Course Project

You are required to complete a project for this course. For the project, you are to choose a theorem having a connection with the theory of quantum information, and prepare a write-up containing a statement and proof of that theorem.

Guidelines

Please make note of the following guidelines, which are intended to guide your choice of a topic as well as the write-up you will prepare.

- The focus of your write-up should be on a theorem and its proof. You should also include relevant definitions and a short summary of whatever background information is needed to understand the statement of your theorem and its proof. (You can feel free to use anything standard within quantum information and computation, and anything from the book associated with the course, without having to explain it.)

If you wish to cover multiple theorems that fall under a common theme or rely on a similar technique, this is fine, but for most projects a single, main theorem is anticipated. If you wish to discuss the motivation or implications of your chosen theorem, please feel free to do this as well—but for this particular project those things will be considered as being secondary.

- There is no length requirement or restriction for the write-up; I am interested in the quality of the content of your write-up, not the quantity. If you turn in a short, brilliant proof of a theorem that was not previously known to have such a proof, that would likely be an outstanding project—but you should not aim to simply compress a known proof in an effort to make it artificially seem simpler. On the other hand, if you feel that you need a large number of pages to carefully explain the statement and proof of your theorem, that is fine too.

- You are free to choose the intended audience of your write-up: beginning or advanced graduate students, researchers focusing on a particular topic, mathematicians, physicists, computer scientists, engineers, etc. You are also free to use whatever notation and assume whatever background knowledge you choose, but (as suggested above) be sure to briefly summarize whatever it is that you are assuming.

- Of course you must provide references to relevant papers, books, and other sources that your write-up is based on.

- Please do not choose a theorem proved either by your supervisor or by me. (It will not be hard at all to avoid my theorems, few as they are!) The theorem you pick must also not be proved in the course book.

- You may not work with anyone else: this will be your project alone. I also prefer that each project is different, so please do not knowingly choose the same theorem as someone else in the class.

- I expect a carefully prepared and type-set write-up that I hope you will take pride in. (For most people this would not need to be said—but after someone gave me a hand-written project two years ago, ripped out of a notebook, I thought it might be worth saying.)
Goals

There are various goals you may strive for in your write-up. Some may be possible and others not realistic depending on the theorem you choose. Some goals to keep in mind are as follows:

- You should certainly try to make the statement and proof of the theorem you choose as clear and understandable as possible.

- If you are able to simplify the known proof (or proofs) of the theorem you choose, or even come up with a completely different proof, this would be great, but it is not expected.

- If you can extend the known proof of your chosen theorem to obtain a more general theorem statement than was previously known, this would also be great.

- Possibly the known proof or proofs of the theorem you choose are very tersely or poorly written, intended for a highly advanced audience, or written for a completely different audience altogether. In such a case, you might aim to expand the proof, filling in missing details and explaining particular steps in greater detail, or to translate the proof into a style that is more understandable to a typical quantum information theorist.

- The known proof or proofs of the theorem you choose may be complicated due to a generality that has secondary importance in the setting of quantum information theory. In such a case, you might be able to come up with a simpler proof of a less general statement that is still interesting within quantum information theory.

Submissions and time-line

- By Thursday, November 16, please submit a statement of the theorem you are proposing to cover, a citation to the primary sources that you will be using, and (if necessary) any other comments regarding your choice that you feel are relevant.

  The purpose of this part of the submission is to allow me to verify that you have chosen a suitable theorem and to give me a chance to offer any feedback that might be helpful.

- Your final write-up is due on Monday, December 18.

Sample Project Topics

The suggestions listed on the pages that follow are meant to provide possible starting points to help you to choose a topic for your project. There is a single, main theorem associated with many (but not all) of these suggestions; in some cases there is a topic that may offer multiple possibilities for a theorem to cover. Note that, in some cases, there may be follow-up work to the papers mentioned below that may be relevant to projects on their topics—so be sure to do a reverse citation look-up (e.g., check Google Scholar to see which papers cite the listed papers you are interested in).

You are not required to choose one of these topics. If you have an interest in a different topic or theorem, please feel free to propose it (provided it is related to the theory of quantum information). You should also feel free to speak with me if there is a particular topic that interests you, but you are not sure what theorem concerning that topic might be appropriate, or for any other reason connected with your choice of a theorem.
1. Reduction criterion implies majorization.

2. Mutually unbiased bases exist in prime-power dimensions.

3. Falsification of the asymptotic quantum Birkhoff conjecture.


5. Faithful squashed entanglement.

6. Quantum de Finetti bounds by information-theoretic arguments.


8. The quantum substate theorem.

10. Unitarity plus causality implies localizability.

11. Continuity of quantum channel capacities.

12. Distilling secret keys from PPT states.


14. The entanglement cost of every entangled state is strictly positive.

15. Conditions for locally broadcasting of a quantum state.

16. Fundamental results concerning min- and max-entropy.

17. An uncertainty relation when a particle is entangled with a quantum memory.
18. The regularized entropy of entanglement is strictly positive for all entangled states.
   


20. Semidefinite programming bounds for distillable entanglement.

   


22. Semidefinite programming hierarchies for nonlocal correlations.

   
   

23. Approximately randomizing quantum channels.

   

24. Local hidden-variable theories for some entangled states.

   
25. Unbounded violation of tripartite Bell inequalities.

26. Results on unextendible product bases.

27. Conditions for the existence of a channel mapping one set of states to another.

28. Criterion for the perfect distinguishability of channels.