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## Index of Symbols

$\Sigma, \Gamma, \Lambda$	Typical names for alphabets (finite and nonempty sets whose elements are viewed as symbols).	1
$\mathbb{C}^\Sigma$	The complex Euclidean space of functions from an alphabet $\Sigma$ to the complex numbers. (Equivalently, the complex Euclidean space of vectors having entries indexed by $\Sigma$ .)	2
$\mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z}$	Typical names for complex Euclidean spaces.	2
$\langle u, v \rangle$	The inner product between vectors $u$ and $v$ .	3
$\ u\ $	The Euclidean norm of a vector $u$ .	4
$\mathcal{S}(\mathcal{X})$	The unit sphere in a complex Euclidean space $\mathcal{X}$ .	4
$\ u\ _p$	The $p$ -norm of a vector $u$ .	4
$\ u\ _\infty$	The $\infty$ -norm of a vector $u$ .	4
$u \perp v, u \perp \mathcal{A}$	Indicates that a vector $u$ is orthogonal to a vector $v$ , or to every element of a set of vectors $\mathcal{A}$ .	4
$e_a$	An element of the vector standard basis, corresponding to a symbol (or index) $a$ .	5

$\Sigma_1 \sqcup \cdots \sqcup \Sigma_n$	The disjoint union of alphabets $\Sigma_1, \dots, \Sigma_n$ .	5
$\mathcal{X}_1 \oplus \cdots \oplus \mathcal{X}_n$	The direct sum of complex Euclidean spaces $\mathcal{X}_1, \dots, \mathcal{X}_n$ .	5
$u_1 \oplus \cdots \oplus u_n$	The direct sum of vectors $u_1, \dots, u_n$ .	5
$\mathcal{X}_1 \otimes \cdots \otimes \mathcal{X}_n$	The tensor product of complex Euclidean spaces $\mathcal{X}_1, \dots, \mathcal{X}_n$ .	6
$u_1 \otimes \cdots \otimes u_n$	The tensor product of vectors $u_1, \dots, u_n$ .	6
$\mathcal{X}^{\otimes n}$	The $n$ -fold tensor product of a complex Euclidean space $\mathcal{X}$ with itself.	7
$u^{\otimes n}$	The $n$ -fold tensor product of a vector $u$ with itself.	7
$\mathbb{R}^\Sigma$	The real Euclidean space of functions from an alphabet $\Sigma$ to the real numbers. (Equivalently, the real Euclidean space of vectors having entries indexed by $\Sigma$ .)	7
$L(\mathcal{X}, \mathcal{Y})$	Space of all linear operators mapping a complex Euclidean space $\mathcal{X}$ to a complex Euclidean space $\mathcal{Y}$ .	8
$E_{a,b}$	An element of the operator standard basis, corresponding to symbols (or indices) $a$ and $b$ .	10
$\bar{A}, \bar{u}$	The entry-wise complex conjugate of an operator $A$ or a vector $u$ .	10
$A^\top, u^\top$	The transpose of an operator $A$ or a vector $u$ .	10
$A^*, u^*$	The adjoint of an operator $A$ or a vector $u$ .	11
$\ker(A)$	The kernel of an operator $A$ .	11
$\text{im}(A)$	The image of an operator $A$ .	11
$\text{rank}(A)$	The rank of an operator $A$ .	12

$A_1 \otimes \cdots \otimes A_n$	The tensor product of operators $A_1, \dots, A_n$ .	13
$A^{\otimes n}$	The $n$ -fold tensor product of an operator $A$ with itself.	14
$L(\mathcal{X})$	Space of linear operators mapping a complex Euclidean space $\mathcal{X}$ to itself.	14
$\mathbb{1}$	The identity operator; denoted $\mathbb{1}_{\mathcal{X}}$ when it is helpful to indicate that it acts on a complex Euclidean space $\mathcal{X}$ .	14
$X^{-1}$	The inverse of an invertible square operator $X$ .	14
$\text{Tr}(X)$	The trace of a square operator $X$ .	15
$\langle A, B \rangle$	The inner product of operators $A$ and $B$ .	15
$\text{Det}(X)$	The determinant of a square operator $X$ .	15
$\text{Sym}(\Sigma)$	The set of permutations, or bijective functions, of the form $\pi : \Sigma \rightarrow \Sigma$ .	15
$\text{sign}(\pi)$	The sign, or parity, of a permutation $\pi$ .	15
$\text{spec}(X)$	The spectrum of a square operator $X$ .	16
$[X, Y]$	The Lie bracket of square operators $X$ and $Y$ .	17
$\text{comm}(\mathcal{A})$	The commutant of a set $\mathcal{A}$ of square operators.	17
$\text{Herm}(\mathcal{X})$	The set of Hermitian operators acting on a complex Euclidean space $\mathcal{X}$ .	17
$\text{Pos}(\mathcal{X})$	The set of positive semidefinite operators acting on a complex Euclidean space $\mathcal{X}$ .	17
$\text{Pd}(\mathcal{X})$	The set of positive definite operators acting on a complex Euclidean space $\mathcal{X}$ .	18

$D(\mathcal{X})$	The set of density operators acting on a complex Euclidean space $\mathcal{X}$ .	18
$\text{Proj}(\mathcal{X})$	The set of projection operators acting on a complex Euclidean space $\mathcal{X}$ .	18
$\Pi_{\mathcal{V}}$	The projection operator whose image is $\mathcal{V}$ .	18
$U(\mathcal{X}, \mathcal{Y})$	The set of isometries mapping a complex Euclidean space $\mathcal{X}$ to a complex Euclidean space $\mathcal{Y}$ .	18
$U(\mathcal{X})$	The set of unitary operators acting on a complex Euclidean space $\mathcal{X}$ .	18
$\text{Diag}(u)$	The diagonal square operator whose diagonal entries are described by the vector $u$ .	19
$\lambda(H)$	The vector of eigenvalues of a Hermitian operator $H$ .	20
$\lambda_k(H)$	The $k$ -th largest eigenvalue of a Hermitian operator $H$ .	20
$X \geq Y$ or $Y \leq X$	Indicates that $X - Y$ is positive semidefinite, for Hermitian operators $X$ and $Y$ .	21
$X > Y$ or $Y < X$	Indicates that $X - Y$ is positive definite, for Hermitian operators $X$ and $Y$ .	21
$T(\mathcal{X}, \mathcal{Y})$	The space of linear maps from $L(\mathcal{X})$ to $L(\mathcal{Y})$ , for complex Euclidean spaces $\mathcal{X}$ and $\mathcal{Y}$ .	21
$\Phi^*$	The adjoint of a map $\Phi \in T(\mathcal{X}, \mathcal{Y})$ .	21
$\Phi_1 \otimes \cdots \otimes \Phi_n$	The tensor product of maps $\Phi_1, \dots, \Phi_n$ .	22
$\Phi^{\otimes n}$	The $n$ -fold tensor product of a map $\Phi$ with itself.	22
$\mathbb{1}_{L(\mathcal{X})}$	The identity map acting on $L(\mathcal{X})$ .	22



$\text{Tr}_{\mathcal{X}}$	The partial trace over a complex Euclidean space $\mathcal{X}$ .	22
$\text{CP}(\mathcal{X}, \mathcal{Y})$	The set of completely positive maps of the form $\Phi \in \mathbb{T}(\mathcal{X}, \mathcal{Y})$ .	23
$\text{vec}(A)$	The vec mapping applied to an operator $A$ .	23
$\sqrt{P}$	The square root of a positive semidefinite operator $P$ .	27
$s(A)$	The vector of singular values of an operator $A$ .	28
$s_k(A)$	The $k$ -th largest singular value of an operator $A$ .	28
$A^+$	The Moore–Penrose pseudo-inverse of an operator $A$ .	30
$\ A\ _p, \ A\ _\infty$	The Schatten $p$ -norm or $\infty$ -norm of an operator $A$ .	32
$\ A\ $	The spectral norm of an operator $A$ . Equivalent to the Schatten $\infty$ -norm of $A$ .	33
$\ A\ _2$	The Frobenius norm of an operator $A$ . Equivalent to the Schatten 2-norm of $A$ .	33
$\ A\ _1$	The trace norm of an operator $A$ . Equivalent to the Schatten 1-norm of $A$ .	34
$\nabla f(x)$	The gradient vector of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at a vector $x \in \mathbb{R}^n$ .	37
$(Df)(x)$	The derivative of a (differentiable) function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at a vector $x \in \mathbb{R}^n$ .	37
$\mathcal{B}(\mathcal{X})$	The unit ball in a complex Euclidean space $\mathcal{X}$ .	38
$\text{Borel}(\mathcal{A})$	The collection of all Borel subsets of a subset $\mathcal{A}$ of a real or complex vector space.	38

$\int f(x) \, d\mu(x)$	The integral of a function $f$ with respect to a Borel measure $\mu$ .	40
$\text{cone}(\mathcal{A})$	The cone generated by a subset $\mathcal{A}$ of a real or complex vector space.	43
$\mathcal{P}(\Sigma)$	The set of probability vectors with entries indexed by an alphabet $\Sigma$ .	44
$\text{conv}(\mathcal{A})$	The convex hull of a subset $\mathcal{A}$ of a real or complex vector space.	44
$E(X)$	The expected value of a random variable $X$ .	48
$\Gamma(\alpha)$	The value of the $\Gamma$ -function at $\alpha$ .	52
$\gamma_n$	The standard Gaussian measure on $\mathbb{R}^n$ .	52
$X, Y, Z$	Typical names for registers.	59
$(X_1, \dots, X_n)$	The compound register formed from registers $X_1, \dots, X_n$ .	59
$\omega_{\mathcal{V}}$	The flat state proportional to the projection onto the subspace $\mathcal{V}$ .	64
$\rho[X_1, \dots, X_n]$	The reduction of a state $\rho$ to registers $X_1, \dots, X_n$ .	69
$C(\mathcal{X}, \mathcal{Y})$	The set of all channels mapping $L(\mathcal{X})$ to $L(\mathcal{Y})$ .	73
$C(\mathcal{X})$	The set of channels mapping $L(\mathcal{X})$ to itself.	73
$K(\Phi)$	The natural representation of a map $\Phi$ .	77
$J(\Phi)$	The Choi representation of a map $\Phi$ .	78
$\Omega$ or $\Omega_{\mathcal{X}}$	Typical name for the completely depolarizing channel acting on $L(\mathcal{X})$ .	93
$\Delta$ or $\Delta_{\mathcal{X}}$	Typical name for the completely dephasing channel acting on $L(\mathcal{X})$ .	94

$F(P, Q)$	The fidelity between positive semidefinite operators $P$ and $Q$ .	139
$B(P, Q \mu)$	The Bhattacharyya coefficient of the nonnegative real vectors obtained by applying a measurement $\mu$ to positive semidefinite operators $P$ and $Q$ .	153
$F(\Phi, P)$	The mapping fidelity of a map $\Phi$ with respect to a positive semidefinite operator $P$ .	160
$W$ or $W_{\mathcal{X}}$	Typical name used to refer to the swap operator acting on a bipartite tensor product space $\mathcal{X} \otimes \mathcal{X}$ .	165
$\ \Phi\ _1$	The induced trace norm of a map $\Phi$ .	167
$\ \ \Phi\ \ _1$	The completely bounded trace norm of a map $\Phi$ .	170
$\mathcal{N}(X)$	The numerical range of a square operator $X$ .	180
$F_{\max}(\Psi_0, \Psi_1)$	The maximum output fidelity of positive maps $\Psi_0$ and $\Psi_1$ .	185
$\mathbb{Z}_n$	The ring of integers modulo $n$ .	212
$W_{a,b}$	A discrete Weyl operator acting on $\mathbb{C}^{\mathbb{Z}_n}$ , for $a, b \in \mathbb{Z}_n$ .	212
$\sigma_x, \sigma_y$ , and $\sigma_z$	The Pauli operators.	213
$A \odot B$	The entry-wise product of operators $A$ and $B$ .	219
$V_\pi$	Permutation operator corresponding to the permutation $\pi$ .	234
$v \prec u$	Indicates that $u$ majorizes $v$ , for real vectors $u$ and $v$ .	235

$r(u)$	The vector obtained by sorting the entries of a real vector $u$ from largest to smallest.	236
$r_k(u)$	The $k$ -th largest entry of a real vector $u$ .	236
$Y \prec X$	Indicates that $X$ majorizes $Y$ , for Hermitian operators $X$ and $Y$ .	241
$S_n$	The symmetric group on $n$ symbols, equivalent to $\text{Sym}(\{1, \dots, n\})$ .	243
$H(u)$	The Shannon entropy of a vector $u$ with nonnegative real number entries.	251
$H(\mathbf{X})$	The Shannon entropy of the probabilistic state of a classical register $\mathbf{X}$ , or the von Neumann entropy of the quantum state of a register $\mathbf{X}$ .	252, 266
$H(\mathbf{X}_1, \dots, \mathbf{X}_n)$	Refers to the Shannon entropy or von Neumann entropy of the compound register $(\mathbf{X}_1, \dots, \mathbf{X}_n)$ .	252, 266
$D(u  v)$	The relative entropy of $u$ with respect to $v$ , for vectors $u$ and $v$ with nonnegative real number entries.	252
$H(\mathbf{X} \mathbf{Y})$	The conditional Shannon entropy or von Neumann entropy of a register $\mathbf{X}$ with respect to a register $\mathbf{Y}$ .	252, 267
$I(\mathbf{X} : \mathbf{Y})$	The mutual information or quantum mutual information between registers $\mathbf{X}$ and $\mathbf{Y}$ .	253, 267
$H(P)$	The von Neumann entropy of a positive semidefinite operator $P$ .	265
$D(P  Q)$	The quantum relative entropy of $P$ with respect to $Q$ , for positive semidefinite operators $P$ and $Q$ .	266

$T_{n,\varepsilon}(p)$	The set of $\varepsilon$ -typical strings of length $n$ with respect to the probability vector $p$ .	286
$\Pi_{n,\varepsilon}$	Projection operator corresponding to the $\varepsilon$ -typical subspace of $\mathcal{X}^{\otimes n}$ with respect to a given state.	291
$I_{\text{acc}}(\eta)$	The accessible information of an ensemble $\eta$ .	295
$\chi(\eta)$	The Holevo information of an ensemble $\eta$ .	297
$\text{Sep}(\mathcal{X} : \mathcal{Y})$	The set of separable operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ , respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ .	311
$\text{SepD}(\mathcal{X} : \mathcal{Y})$	The set of separable density operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ , respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ .	311
$\text{Ent}_r(\mathcal{X} : \mathcal{Y})$	The set of operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ having entanglement rank bounded by $r$ , with respect to the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ .	322
$\text{SepCP}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$	The set of separable maps from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$ , respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$ .	325
$\text{SepC}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$	The set of separable channels from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$ , respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$ .	326
$\text{LOCC}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$	The set of LOCC channels from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$ , respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ and between $\mathcal{Z}$ and $\mathcal{W}$ .	330
$E_{\text{D}}(\mathbf{X} : \mathbf{Y})$	The distillable entanglement of the state of a pair of registers $(\mathbf{X}, \mathbf{Y})$ .	347

$E_c(\mathbf{X} : \mathbf{Y})$	The entanglement cost of the state of a pair of registers $(\mathbf{X}, \mathbf{Y})$ .	347
$\text{PPT}(\mathcal{X} : \mathcal{Y})$	The set of PPT operators acting on $\mathcal{X} \otimes \mathcal{Y}$ , respecting the bipartition between $\mathcal{X}$ and $\mathcal{Y}$ .	353
$E_F(\mathbf{X} : \mathbf{Y})$	The entanglement of formation of the state of a pair of registers $(\mathbf{X}, \mathbf{Y})$ .	385
$W_\pi$	A unitary operator acting on $\mathcal{X}^{\otimes n}$ , for a complex Euclidean space $\mathcal{X}$ , that permutes tensor factors according to the permutation $\pi$ .	391
$\mathcal{X}^{\odot n}$	The symmetric subspace of $\mathcal{X}^{\otimes n}$ , for $\mathcal{X}$ a complex Euclidean space. Also denoted $\mathcal{X}_1 \odot \cdots \odot \mathcal{X}_n$ when $\mathcal{X}_1, \dots, \mathcal{X}_n$ are identical copies of $\mathcal{X}$ .	392
$\text{Bag}(n, \Sigma)$	The set of functions describing a bag of $n$ items, each labeled by an element of an alphabet $\Sigma$ .	393
$\mathbb{N}$	The set of nonnegative integers $\{0, 1, 2, \dots\}$ .	393
$\Sigma_\phi^n$	The subset of $\Sigma^n$ consistent with a given function $\phi \in \text{Bag}(n, \Sigma)$ .	393
$\mathcal{X}^{\otimes n}$	The anti-symmetric subspace of $\mathcal{X}^{\otimes n}$ , for $\mathcal{X}$ a complex Euclidean space.	398
$L(\mathcal{X})^{\otimes n}$	The algebra of permutation-invariant operators acting on $\mathcal{X}^{\otimes n}$ , for $\mathcal{X}$ a complex Euclidean space.	400
$\mu$	Symbol used to denote uniform spherical measure.	408
$\eta$	Symbol used to denote Haar measure.	411

$H_{\min}(\Phi)$	The minimum output entropy of a channel $\Phi$ .	451
$C(\Phi)$	The classical capacity of a channel $\Phi$ .	466
$C_{\text{E}}(\Phi)$	The entanglement-assisted classical capacity of a channel $\Phi$ .	469
$\chi(\Phi)$	The Holevo capacity of a channel $\Phi$ .	470
$\chi_{\text{E}}(\Phi)$	The entanglement-assisted Holevo capacity of a channel $\Phi$ .	474
$I_{\text{C}}(\rho; \Phi)$	The coherent information of a state $\rho$ through a channel $\Phi$ .	474
$I_{\text{C}}(\Phi)$	The maximum coherent information of a channel $\Phi$ .	474
$K_{a_1 \cdots a_n, \varepsilon}(p)$	The set of $\varepsilon$ -typical strings of length $n$ , conditioned on a string $a_1 \cdots a_n$ , with respect to the probability vector $p$ .	479
$\Lambda_{a_1 \cdots a_n, \varepsilon}$	Projection onto the $\varepsilon$ -typical subspace of $\mathcal{X}^{\otimes n}$ , for $\mathcal{X}$ a complex Euclidean space, conditioned on a string $a_1 \cdots a_n$ .	481
$S_{n, \varepsilon}(p)$	The set of $\varepsilon$ -strongly typical strings of length $n$ with respect to the probability vector $p$ .	498
$Q(\Phi)$	The quantum capacity of a channel $\Phi$ .	513
$Q_{\text{EG}}(\Phi)$	The entanglement generation capacity of a channel $\Phi$ .	514
$Q_{\text{E}}(\Phi)$	The entanglement-assisted quantum capacity of a channel $\Phi$ .	519
$\Phi_0 \oplus \Phi_1$	The direct sum of maps $\Phi_0$ and $\Phi_1$ .	540

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