## CS 360 Assignment 1

Due Date Thurs, Feb 1st, at the beginning of class.

All questions are worth the same amount. Please ensure that your name and student number appear, in ink, on each page of your assignment.

Work is to be done individually.

## Assignment Questions

1. Consider the set of strings $A=\left\{w \in \Sigma^{*} \mid w\right.$ has an odd number of $a$ symbols and three $b$ symbols $\}$. $A$ can be seen as the intersection of two sets, one set with a restriction on the occurrence of $a$ and the other set with a restriction on the occurrence of $b$. Give DFAs for each of the two different sets and then construct a DFA that recognizes the intersection of the two languages, i.e., that recognizes $A$.
2. Question 1.21 from the course textbook. Consider the following DFA, with state set $\left\{q_{1}, q_{2}, q_{3}\right\}, \Sigma=\{a, b\}$, initial state $q_{1}$, and set of accept states $\left\{q_{1}, q_{2}\right\}$. The transition function is given as follows: $\delta\left(q_{1}, a\right)=q_{2}, \delta\left(q_{1}, b\right)=q_{2}, \delta\left(q_{2}, a\right)=q_{2}, \delta\left(q_{2}, b\right)=$ $q_{3}, \delta\left(q_{3}, a\right)=q_{1}, \delta\left(q_{3}, b\right)=q_{2}$. Give a regular expression that describes the same language as the automaton. Explain/justify your answer.
3. Problem 1.31 from the course textbook. Let $w=w_{1} w_{2} \ldots w_{n}$ be a string in $\Sigma^{*}$. Then the reverse of $w$ is the string $w^{R}=w_{n} w_{n-1} \ldots w_{1}$. Then if $A \subseteq \Sigma^{*}$ then $A^{R}=\left\{w^{R} \mid w \in\right.$ $A\}$. Show that if $A$ is a regular language then $A^{R}$ is a regular language.
4. Let $M=\left(Q, \Sigma, \delta, q_{0}, F\right)$ be a DFA where $Q$ contains $k$ states. Show that if $L(M) \neq \emptyset$ then $L(M)$ must contain at least one string of length at most $k-1$.

5 . Let $L$ be a regular language over the alphabet $\Sigma$. Describe a procedure to determine whether $L=\Sigma^{*}$.
6. Either prove or disprove the following statement about regular expressions $(R \circ S)^{*}=R^{*} \circ S^{*}$.
7. Show that the language $A=\left\{a^{k} \mid k\right.$ is a multiple of 3$\}$ is a regular language.

