CS 360 Assignment 2

Due Date Thurs, Mar 1st, at the beginning of class.

All questions are worth the same amount. Please ensure that your name and student number appear, in ink, on each page of your assignment.

Work is to be done individually.

Assignment Questions

1. Let $\Sigma = \{a, b\}$. Show that $\{a^n w a^n \mid n \geq 1 \text{ and } w \in \Sigma^*\}$ is regular.
2. Let $\Sigma = \{a, b\}$. Show that $\{a^n b w a^n \mid n \geq 1 \text{ and } w \in \Sigma^*\}$ is not regular.
3. Let $\Sigma = \{0, 1, 2\}$, show that the language $\{0^i 1^j 2^k \mid i = 2j \text{ or } j = 2k\}$ is a context free language.
4. (See Sipser 2.9) Describe a PDA that recognizes the language $\{a^i b^j c^k \mid i = j \text{ or } j = k\}$.
5. (See Sipser 2.44) If A and B are regular languages show that $\{vw \mid v \in A, w \in B \text{ and } |v| = |w|\}$ is context free.
6. (From our Sipser text book, problem 2.19). Consider G given by the grammar:
   $S \rightarrow\ aSb \mid bY \mid Ya$
   $Y \rightarrow\ bY \mid aY \mid \epsilon$
   Give an English description of the language produced by G. Give a CFG of the complement language.
7. A string, w, of left and right parentheses is balanced, if $w = \epsilon$ or if there is some left parenthesis that can be matched to a right parenthesis on its immediate right, the two matching parentheses can then be removed to form w' and then w' is again balanced. If this removal process can be repeated until the string w is transformed into the empty string, then w is balanced. Show that if a string of parentheses is balanced then it is generated by the following grammar:
   $B \rightarrow BB \mid (B) \mid \epsilon$.