References


References


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Dyson, F. 1962b. Statistical theory of the energy levels of complex systems. II. *Journal of Mathematical Physics*, 3(1), 157–165.
Dyson, F. 1962c. Statistical theory of the energy levels of complex systems. III. *Journal of Mathematical Physics*, 3(1), 166–175.


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<tr>
<td>Σ, Γ, Λ</td>
<td>Typical names for alphabets (finite and nonempty sets whose elements are viewed as symbols).</td>
<td>1</td>
</tr>
<tr>
<td>( \mathbb{C}^\Sigma )</td>
<td>The complex Euclidean space of functions from an alphabet ( \Sigma ) to the complex numbers. (Equivalently, the complex Euclidean space of vectors having entries indexed by ( \Sigma ).)</td>
<td>2</td>
</tr>
<tr>
<td>( \mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z} )</td>
<td>Typical names for complex Euclidean spaces.</td>
<td>2</td>
</tr>
<tr>
<td>( \langle u, v \rangle )</td>
<td>The inner product between vectors ( u ) and ( v ).</td>
<td>3</td>
</tr>
<tr>
<td>( | u | )</td>
<td>The Euclidean norm of a vector ( u ).</td>
<td>4</td>
</tr>
<tr>
<td>( \mathcal{S}(\mathcal{X}) )</td>
<td>The unit sphere in a complex Euclidean space ( \mathcal{X} ).</td>
<td>4</td>
</tr>
<tr>
<td>( | u |_p )</td>
<td>The ( p )-norm of a vector ( u ).</td>
<td>4</td>
</tr>
<tr>
<td>( | u |_\infty )</td>
<td>The ( \infty )-norm of a vector ( u ).</td>
<td>4</td>
</tr>
<tr>
<td>( u \perp v, u \perp A )</td>
<td>Indicates that a vector ( u ) is orthogonal to a vector ( v ), or to every element of a set of vectors ( A ).</td>
<td>4</td>
</tr>
<tr>
<td>( e_a )</td>
<td>An element of the vector standard basis, corresponding to a symbol (or index) ( a ).</td>
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<tr>
<td>$\Sigma_1 \sqcup \cdots \sqcup \Sigma_n$</td>
<td>The disjoint union of alphabets $\Sigma_1, \ldots, \Sigma_n$.</td>
</tr>
<tr>
<td>$\mathcal{X}_1 \oplus \cdots \oplus \mathcal{X}_n$</td>
<td>The direct sum of complex Euclidean spaces $\mathcal{X}_1, \ldots, \mathcal{X}_n$.</td>
</tr>
<tr>
<td>$u_1 \oplus \cdots \oplus u_n$</td>
<td>The direct sum of vectors $u_1, \ldots, u_n$.</td>
</tr>
<tr>
<td>$\mathcal{X}_1 \otimes \cdots \otimes \mathcal{X}_n$</td>
<td>The tensor product of complex Euclidean spaces $\mathcal{X}_1, \ldots, \mathcal{X}_n$.</td>
</tr>
<tr>
<td>$u_1 \otimes \cdots \otimes u_n$</td>
<td>The tensor product of vectors $u_1, \ldots, u_n$.</td>
</tr>
<tr>
<td>$\mathcal{X}^\otimes n$</td>
<td>The $n$-fold tensor product of a complex Euclidean space $\mathcal{X}$ with itself.</td>
</tr>
<tr>
<td>$u^\otimes n$</td>
<td>The $n$-fold tensor product of a vector $u$ with itself.</td>
</tr>
<tr>
<td>$\mathbb{R}^{\Sigma}$</td>
<td>The real Euclidean space of functions from an alphabet $\Sigma$ to the real numbers. (Equivalently, the real Euclidean space of vectors having entries indexed by $\Sigma$.)</td>
</tr>
<tr>
<td>$L(\mathcal{X}, \mathcal{Y})$</td>
<td>Space of all linear operators mapping a complex Euclidean space $\mathcal{X}$ to a complex Euclidean space $\mathcal{Y}$.</td>
</tr>
<tr>
<td>$E_{a,b}$</td>
<td>An element of the operator standard basis, corresponding to symbols (or indices) $a$ and $b$.</td>
</tr>
<tr>
<td>$\overline{A}, \overline{u}$</td>
<td>The entry-wise complex conjugate of an operator $A$ or a vector $u$.</td>
</tr>
<tr>
<td>$A^\top, u^\top$</td>
<td>The transpose of an operator $A$ or a vector $u$.</td>
</tr>
<tr>
<td>$A^<em>, u^</em>$</td>
<td>The adjoint of an operator $A$ or a vector $u$.</td>
</tr>
<tr>
<td>$\ker(A)$</td>
<td>The kernel of an operator $A$.</td>
</tr>
<tr>
<td>$\text{im}(A)$</td>
<td>The image of an operator $A$.</td>
</tr>
<tr>
<td>$\text{rank}(A)$</td>
<td>The rank of an operator $A$.</td>
</tr>
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<td>Symbol</td>
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</tr>
<tr>
<td>$A_1 \otimes \cdots \otimes A_n$</td>
<td>The tensor product of operators $A_1, \ldots, A_n$.</td>
</tr>
<tr>
<td>$A^{\otimes n}$</td>
<td>The $n$-fold tensor product of an operator $A$ with itself.</td>
</tr>
<tr>
<td>$L(\mathcal{X})$</td>
<td>Space of linear operators mapping a complex Euclidean space $\mathcal{X}$ to itself.</td>
</tr>
<tr>
<td>$1$</td>
<td>The identity operator; denoted $1_{\mathcal{X}}$ when it is helpful to indicate that it acts on a complex Euclidean space $\mathcal{X}$.</td>
</tr>
<tr>
<td>$X^{-1}$</td>
<td>The inverse of an invertible square operator $X$.</td>
</tr>
<tr>
<td>$\text{Tr}(X)$</td>
<td>The trace of a square operator $X$.</td>
</tr>
<tr>
<td>$\langle A, B \rangle$</td>
<td>The inner product of operators $A$ and $B$.</td>
</tr>
<tr>
<td>$\text{Det}(X)$</td>
<td>The determinant of a square operator $X$.</td>
</tr>
<tr>
<td>$\text{Sym}(\Sigma)$</td>
<td>The set of permutations, or bijective functions, of the form $\pi : \Sigma \to \Sigma$.</td>
</tr>
<tr>
<td>$\text{sign}(\pi)$</td>
<td>The sign, or parity, of a permutation $\pi$.</td>
</tr>
<tr>
<td>$\text{spec}(X)$</td>
<td>The spectrum of a square operator $X$.</td>
</tr>
<tr>
<td>$[X, Y]$</td>
<td>The Lie bracket of square operators $X$ and $Y$.</td>
</tr>
<tr>
<td>$\text{comm}(\mathcal{A})$</td>
<td>The commutant of a set $\mathcal{A}$ of square operators.</td>
</tr>
<tr>
<td>$\text{Herm}(\mathcal{X})$</td>
<td>The set of Hermitian operators acting on a complex Euclidean space $\mathcal{X}$.</td>
</tr>
<tr>
<td>$\text{Pos}(\mathcal{X})$</td>
<td>The set of positive semidefinite operators acting on a complex Euclidean space $\mathcal{X}$.</td>
</tr>
<tr>
<td>$\text{Pd}(\mathcal{X})$</td>
<td>The set of positive definite operators acting on a complex Euclidean space $\mathcal{X}$.</td>
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<td>D(\mathcal{X})</td>
<td>The set of density operators acting on a complex Euclidean space ( \mathcal{X} ).</td>
<td>18</td>
</tr>
<tr>
<td>\text{Proj}(\mathcal{X})</td>
<td>The set of projection operators acting on a complex Euclidean space ( \mathcal{X} ).</td>
<td>18</td>
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<tr>
<td>\Pi_{\mathcal{V}}</td>
<td>The projection operator whose image is ( \mathcal{V} ).</td>
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<tr>
<td>U(\mathcal{X},\mathcal{Y})</td>
<td>The set of isometries mapping a complex Euclidean space ( \mathcal{X} ) to a complex Euclidean space ( \mathcal{Y} ).</td>
<td>18</td>
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<tr>
<td>U(\mathcal{X})</td>
<td>The set of unitary operators acting on a complex Euclidean space ( \mathcal{X} ).</td>
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<tr>
<td>\text{Diag}(u)</td>
<td>The diagonal square operator whose diagonal entries are described by the vector ( u ).</td>
<td>19</td>
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<tr>
<td>\lambda(H)</td>
<td>The vector of eigenvalues of a Hermitian operator ( H ).</td>
<td>20</td>
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<tr>
<td>\lambda_k(H)</td>
<td>The ( k )-th largest eigenvalue of a Hermitian operator ( H ).</td>
<td>20</td>
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<tr>
<td>( X \geq Y ) or ( Y \leq X )</td>
<td>Indicates that ( X - Y ) is positive semidefinite, for Hermitian operators ( X ) and ( Y ).</td>
<td>21</td>
</tr>
<tr>
<td>( X &gt; Y ) or ( Y &lt; X )</td>
<td>Indicates that ( X - Y ) is positive definite, for Hermitian operators ( X ) and ( Y ).</td>
<td>21</td>
</tr>
<tr>
<td>T(\mathcal{X},\mathcal{Y})</td>
<td>The space of linear maps from ( L(\mathcal{X}) ) to ( L(\mathcal{Y}) ), for complex Euclidean spaces ( \mathcal{X} ) and ( \mathcal{Y} ).</td>
<td>21</td>
</tr>
<tr>
<td>\Phi^*</td>
<td>The adjoint of a map ( \Phi \in T(\mathcal{X},\mathcal{Y}) ).</td>
<td>21</td>
</tr>
<tr>
<td>\Phi_1 \otimes \cdots \otimes \Phi_n</td>
<td>The tensor product of maps ( \Phi_1, \ldots, \Phi_n ).</td>
<td>22</td>
</tr>
<tr>
<td>\Phi^{\otimes n}</td>
<td>The ( n )-fold tensor product of a map ( \Phi ) with itself.</td>
<td>22</td>
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<tr>
<td>1_{L(\mathcal{X})}</td>
<td>The identity map acting on ( L(\mathcal{X}) ).</td>
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<tr>
<td>$\text{Tr}_\mathcal{X}$</td>
<td>The partial trace over a complex Euclidean space $\mathcal{X}$.</td>
<td>22</td>
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<tr>
<td>$\text{CP}(\mathcal{X}, \mathcal{Y})$</td>
<td>The set of completely positive maps of the form $\Phi \in \mathcal{T}(\mathcal{X}, \mathcal{Y})$.</td>
<td>23</td>
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<tr>
<td>$\text{vec}(A)$</td>
<td>The vec mapping applied to an operator $A$.</td>
<td>23</td>
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<tr>
<td>$\sqrt{P}$</td>
<td>The square root of a positive semidefinite operator $P$.</td>
<td>27</td>
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<tr>
<td>$s(A)$</td>
<td>The vector of singular values of an operator $A$.</td>
<td>28</td>
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<tr>
<td>$s_k(A)$</td>
<td>The $k$-th largest singular value of an operator $A$.</td>
<td>28</td>
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<tr>
<td>$A^+$</td>
<td>The Moore–Penrose pseudo-inverse of an operator $A$.</td>
<td>30</td>
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<tr>
<td>$|A|<em>p, |A|</em>\infty$</td>
<td>The Schatten $p$-norm or $\infty$-norm of an operator $A$.</td>
<td>32</td>
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<tr>
<td>$|A|$</td>
<td>The spectral norm of an operator $A$. Equivalent to the Schatten $\infty$-norm of $A$.</td>
<td>33</td>
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<tr>
<td>$|A|_2$</td>
<td>The Frobenius norm of an operator $A$. Equivalent to the Schatten 2-norm of $A$.</td>
<td>33</td>
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<tr>
<td>$|A|_1$</td>
<td>The trace norm of an operator $A$. Equivalent to the Schatten 1-norm of $A$.</td>
<td>34</td>
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<tr>
<td>$\nabla f(x)$</td>
<td>The gradient vector of a function $f : \mathbb{R}^n \to \mathbb{R}$ at a vector $x \in \mathbb{R}^n$.</td>
<td>37</td>
</tr>
<tr>
<td>$(Df)(x)$</td>
<td>The derivative of a (differentiable) function $f : \mathbb{R}^n \to \mathbb{R}$ at a vector $x \in \mathbb{R}^n$.</td>
<td>37</td>
</tr>
<tr>
<td>$\mathcal{B}(\mathcal{X})$</td>
<td>The unit ball in a complex Euclidean space $\mathcal{X}$.</td>
<td>38</td>
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<tr>
<td>$\text{Borel}(\mathcal{A})$</td>
<td>The collection of all Borel subsets of a subset $\mathcal{A}$ of a real or complex vector space.</td>
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<tr>
<td>( \int f(x) , d\mu(x) )</td>
<td>The integral of a function ( f ) with respect to a Borel measure ( \mu ).</td>
<td>40</td>
</tr>
<tr>
<td>cone(( A ))</td>
<td>The cone generated by a subset ( A ) of a real or complex vector space.</td>
<td>43</td>
</tr>
<tr>
<td>( \mathcal{P}(\Sigma) )</td>
<td>The set of probability vectors with entries indexed by an alphabet ( \Sigma ).</td>
<td>44</td>
</tr>
<tr>
<td>conv(( A ))</td>
<td>The convex hull of a subset ( A ) of a real or complex vector space.</td>
<td>44</td>
</tr>
<tr>
<td>( E(X) )</td>
<td>The expected value of a random variable ( X ).</td>
<td>48</td>
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<tr>
<td>( \Gamma(\alpha) )</td>
<td>The value of the ( \Gamma )-function at ( \alpha ).</td>
<td>52</td>
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<tr>
<td>( \gamma_n )</td>
<td>The standard Gaussian measure on ( \mathbb{R}^n ).</td>
<td>52</td>
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<tr>
<td>( X, Y, Z )</td>
<td>Typical names for registers.</td>
<td>59</td>
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<tr>
<td>((X_1, \ldots, X_n))</td>
<td>The compound register formed from registers ( X_1, \ldots, X_n ).</td>
<td>59</td>
</tr>
<tr>
<td>( \omega_\mathcal{V} )</td>
<td>The flat state proportional to the projection onto the subspace ( \mathcal{V} ).</td>
<td>64</td>
</tr>
<tr>
<td>( \rho[X_1, \ldots, X_n] )</td>
<td>The reduction of a state ( \rho ) to registers ( X_1, \ldots, X_n ).</td>
<td>69</td>
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<tr>
<td>( C(\mathcal{X}, \mathcal{Y}) )</td>
<td>The set of all channels mapping ( \mathcal{L}(\mathcal{X}) ) to ( \mathcal{L}(\mathcal{Y}) ).</td>
<td>73</td>
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<tr>
<td>( C(\mathcal{X}) )</td>
<td>The set of channels mapping ( \mathcal{L}(\mathcal{X}) ) to itself.</td>
<td>73</td>
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<tr>
<td>( K(\Phi) )</td>
<td>The natural representation of a map ( \Phi ).</td>
<td>77</td>
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<tr>
<td>( J(\Phi) )</td>
<td>The Choi representation of a map ( \Phi ).</td>
<td>78</td>
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<tr>
<td>( \Omega ) or ( \Omega_{\mathcal{X}} )</td>
<td>Typical name for the completely depolarizing channel acting on ( \mathcal{L}(\mathcal{X}) ).</td>
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<tr>
<td>( \Delta ) or ( \Delta_{\mathcal{X}} )</td>
<td>Typical name for the completely dephasing channel acting on ( \mathcal{L}(\mathcal{X}) ).</td>
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<tr>
<td>F(P,Q)</td>
<td>The fidelity between positive semidefinite operators P and Q.</td>
<td>139</td>
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<tr>
<td>B(P,Q</td>
<td>µ)</td>
<td>The Bhattacharyya coefficient of the nonnegative real vectors obtained by applying a measurement µ to positive semidefinite operators P and Q.</td>
</tr>
<tr>
<td>F(Φ,P)</td>
<td>The mapping fidelity of a map Φ with respect to a positive semidefinite operator P.</td>
<td>160</td>
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<tr>
<td>W or WX</td>
<td>Typical name used to refer to the swap operator acting on a bipartite tensor product space X⊗X.</td>
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<td></td>
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<tr>
<td></td>
<td>The completely bounded trace norm of a map Φ.</td>
<td>170</td>
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<tr>
<td>N(X)</td>
<td>The numerical range of a square operator X.</td>
<td>180</td>
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<tr>
<td>Fmax(Ψ0,Ψ1)</td>
<td>The maximum output fidelity of positive maps Ψ0 and Ψ1.</td>
<td>185</td>
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<tr>
<td>Zn</td>
<td>The ring of integers modulo n.</td>
<td>212</td>
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<tr>
<td>Wa,b</td>
<td>A discrete Weyl operator acting on C^Zn, for a,b ∈ Zn.</td>
<td>212</td>
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<tr>
<td>σx, σy, and σz</td>
<td>The Pauli operators.</td>
<td>213</td>
</tr>
<tr>
<td>A ⊗ B</td>
<td>The entry-wise product of operators A and B.</td>
<td>219</td>
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<tr>
<td>Vπ</td>
<td>Permutation operator corresponding to the permutation π.</td>
<td>234</td>
</tr>
<tr>
<td>v ≺ u</td>
<td>Indicates that u majorizes v, for real vectors u and v.</td>
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</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
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<tr>
<td>$r(u)$</td>
<td>The vector obtained by sorting the entries of a real vector $u$ from largest to smallest.</td>
<td>236</td>
</tr>
<tr>
<td>$r_k(u)$</td>
<td>The $k$-th largest entry of a real vector $u$.</td>
<td>236</td>
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<tr>
<td>$Y \prec X$</td>
<td>Indicates that $X$ majorizes $Y$, for Hermitian operators $X$ and $Y$.</td>
<td>241</td>
</tr>
<tr>
<td>$S_n$</td>
<td>The symmetric group on $n$ symbols, equivalent to $\text{Sym} {1, \ldots, n}$.</td>
<td>243</td>
</tr>
<tr>
<td>$H(u)$</td>
<td>The Shannon entropy of a vector $u$ with nonnegative real number entries.</td>
<td>251</td>
</tr>
<tr>
<td>$H(X)$</td>
<td>The Shannon entropy of the probabilistic state of a classical register $X$, or the von Neumann entropy of the quantum state of a register $X$.</td>
<td>252, 266</td>
</tr>
<tr>
<td>$H(X_1, \ldots, X_n)$</td>
<td>Refers to the Shannon entropy or von Neumann entropy of the compound register $(X_1, \ldots, X_n)$.</td>
<td>252, 266</td>
</tr>
<tr>
<td>$D(u|v)$</td>
<td>The relative entropy of $u$ with respect to $v$, for vectors $u$ and $v$ with nonnegative real number entries.</td>
<td>252</td>
</tr>
<tr>
<td>$H(X</td>
<td>Y)$</td>
<td>The conditional Shannon entropy or von Neumann entropy of a register $X$ with respect to a register $Y$.</td>
</tr>
<tr>
<td>$I(X : Y)$</td>
<td>The mutual information or quantum mutual information between registers $X$ and $Y$.</td>
<td>253, 267</td>
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<tr>
<td>$H(P)$</td>
<td>The von Neumann entropy of a positive semidefinite operator $P$.</td>
<td>265</td>
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<tr>
<td>$D(P|Q)$</td>
<td>The quantum relative entropy of $P$ with respect to $Q$, for positive semidefinite operators $P$ and $Q$.</td>
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<tr>
<td>$T_{n,\varepsilon}(p)$</td>
<td>The set of $\varepsilon$-typical strings of length $n$ with respect to the probability vector $p$.</td>
<td>286</td>
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<tr>
<td>$\Pi_{n,\varepsilon}$</td>
<td>Projection operator corresponding to the $\varepsilon$-typical subspace of $\mathcal{X}^\otimes n$ with respect to a given state.</td>
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<tr>
<td>$I_{\text{acc}}(\eta)$</td>
<td>The accessible information of an ensemble $\eta$.</td>
<td>295</td>
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<tr>
<td>$\chi(\eta)$</td>
<td>The Holevo information of an ensemble $\eta$.</td>
<td>297</td>
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<tr>
<td>$\text{Sep}(\mathcal{X} : \mathcal{Y})$</td>
<td>The set of separable operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.</td>
<td>311</td>
</tr>
<tr>
<td>$\text{SepD}(\mathcal{X} : \mathcal{Y})$</td>
<td>The set of separable density operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.</td>
<td>311</td>
</tr>
<tr>
<td>$\text{Ent}_r(\mathcal{X} : \mathcal{Y})$</td>
<td>The set of operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ having entanglement rank bounded by $r$, with respect to the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.</td>
<td>322</td>
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<tr>
<td>$\text{SepCP}(\mathcal{X} , \mathcal{Z} : \mathcal{Y} , \mathcal{W})$</td>
<td>The set of separable maps from $\text{L}(\mathcal{X} \otimes \mathcal{Y})$ to $\text{L}(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$.</td>
<td>325</td>
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<tr>
<td>$\text{SepC}(\mathcal{X} , \mathcal{Z} : \mathcal{Y} , \mathcal{W})$</td>
<td>The set of separable channels from $\text{L}(\mathcal{X} \otimes \mathcal{Y})$ to $\text{L}(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$.</td>
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</tr>
<tr>
<td>$\text{LOCC}(\mathcal{X} , \mathcal{Z} : \mathcal{Y} , \mathcal{W})$</td>
<td>The set of LOCC channels from $\text{L}(\mathcal{X} \otimes \mathcal{Y})$ to $\text{L}(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$.</td>
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</tr>
<tr>
<td>$E_D(\mathcal{X} : \mathcal{Y})$</td>
<td>The distillable entanglement of the state of a pair of registers $(\mathcal{X}, \mathcal{Y})$.</td>
<td>347</td>
</tr>
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**Index of Symbols**

- **$E_C(X : Y)$**  
The entanglement cost of the state of a pair of registers $(X, Y)$.  

- **$\text{PPT}(\mathcal{X} : \mathcal{Y})$**  
The set of PPT operators acting on $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$.  

- **$E_f(X : Y)$**  
The entanglement of formation of the state of a pair of registers $(X, Y)$.  

- **$W_\pi$**  
A unitary operator acting on $\mathcal{X}^\otimes n$, for a complex Euclidean space $\mathcal{X}$, that permutes tensor factors according to the permutation $\pi$.  

- **$\mathcal{X}^\otimes n$**  
The symmetric subspace of $\mathcal{X}^\otimes n$, for $\mathcal{X}$ a complex Euclidean space. Also denoted $\mathcal{X}_1 \otimes \cdots \otimes \mathcal{X}_n$ when $\mathcal{X}_1, \ldots, \mathcal{X}_n$ are identical copies of $\mathcal{X}$.  

- **$\text{Bag}(n, \Sigma)$**  
The set of functions describing a bag of $n$ items, each labeled by an element of an alphabet $\Sigma$.  

- **$\mathbb{N}$**  
The set of nonnegative integers $\{0, 1, 2, \ldots\}$.  

- **$\Sigma^n_\phi$**  
The subset of $\Sigma^n$ consistent with a given function $\phi \in \text{Bag}(n, \Sigma)$.  

- **$\mathcal{X}^\otimes n$**  
The anti-symmetric subspace of $\mathcal{X}^\otimes n$, for $\mathcal{X}$ a complex Euclidean space.  

- **$L(\mathcal{X})^\otimes n$**  
The algebra of permutation-invariant operators acting on $\mathcal{X}^\otimes n$, for $\mathcal{X}$ a complex Euclidean space.  

- **$\mu$**  
Symbol used to denote uniform spherical measure.  

- **$\eta$**  
Symbol used to denote Haar measure.
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<tr>
<th>Symbol</th>
<th>Description</th>
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<td>$H_{\text{min}}(\Phi)$</td>
<td>The minimum output entropy of a channel $\Phi$.</td>
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<td>$C(\Phi)$</td>
<td>The classical capacity of a channel $\Phi$.</td>
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<tr>
<td>$C_{E}(\Phi)$</td>
<td>The entanglement-assisted classical capacity of a channel $\Phi$.</td>
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<tr>
<td>$\chi(\Phi)$</td>
<td>The Holevo capacity of a channel $\Phi$.</td>
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<tr>
<td>$\chi_{E}(\Phi)$</td>
<td>The entanglement-assisted Holevo capacity of a channel $\Phi$.</td>
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</tr>
<tr>
<td>$I_{c}(\rho;\Phi)$</td>
<td>The coherent information of a state $\rho$ through a channel $\Phi$.</td>
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</tr>
<tr>
<td>$I_{c}(\Phi)$</td>
<td>The maximum coherent information of a channel $\Phi$.</td>
<td>474</td>
</tr>
<tr>
<td>$K_{a_1 \ldots a_n,\epsilon}(p)$</td>
<td>The set of $\epsilon$-typical strings of length $n$, conditioned on a string $a_1 \cdots a_n$, with respect to the probability vector $p$.</td>
<td>479</td>
</tr>
<tr>
<td>$\Lambda_{a_1 \ldots a_n,\epsilon}$</td>
<td>Projection onto the $\epsilon$-typical subspace of $X^\otimes n$, for $X$ a complex Euclidean space, conditioned on a string $a_1 \cdots a_n$.</td>
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</tr>
<tr>
<td>$S_{n,\epsilon}(p)$</td>
<td>The set of $\epsilon$-strongly typical strings of length $n$ with respect to the probability vector $p$.</td>
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<tr>
<td>$Q(\Phi)$</td>
<td>The quantum capacity of a channel $\Phi$.</td>
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<td>$Q_{E,G}(\Phi)$</td>
<td>The entanglement generation capacity of a channel $\Phi$.</td>
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<tr>
<td>$Q_{E}(\Phi)$</td>
<td>The entanglement-assisted quantum capacity of a channel $\Phi$.</td>
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</tr>
<tr>
<td>$\Phi_0 \oplus \Phi_1$</td>
<td>The direct sum of maps $\Phi_0$ and $\Phi_1$.</td>
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