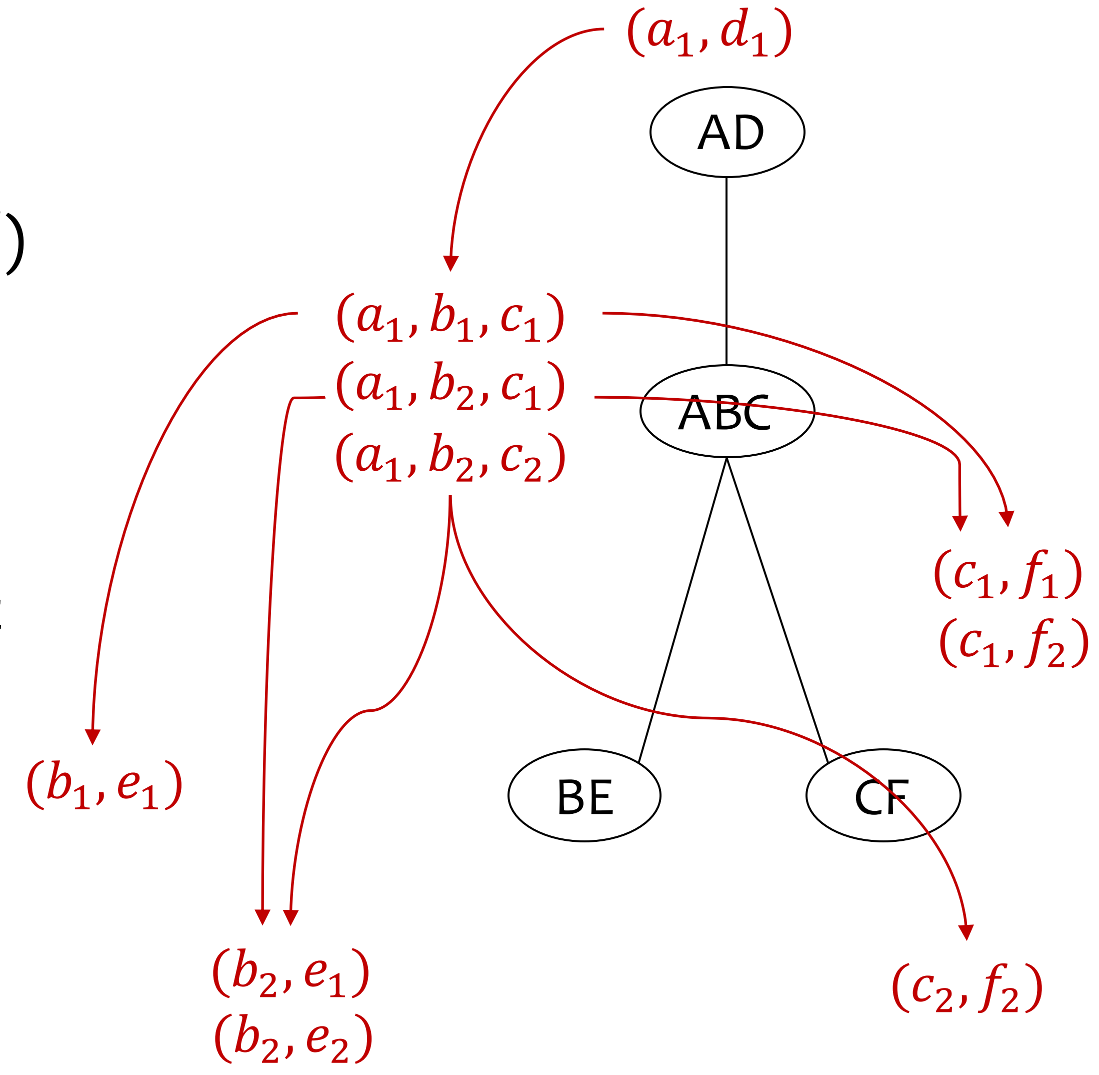
- No more tuples can be removed but some intermediate join result does not participate in any full join result
- How to find non-dangling tuples for cyclic joins? Open questions!

A Practical version of Yannakakis Algorithm

Take an arbitrary join tree

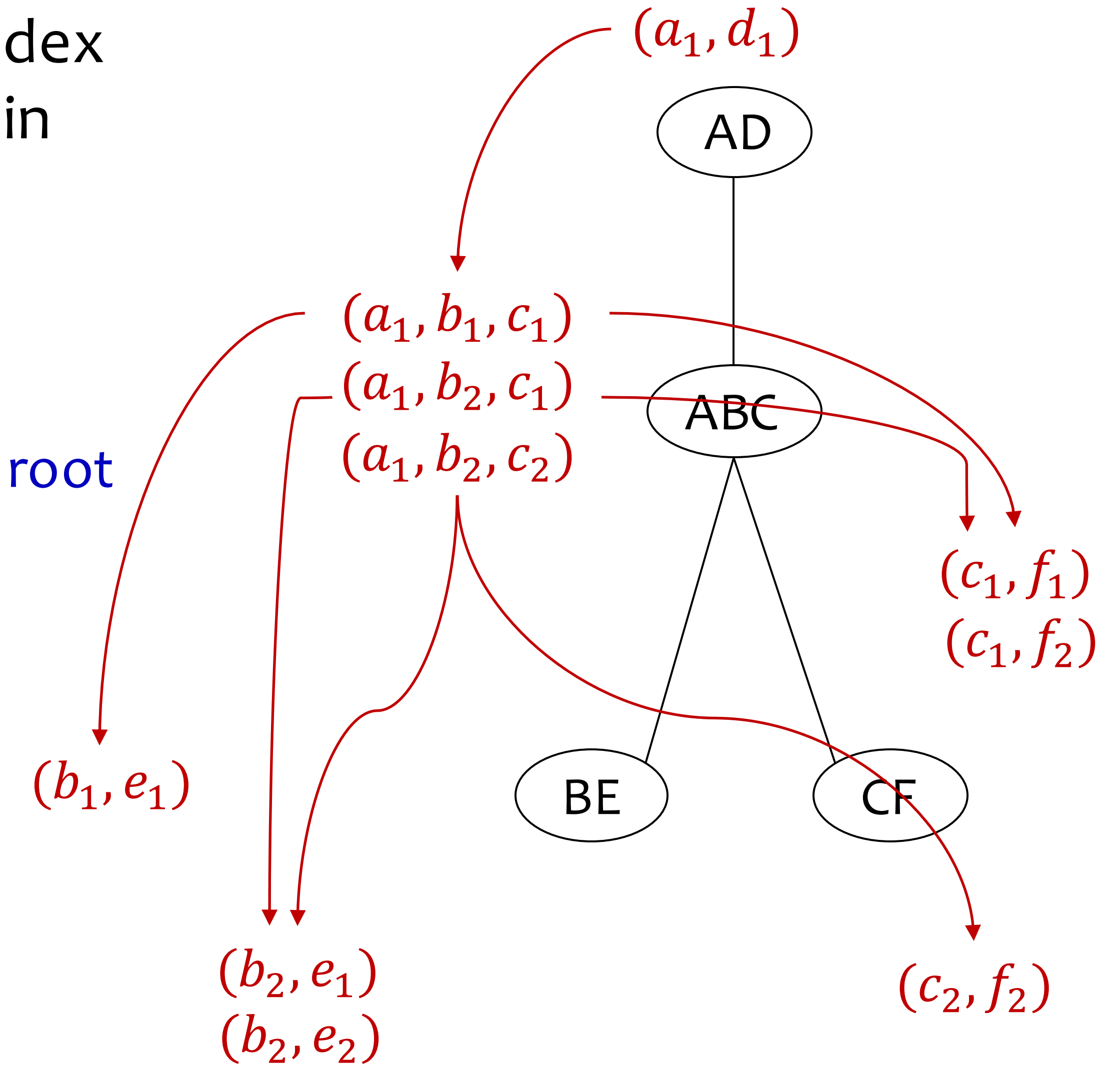
- In a bottom-up phase:
  - pick a non-visited node e (with parent e')
  - update $R_{e'}$ with $R_{e'} \ltimes R_e$

- For each tuple t in the root node R_r:
  - Probe tuples in a top-down manner that can be joined with t



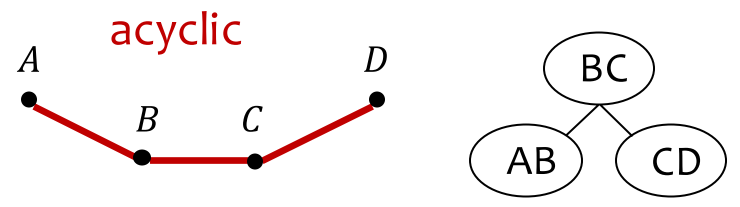
A Practical version of Yannakakis Algorithm

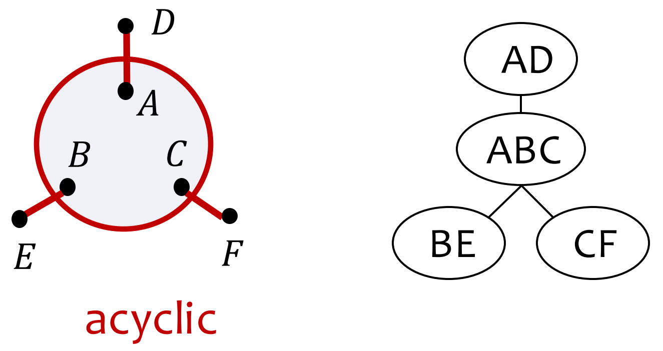
- After $O(N)$ linear preprocessing time, an index of linear size can be built such that every join result can be enumerated with $O(1)$ delay
- Application in dynamic query processing
  - How to maintain non-dangling tuples in the root node and probe indexes?
- Total complexity: $O(N + OUT)$

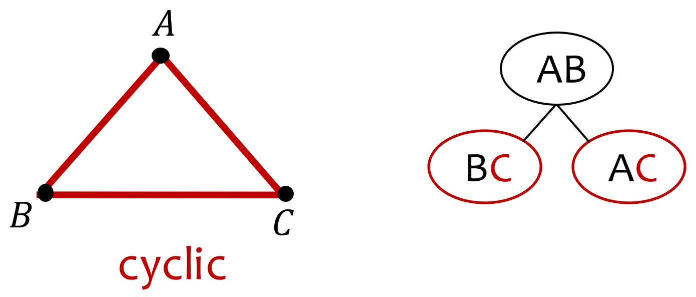


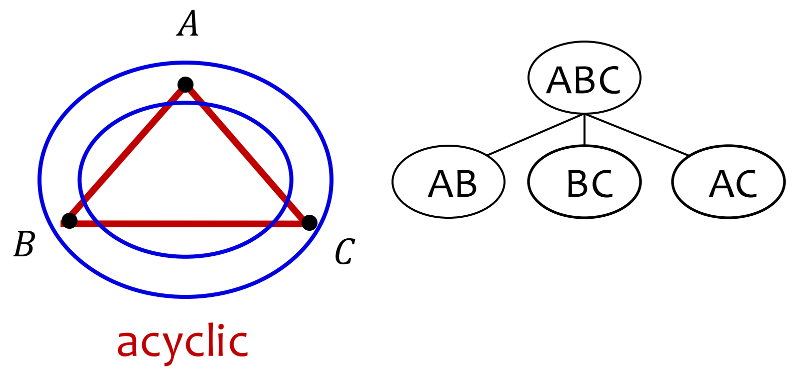
Recap on Acyclicity

- A join query $Q = (V, E)$ is acyclic if it has a join tree T such that
  - one-to-one correspondence between nodes in T with the relations in E;
  - for any attribute $A \in V$, all nodes containing A form a connected subtree.



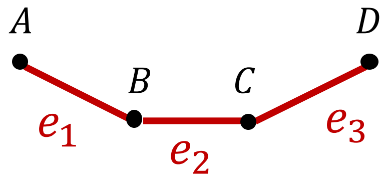




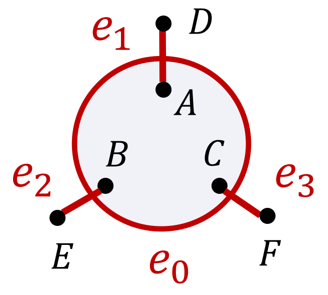


Discussion: How to Build a Join Tree

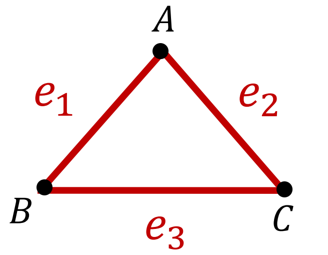
- A query $Q = (V, E)$ is acyclic if GYO reduction results in an empty query:
  - If there is an attribute $A \in V$ that only appears in one relation say e, then remove A from e;
  - If there is a pair of relations $e, e' \in E$ such that $e \subseteq e'$, then remove e from E.

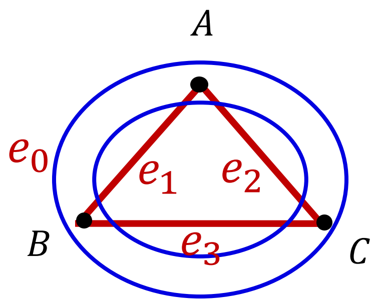


GYO sequence:
$A, e_1, B, e_2, C, D, e_3$



GYO sequence:
$D, e_1, E, e_2, F, e_3, A, B, C, e_0$

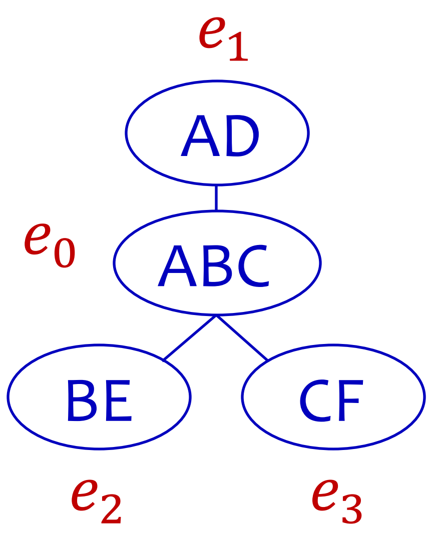




GYO sequence:
$e_1, e_2, e_3, A, B, C, e_0$

Equivalence Between Two Definitions

- Q is acyclic if it has a join tree T.



Always peel off a leaf node

- Q is acyclic if GYO reduction results in an empty query

GYO sequence:

$D, e_1, E, e_2, F, e_3, A, B, C, e_0$

$e_1 \subseteq e_0$ $\quad$ $e_2 \subseteq e_0$ $\quad$ $e_3 \subseteq e_0$

If $e \subseteq e'$ in the GYO sequence, add e as a child node of e'

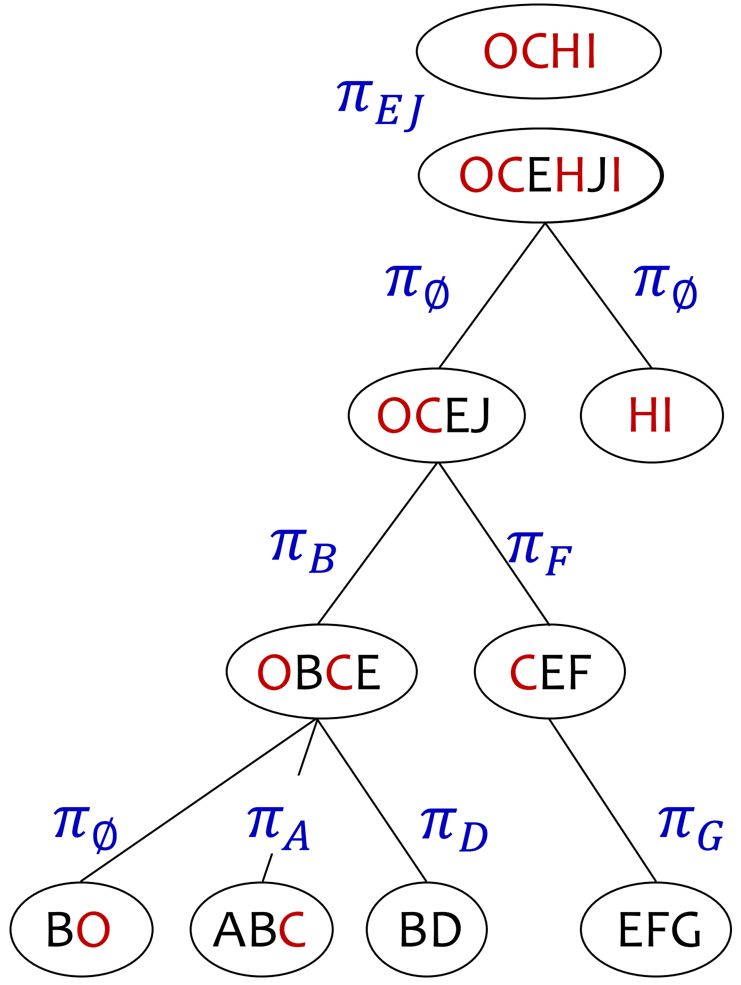
Yannakakis for Acyclic Join-Project Queries

- Chain Matrix Multiplication:
  - $\pi_{AF} R(A, B) \bowtie S(B, C) \bowtie T(C, D) \bowtie W(D, E) \bowtie U(E, F)$

- Parenthesis algorithm

Yannakakis for Acyclic Join-Project Queries

- Let $\boldsymbol{y}$ be the set of output attributes
- Semi-join Reducer
  - If $\boldsymbol{y} = \emptyset$, output true if and only if $R_r \neq \emptyset$
- In a bottom-up phase:
  - Project as early as possible!
  - For a non-output attribute, if it does not appear in any node above, then project it away

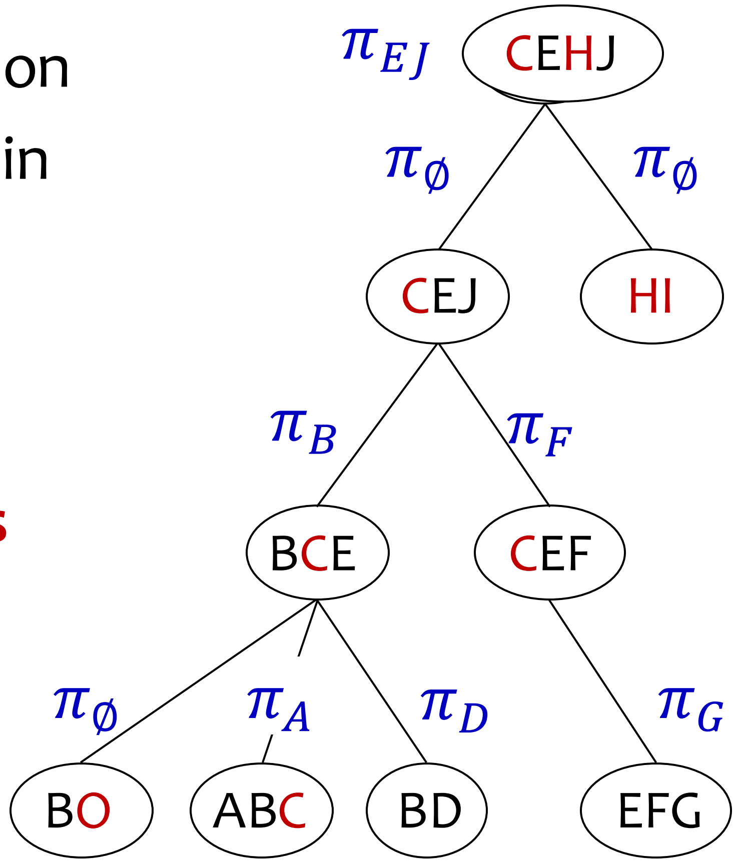


output attributes

Yannakakis for Acyclic Join-Project Query

- Consider an arbitrary node e and its parent u
  - Let v be the attributes at node e after join-projection
  - All attributes in v are either output attributes or join attribute with u
- How to bound the intermediate join size?
  - $R_u \bowtie R_v \subseteq R_u \times (\pi_{v-u} Q)$

    since $v - u$ **are output attributes**
    and no dangling tuples
  - $|R_u \bowtie R_v| \le |R_u| \cdot |Q| \le N \cdot OUT$
- Time complexity: $O(N \cdot OUT)$



output attributes

Yannakakis for Acyclic Join-Aggregate Query

- Annotated Relations are functions mapping tuples to elements from Z

Annotation of a join result t: $w(t) = \prod_e w(\pi_e t)$

$R_1(x_1, x_2)$

x_1	x_2	w
a_1	b_1	2
a_2	b_1	3

$R_2(x_2, x_3)$

x_2	x_3	w
b_1	c_1	2
b_1	C_2	1

$R_3(x_1, x_3)$

x_1	x_3	w
a_1	c_1	1
a_1	c_2	3
a_2	c_2	3

$R_1(x_1, x_2) \bowtie R_2(x_2, x_3) \bowtie R_3(x_1, x_3)$

x_1	x_2	x_3	w
a_1	b_1	c_1	$2 \cdot 2 \cdot 1 = 4$
a_1	b_1	c_2	$2 \cdot 1 \cdot 3 = 6$
a_2	b_1	c_2	$3 \cdot 1 \cdot 3 = 9$

- Count the full join size: $w(\cdot) = 1$

$\sum_{x_1, x_2, x_3} R_1(x_1, x_2) \bowtie R_2(x_2, x_3) \bowtie R_3(x_1, x_3)$

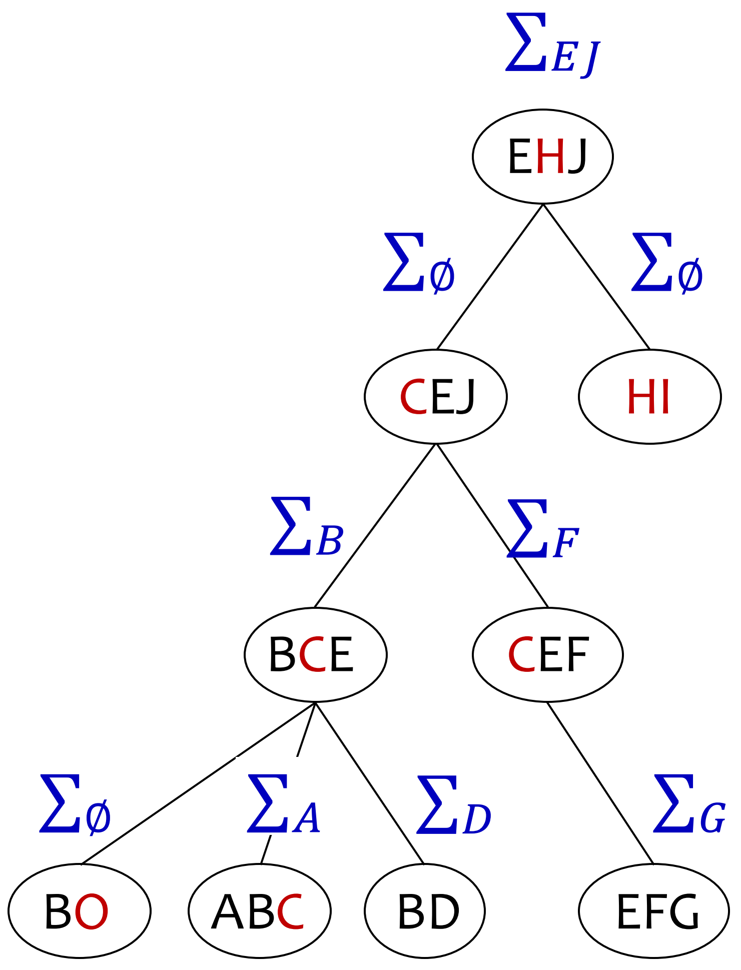
$\emptyset$	w
()	$4 + 6 + 9 = 19$

Annotation of a query result $t \in q(D)$:

$$w(t) = \sum_{t' \in \bowtie_e R_e : \pi_{\mathbf{y}} t' = t} w(t')$$

Yannakakis for Acyclic Join-Aggregate Query

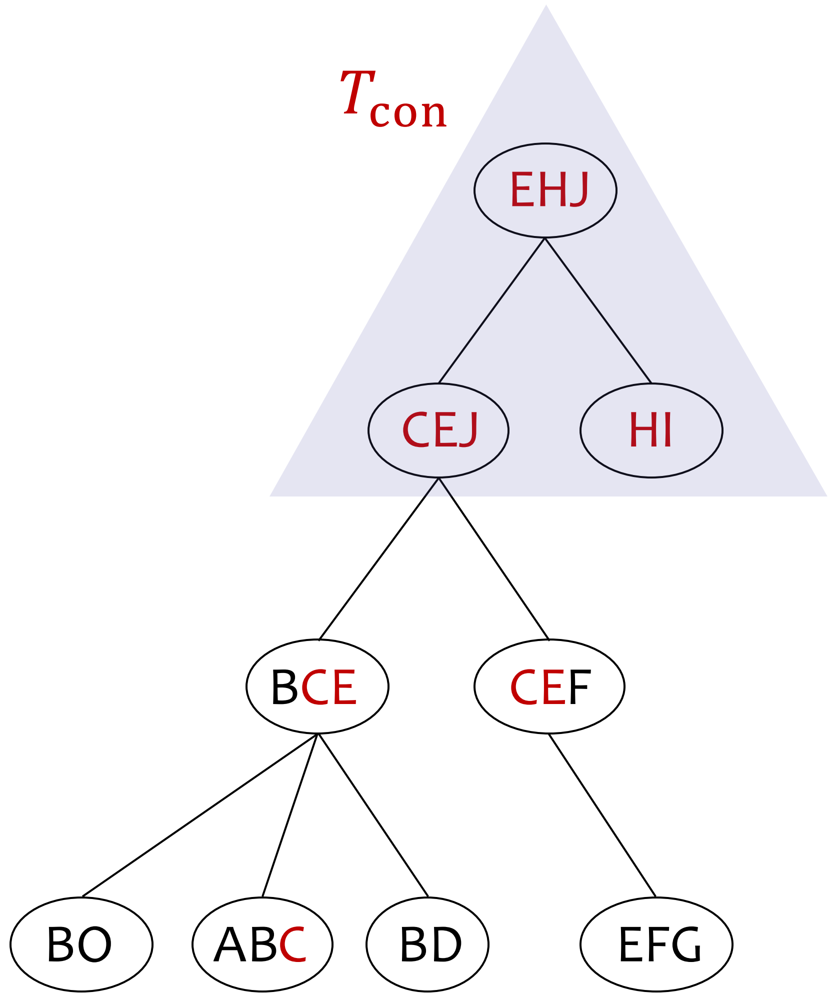
- Semi-join Reducers
- In a bottom-up phase:
  - Aggregate as early as possible!
  - For a non-output attribute, if it does not appear in any node above, then aggregate it away
- Same analysis as join-project queries!
- Semi-ring model usually does not have the inverse of the addition



output attributes

Free-Connex Query

- A join-project query Q is free-connex if it has a join tree T such that
  - There exists a connected subtree $T_{\text{con}} \subseteq T$ containing the root node r of T and the union of nodes in T_{con} is exactly the output attributes



output attributes

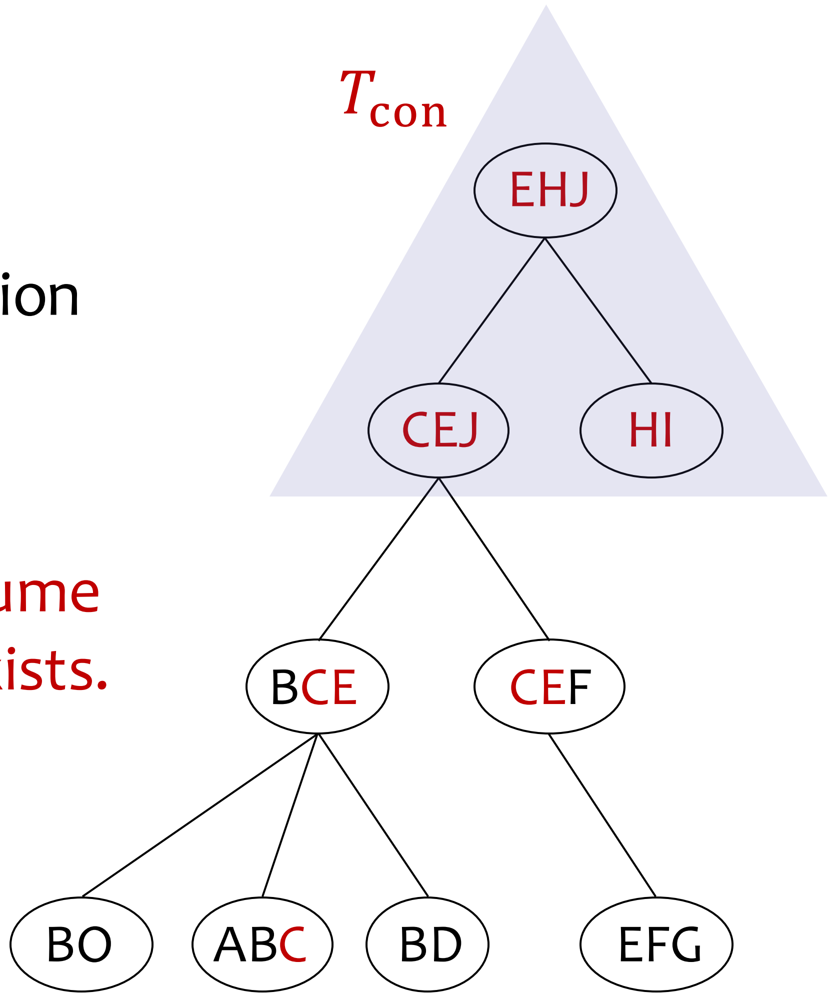
Free-Connex Query

- Let $\boldsymbol{y}$ be the set of output attributes
- Consider an arbitrary node e and its parent u
  - Let v be the attributes at node e after join-projection
  - Either $v \subseteq u$ or $v \subseteq \boldsymbol{y}$!

Case 1: $u \subseteq \boldsymbol{y}$. We have $v \subseteq \boldsymbol{y}$.

Case 2: $u - \boldsymbol{y} \neq \emptyset$. We have $v \subseteq u$. Suppose not, assume $\mathrm{A} \in v - u$. There must be $\mathrm{A} \in e \cap \boldsymbol{y}$. No such T_{con} exists.

- How to bound the intermediate join results?
  - If $v \subseteq u$, $|R_u \bowtie R_v| \leq |R_u| = N$
  - If $v \subseteq \boldsymbol{y}$, $|R_u \bowtie R_v| \leq |Q| = OUT$

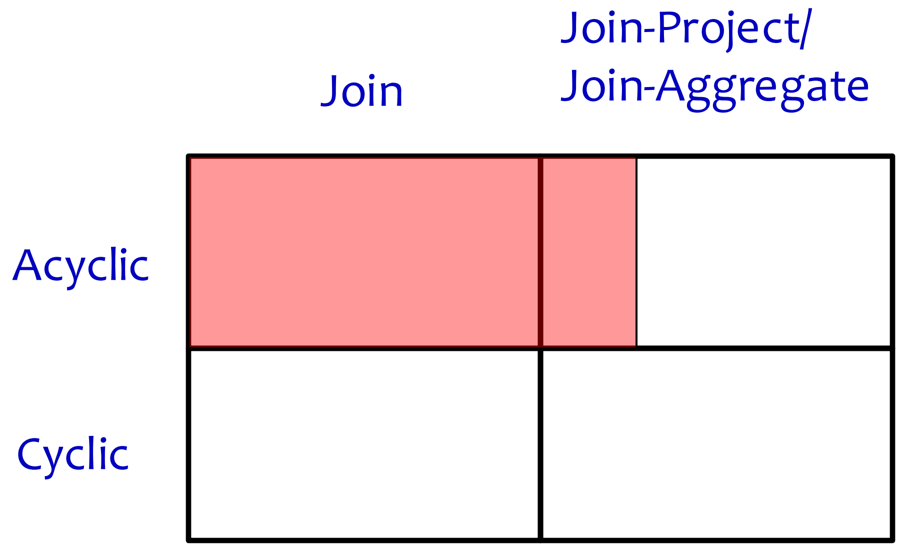


output attributes

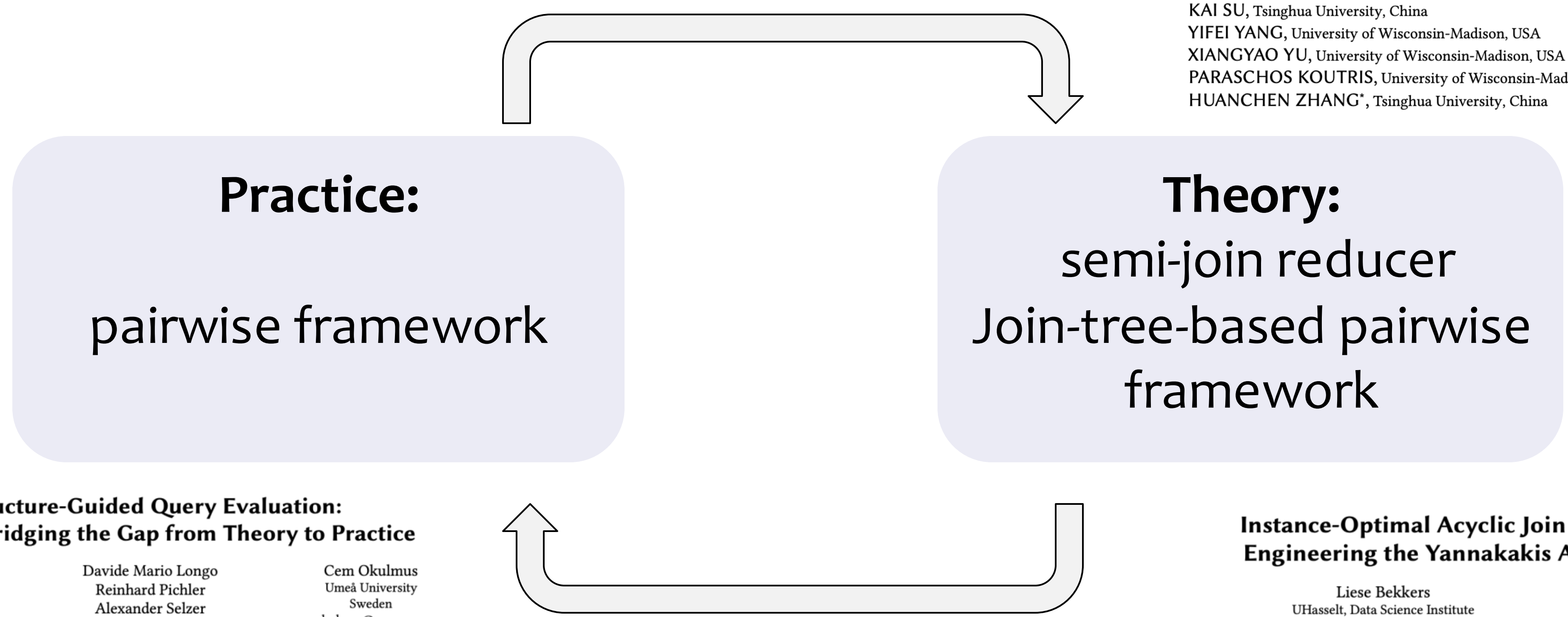
Summary of Yannakakis Algorithm

- Semi-join reducer: remove dangling tuples in $O(N)$ time

- Complexity:
  - Free-connex Join-Project or Join-Aggregate: $O(N + OUT)$
    - Acyclic Joins
  - Acyclic Join-Project or Join-Aggregate: $O(N \cdot OUT)$



Summary of Traditional Query Processing



Practice:

pairwise framework

Theory:

semi-join reducer

Join-tree-based pairwise framework

Debunking the Myth of Join Ordering: Toward Robust SQL Analytics

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Structure-Guided Query Evaluation: Towards Bridging the Gap from Theory to Practice

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Predicate Transfer: Efficient Pre-Filtering on Multi-Join Queries

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Yannakakis$^+$: Practical Acyclic Query Evaluation with Theoretical Guarantees

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Instance-Optimal Acyclic Join Processing Without Regret: Engineering the Yannakakis Algorithm in Column Stores

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