CS 360 Assignment 4

Due Date Thursday, December 1st, at the beginning of class.

All questions are worth the same amount. Please ensure that your name and student number appear, in ink, on each page of your assignment.

Work is to be done individually.

Assignment Questions

1. Show that the set $S = \{(i, j) \mid i, j \text{ are Natural numbers}\}$ is countable.

2. Explain why a language $A \subseteq \Sigma^*$ is Turing-recognizable if and only if $A \leq_m A_{TM}$.
   Where $A_{TM} = \{< M, w > \mid M \text{ is a Turing machine and } M \text{ accepts } w\}$.

3. Consider the language $A = \{< R, S > \mid \text{ such that } R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$.
   Show that $A$ is decidable.

4. If $A = \{< M_1, M_2, k > \mid M_1, M_2 \text{ are TMs and } L(M_1) \cap L(M_2) \text{ contains at least } k \text{ strings}\}$.
   Show $A$ is Turing-recognizable.

5. Let $A = \{< M > \mid M \text{ is a TM that accepts } w^R \text{ if it accepts } w\}$. Show that $A$ is not decidable.