


References


Chitambar, E., Leung, D., Mančinska, L., Ozols, M., and Winter, A. 2014. Everything you always wanted to know about LOCC (but were afraid to ask). Communications in Mathematical Physics, 328(1), 303–326.


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Dyson, F. 1962b. Statistical theory of the energy levels of complex systems. II. *Journal of Mathematical Physics*, 3(1), 157–165.
Dyson, F. 1962c. Statistical theory of the energy levels of complex systems. III. *Journal of Mathematical Physics*, 3(1), 166–175.


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<tr>
<td>Σ, Γ, Λ</td>
<td>Typical names for alphabets (finite and nonempty sets whose elements are viewed as symbols).</td>
</tr>
<tr>
<td>( \mathbb{C}^\Sigma )</td>
<td>The complex Euclidean space of functions from an alphabet ( \Sigma ) to the complex numbers. (Equivalently, the complex Euclidean space of vectors having entries indexed by ( \Sigma ).)</td>
</tr>
<tr>
<td>( \mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z} )</td>
<td>Typical names for complex Euclidean spaces.</td>
</tr>
<tr>
<td>( \langle u, v \rangle )</td>
<td>The inner product between vectors ( u ) and ( v ).</td>
</tr>
<tr>
<td>( | u | )</td>
<td>The Euclidean norm of a vector ( u ).</td>
</tr>
<tr>
<td>( S(\mathcal{X}) )</td>
<td>The unit sphere in a complex Euclidean space ( \mathcal{X} ).</td>
</tr>
<tr>
<td>( | u |_p )</td>
<td>The ( p )-norm of a vector ( u ).</td>
</tr>
<tr>
<td>( | u |_\infty )</td>
<td>The ( \infty )-norm of a vector ( u ).</td>
</tr>
<tr>
<td>( u \perp v, u \perp A )</td>
<td>Indicates that a vector ( u ) is orthogonal to a vector ( v ), or to every element of a set of vectors ( A ).</td>
</tr>
<tr>
<td>( e_a )</td>
<td>An element of the vector standard basis, corresponding to a symbol (or index) ( a ).</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\Sigma_1 \sqcup \cdots \sqcup \Sigma_n$</td>
<td>The disjoint union of alphabets $\Sigma_1, \ldots, \Sigma_n$.</td>
</tr>
<tr>
<td>$X_1 \oplus \cdots \oplus X_n$</td>
<td>The direct sum of complex Euclidean spaces $X_1, \ldots, X_n$.</td>
</tr>
<tr>
<td>$u_1 \oplus \cdots \oplus u_n$</td>
<td>The direct sum of vectors $u_1, \ldots, u_n$.</td>
</tr>
<tr>
<td>$X_1 \otimes \cdots \otimes X_n$</td>
<td>The tensor product of complex Euclidean spaces $X_1, \ldots, X_n$.</td>
</tr>
<tr>
<td>$u_1 \otimes \cdots \otimes u_n$</td>
<td>The tensor product of vectors $u_1, \ldots, u_n$.</td>
</tr>
<tr>
<td>$X \otimes^n$</td>
<td>The $n$-fold tensor product of a complex Euclidean space $X$ with itself.</td>
</tr>
<tr>
<td>$u \otimes^n$</td>
<td>The $n$-fold tensor product of a vector $u$ with itself.</td>
</tr>
<tr>
<td>$\mathbb{R}^\Sigma$</td>
<td>The real Euclidean space of functions from an alphabet $\Sigma$ to the real numbers. (Equivalently, the real Euclidean space of vectors having entries indexed by $\Sigma$.)</td>
</tr>
<tr>
<td>$L(X, Y)$</td>
<td>Space of all linear operators mapping a complex Euclidean space $X$ to a complex Euclidean space $Y$.</td>
</tr>
<tr>
<td>$E_{a,b}$</td>
<td>An element of the operator standard basis, corresponding to symbols (or indices) $a$ and $b$.</td>
</tr>
<tr>
<td>$\overline{A}, \overline{u}$</td>
<td>The entry-wise complex conjugate of an operator $A$ or a vector $u$.</td>
</tr>
<tr>
<td>$A^\top, u^\top$</td>
<td>The transpose of an operator $A$ or a vector $u$.</td>
</tr>
<tr>
<td>$A^<em>, u^</em>$</td>
<td>The adjoint of an operator $A$ or a vector $u$.</td>
</tr>
<tr>
<td>$\ker(A)$</td>
<td>The kernel of an operator $A$.</td>
</tr>
<tr>
<td>$\im(A)$</td>
<td>The image of an operator $A$.</td>
</tr>
<tr>
<td>$\rank(A)$</td>
<td>The rank of an operator $A$.</td>
</tr>
<tr>
<td>Symbol</td>
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<tr>
<td>--------</td>
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</tr>
<tr>
<td>$A_1 \otimes \cdots \otimes A_n$</td>
<td>The tensor product of operators $A_1, \ldots, A_n$.</td>
</tr>
<tr>
<td>$A^\otimes n$</td>
<td>The $n$-fold tensor product of an operator $A$ with itself.</td>
</tr>
<tr>
<td>$L(\mathcal{X})$</td>
<td>Space of linear operators mapping a complex Euclidean space $\mathcal{X}$ to itself.</td>
</tr>
<tr>
<td>$1$</td>
<td>The identity operator; denoted $1_{\mathcal{X}}$ when it is helpful to indicate that it acts on a complex Euclidean space $\mathcal{X}$.</td>
</tr>
<tr>
<td>$X^{-1}$</td>
<td>The inverse of an invertible square operator $X$.</td>
</tr>
<tr>
<td>$\text{Tr}(X)$</td>
<td>The trace of a square operator $X$.</td>
</tr>
<tr>
<td>$\langle A, B \rangle$</td>
<td>The inner product of operators $A$ and $B$.</td>
</tr>
<tr>
<td>$\text{Det}(X)$</td>
<td>The determinant of a square operator $X$.</td>
</tr>
<tr>
<td>$\text{Sym}(\Sigma)$</td>
<td>The set of permutations, or bijective functions, of the form $\pi : \Sigma \rightarrow \Sigma$.</td>
</tr>
<tr>
<td>$\text{sign}(\pi)$</td>
<td>The sign, or parity, of a permutation $\pi$.</td>
</tr>
<tr>
<td>$\text{spec}(X)$</td>
<td>The spectrum of a square operator $X$.</td>
</tr>
<tr>
<td>$[X, Y]$</td>
<td>The Lie bracket of square operators $X$ and $Y$.</td>
</tr>
<tr>
<td>$\text{comm}(\mathcal{A})$</td>
<td>The commutant of a set $\mathcal{A}$ of square operators.</td>
</tr>
<tr>
<td>$\text{Herm}(\mathcal{X})$</td>
<td>The set of Hermitian operators acting on a complex Euclidean space $\mathcal{X}$.</td>
</tr>
<tr>
<td>$\text{Pos}(\mathcal{X})$</td>
<td>The set of positive semidefinite operators acting on a complex Euclidean space $\mathcal{X}$.</td>
</tr>
<tr>
<td>$\text{Pd}(\mathcal{X})$</td>
<td>The set of positive definite operators acting on a complex Euclidean space $\mathcal{X}$.</td>
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<td>D(\mathcal{X})</td>
<td>The set of density operators acting on a complex Euclidean space ( \mathcal{X} ).</td>
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<tr>
<td>Proj(\mathcal{X})</td>
<td>The set of projection operators acting on a complex Euclidean space ( \mathcal{X} ).</td>
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<tr>
<td>\Pi_\mathcal{V}</td>
<td>The projection operator whose image is ( \mathcal{V} ).</td>
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<tr>
<td>U(\mathcal{X}, \mathcal{Y})</td>
<td>The set of isometries mapping a complex Euclidean space ( \mathcal{X} ) to a complex Euclidean space ( \mathcal{Y} ).</td>
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<tr>
<td>U(\mathcal{X})</td>
<td>The set of unitary operators acting on a complex Euclidean space ( \mathcal{X} ).</td>
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<tr>
<td>Diag(( u ))</td>
<td>The diagonal square operator whose diagonal entries are described by the vector ( u ).</td>
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<tr>
<td>\lambda(H)</td>
<td>The vector of eigenvalues of a Hermitian operator ( H ).</td>
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<tr>
<td>\lambda_k(H)</td>
<td>The ( k )-th largest eigenvalue of a Hermitian operator ( H ).</td>
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<tr>
<td>X \geq Y or Y \leq X</td>
<td>Indicates that ( X - Y ) is positive semidefinite, for Hermitian operators ( X ) and ( Y ).</td>
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<tr>
<td>X &gt; Y or Y &lt; X</td>
<td>Indicates that ( X - Y ) is positive definite, for Hermitian operators ( X ) and ( Y ).</td>
<td>21</td>
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<tr>
<td>T(\mathcal{X}, \mathcal{Y})</td>
<td>The space of linear maps from ( L(\mathcal{X}) ) to ( L(\mathcal{Y}) ), for complex Euclidean spaces ( \mathcal{X} ) and ( \mathcal{Y} ).</td>
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<tr>
<td>\Phi^*</td>
<td>The adjoint of a map ( \Phi \in T(\mathcal{X}, \mathcal{Y}) ).</td>
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<tr>
<td>\Phi_1 \otimes \cdots \otimes \Phi_n</td>
<td>The tensor product of maps ( \Phi_1, \ldots, \Phi_n ).</td>
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<tr>
<td>\Phi^{\otimes n}</td>
<td>The ( n )-fold tensor product of a map ( \Phi ) with itself.</td>
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<tr>
<td>\mathbb{1}_{L(\mathcal{X})}</td>
<td>The identity map acting on ( L(\mathcal{X}) ).</td>
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<tr>
<td>(\text{Tr}_X)</td>
<td>The partial trace over a complex Euclidean space (X).</td>
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<tr>
<td>(\text{CP}(X,Y))</td>
<td>The set of completely positive maps of the form (\Phi \in T(X,Y)).</td>
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<tr>
<td>(\text{vec}(A))</td>
<td>The vec mapping applied to an operator (A).</td>
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<tr>
<td>(\sqrt{P})</td>
<td>The square root of a positive semidefinite operator (P).</td>
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<tr>
<td>(s(A))</td>
<td>The vector of singular values of an operator (A).</td>
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<tr>
<td>(s_k(A))</td>
<td>The (k)-th largest singular value of an operator (A).</td>
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<tr>
<td>(A^+)</td>
<td>The Moore–Penrose pseudo-inverse of an operator (A).</td>
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<tr>
<td>(|A|<em>p, |A|</em>\infty)</td>
<td>The Schatten (p)-norm or (\infty)-norm of an operator (A).</td>
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<tr>
<td>(|A|)</td>
<td>The spectral norm of an operator (A). Equivalent to the Schatten (\infty)-norm of (A).</td>
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<tr>
<td>(|A|_2)</td>
<td>The Frobenius norm of an operator (A). Equivalent to the Schatten 2-norm of (A).</td>
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<tr>
<td>(|A|_1)</td>
<td>The trace norm of an operator (A). Equivalent to the Schatten 1-norm of (A).</td>
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<tr>
<td>(\nabla f(x))</td>
<td>The gradient vector of a function (f : \mathbb{R}^n \to \mathbb{R}) at a vector (x \in \mathbb{R}^n).</td>
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<tr>
<td>((Df)(x))</td>
<td>The derivative of a (differentiable) function (f : \mathbb{R}^n \to \mathbb{R}) at a vector (x \in \mathbb{R}^n).</td>
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<tr>
<td>(B(X))</td>
<td>The unit ball in a complex Euclidean space (X).</td>
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<tr>
<td>(\text{Borel}(A))</td>
<td>The collection of all Borel subsets of a subset (A) of a real or complex vector space.</td>
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\[
\int f(x) \, d\mu(x)
\] The integral of a function \( f \) with respect to a Borel measure \( \mu \).  

cone(\( \mathcal{A} \)) The cone generated by a subset \( \mathcal{A} \) of a real or complex vector space.  

\( \mathcal{P}(\Sigma) \) The set of probability vectors with entries indexed by an alphabet \( \Sigma \).  

conv(\( \mathcal{A} \)) The convex hull of a subset \( \mathcal{A} \) of a real or complex vector space.  

\( \mathbb{E}(X) \) The expected value of a random variable \( X \).  

\( \Gamma(\alpha) \) The value of the \( \Gamma \)-function at \( \alpha \).  

\( \gamma_n \) The standard Gaussian measure on \( \mathbb{R}^n \).  

\( X, Y, Z \) Typical names for registers.  

\( (X_1, \ldots, X_n) \) The compound register formed from registers \( X_1, \ldots, X_n \).  

\( \omega_V \) The flat state proportional to the projection onto the subspace \( V \).  

\( \rho[X_1, \ldots, X_n] \) The reduction of a state \( \rho \) to registers \( X_1, \ldots, X_n \).  

\( C(\mathcal{X}, \mathcal{Y}) \) The set of all channels mapping \( L(\mathcal{X}) \) to \( L(\mathcal{Y}) \).  

\( C(\mathcal{X}) \) The set of channels mapping \( L(\mathcal{X}) \) to itself.  

\( K(\Phi) \) The natural representation of a map \( \Phi \).  

\( J(\Phi) \) The Choi representation of a map \( \Phi \).  

\( \Omega \) or \( \Omega_{\mathcal{X}} \) Typical name for the completely depolarizing channel acting on \( L(\mathcal{X}) \).  

\( \Delta \) or \( \Delta_{\mathcal{X}} \) Typical name for the completely dephasing channel acting on \( L(\mathcal{X}) \).
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<td>$F(P, Q)$</td>
<td>The fidelity between positive semidefinite operators $P$ and $Q$.</td>
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<tr>
<td>$B(P, Q</td>
<td>\mu)$</td>
<td>The Bhattacharyya coefficient of the nonnegative real vectors obtained by applying a measurement $\mu$ to positive semidefinite operators $P$ and $Q$.</td>
</tr>
<tr>
<td>$F(\Phi, P)$</td>
<td>The mapping fidelity of a map $\Phi$ with respect to a positive semidefinite operator $P$.</td>
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<tr>
<td>$W$ or $W_X$</td>
<td>Typical name used to refer to the swap operator acting on a bipartite tensor product space $X \otimes X$.</td>
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<tr>
<td>$|\Phi|_1$</td>
<td>The induced trace norm of a map $\Phi$.</td>
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<td>$</td>
<td></td>
<td></td>
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<tr>
<td>$\mathcal{N}(X)$</td>
<td>The numerical range of a square operator $X$.</td>
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<tr>
<td>$F_{\text{max}}(\Psi_0, \Psi_1)$</td>
<td>The maximum output fidelity of positive maps $\Psi_0$ and $\Psi_1$.</td>
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<tr>
<td>$\mathbb{Z}_n$</td>
<td>The ring of integers modulo $n$.</td>
<td>212</td>
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<tr>
<td>$W_{a,b}$</td>
<td>A discrete Weyl operator acting on $\mathbb{C}^{\mathbb{Z}_n}$, for $a, b \in \mathbb{Z}_n$.</td>
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<tr>
<td>$\sigma_x, \sigma_y, \text{and } \sigma_z$</td>
<td>The Pauli operators.</td>
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<tr>
<td>$A \odot B$</td>
<td>The entry-wise product of operators $A$ and $B$.</td>
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<tr>
<td>$V_\pi$</td>
<td>Permutation operator corresponding to the permutation $\pi$.</td>
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<tr>
<td>$v \prec u$</td>
<td>Indicates that $u$ majorizes $v$, for real vectors $u$ and $v$.</td>
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<tr>
<td>$r(u)$</td>
<td>The vector obtained by sorting the entries of a real vector $u$ from largest to smallest.</td>
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<tr>
<td>$r_k(u)$</td>
<td>The $k$-th largest entry of a real vector $u$.</td>
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<tr>
<td>$Y \prec X$</td>
<td>Indicates that $X$ majorizes $Y$, for Hermitian operators $X$ and $Y$.</td>
<td>241</td>
</tr>
<tr>
<td>$S_n$</td>
<td>The symmetric group on $n$ symbols, equivalent to $\text{Sym}([1, \ldots, n])$.</td>
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<tr>
<td>$H(u)$</td>
<td>The Shannon entropy of a vector $u$ with nonnegative real number entries.</td>
<td>251</td>
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<tr>
<td>$H(X)$</td>
<td>The Shannon entropy of the probabilistic state of a classical register $X$, or the von Neumann entropy of the quantum state of a register $X$.</td>
<td>252, 266</td>
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<tr>
<td>$H(X_1, \ldots, X_n)$</td>
<td>Refers to the Shannon entropy or von Neumann entropy of the compound register $(X_1, \ldots, X_n)$.</td>
<td>252, 266</td>
</tr>
<tr>
<td>$D(u|v)$</td>
<td>The relative entropy of $u$ with respect to $v$, for vectors $u$ and $v$ with nonnegative real number entries.</td>
<td>252</td>
</tr>
<tr>
<td>$H(X</td>
<td>Y)$</td>
<td>The conditional Shannon entropy or von Neumann entropy of a register $X$ with respect to a register $Y$.</td>
</tr>
<tr>
<td>$I(X:Y)$</td>
<td>The mutual information or quantum mutual information between registers $X$ and $Y$.</td>
<td>253, 267</td>
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<tr>
<td>$H(P)$</td>
<td>The von Neumann entropy of a positive semidefinite operator $P$.</td>
<td>265</td>
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<tr>
<td>$D(P|Q)$</td>
<td>The quantum relative entropy of $P$ with respect to $Q$, for positive semidefinite operators $P$ and $Q$.</td>
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<tr>
<td>$T_{n,\varepsilon}(p)$</td>
<td>The set of $\varepsilon$-typical strings of length $n$ with respect to the probability vector $p$.</td>
<td>286</td>
</tr>
<tr>
<td>$\Pi_{n,\varepsilon}$</td>
<td>Projection operator corresponding to the $\varepsilon$-typical subspace of $\mathcal{X}^\otimes n$ with respect to a given state.</td>
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<tr>
<td>$I_{\text{acc}}(\eta)$</td>
<td>The accessible information of an ensemble $\eta$.</td>
<td>295</td>
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<tr>
<td>$\chi(\eta)$</td>
<td>The Holevo information of an ensemble $\eta$.</td>
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<tr>
<td>$\text{Sep}(\mathcal{X} : \mathcal{Y})$</td>
<td>The set of separable operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.</td>
<td>311</td>
</tr>
<tr>
<td>$\text{SepD}(\mathcal{X} : \mathcal{Y})$</td>
<td>The set of separable density operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.</td>
<td>311</td>
</tr>
<tr>
<td>$\text{Ent}_r(\mathcal{X} : \mathcal{Y})$</td>
<td>The set of operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ having entanglement rank bounded by $r$, with respect to the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.</td>
<td>322</td>
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<td>$\text{SepCP}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$</td>
<td>The set of separable maps from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$.</td>
<td>325</td>
</tr>
<tr>
<td>$\text{SepC}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$</td>
<td>The set of separable channels from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$.</td>
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<tr>
<td>$\text{LOCC}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$</td>
<td>The set of LOCC channels from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$.</td>
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<tr>
<td>$E_D(\mathcal{X} : \mathcal{Y})$</td>
<td>The distillable entanglement of the state of a pair of registers $(\mathcal{X}, \mathcal{Y})$.</td>
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E_c(X : Y) The entanglement cost of the state of a pair of registers (X, Y). 347
PPT(X : Y) The set of PPT operators acting on X ⊗ Y, respecting the bipartition between X and Y. 353
E_f(X : Y) The entanglement of formation of the state of a pair of registers (X, Y). 385
W_π A unitary operator acting on X ⊗ n, for a complex Euclidean space X, that permutes tensor factors according to the permutation π. 391
X⊗n The symmetric subspace of X⊗n, for X a complex Euclidean space. Also denoted X_1 ⊗ ⋯ ⊗ X_n when X_1, …, X_n are identical copies of X. 392
Bag(n, Σ) The set of functions describing a bag of n items, each labeled by an element of an alphabet Σ. 393
N The set of nonnegative integers \{0, 1, 2, …\}. 393
Σ^n_φ The subset of Σ^n consistent with a given function φ ∈ Bag(n, Σ). 393
X⊗n The anti-symmetric subspace of X⊗n, for X a complex Euclidean space. 398
L(X)⊗n The algebra of permutation-invariant operators acting on X⊗n, for X a complex Euclidean space. 400
µ Symbol used to denote uniform spherical measure. 408
η Symbol used to denote Haar measure. 411
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<tr>
<th>Symbol</th>
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<tr>
<td>( H_{\text{min}}(\Phi) )</td>
<td>The minimum output entropy of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( C(\Phi) )</td>
<td>The classical capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( C_E(\Phi) )</td>
<td>The entanglement-assisted classical capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( \chi(\Phi) )</td>
<td>The Holevo capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( \chi_E(\Phi) )</td>
<td>The entanglement-assisted Holevo capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( I_c(\rho; \Phi) )</td>
<td>The coherent information of a state ( \rho ) through a channel ( \Phi ).</td>
</tr>
<tr>
<td>( I_c(\Phi) )</td>
<td>The maximum coherent information of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( K_{a_1 \cdots a_n, \varepsilon}(p) )</td>
<td>The set of ( \varepsilon )-typical strings of length ( n ), conditioned on a string ( a_1 \cdots a_n ), with respect to the probability vector ( p ).</td>
</tr>
<tr>
<td>( \Lambda_{a_1 \cdots a_n, \varepsilon} )</td>
<td>Projection onto the ( \varepsilon )-typical subspace of ( \mathcal{X}^\otimes n ), for ( \mathcal{X} ) a complex Euclidean space, conditioned on a string ( a_1 \cdots a_n ).</td>
</tr>
<tr>
<td>( S_{n, \varepsilon}(p) )</td>
<td>The set of ( \varepsilon )-strongly typical strings of length ( n ) with respect to the probability vector ( p ).</td>
</tr>
<tr>
<td>( Q(\Phi) )</td>
<td>The quantum capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( Q_{EG}(\Phi) )</td>
<td>The entanglement generation capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( Q_E(\Phi) )</td>
<td>The entanglement-assisted quantum capacity of a channel ( \Phi ).</td>
</tr>
<tr>
<td>( \Phi_0 \oplus \Phi_1 )</td>
<td>The direct sum of maps ( \Phi_0 ) and ( \Phi_1 ).</td>
</tr>
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