## Relational Databases

## - Basic concepts

- Data model: organize data as tables
- A relational database is a set of tables
- Advantages
- Simple concepts
- Solid mathematical foundation
- set theory
- Powerful query languages
- Efficient query optimization strategies
- Design theory
- Industry standard
- Relational model
- SQL language


## Relational Model

## Relation

- A relation $R$ with attributes $A=\left\{A_{1}, A_{2}, \ldots, A_{n}\right\}$ defined over $n$ domains $D=\left\{D_{1}, D_{2}, \ldots, D_{n}\right\}$ (not necessarily distinct) with values $\left\{\operatorname{Dom}_{1}, \operatorname{Dom}_{2}, \ldots, \operatorname{Dom}_{n}\right\}$ is a finite, time varying set of $n$-tuples $<d_{1}$, $d_{2}, \ldots, d_{n}>$ such that $d_{1} \in \operatorname{Dom}_{1}, d_{2} \in \operatorname{Dom}_{2}, \ldots, d_{n} \in \operatorname{Dom}_{n}$ and $A_{1}$ $\in D_{1}, A_{2} \in D_{2}, \ldots, A_{n} \in D_{n}$.
- Notation: $R\left(A_{1}, A_{2}, \ldots, A_{n}\right)$ or $R\left(A_{1}: D_{1}, A_{2}: D_{2}, \ldots, A_{n}: D_{n}\right)$
- Alternatively, given $R$ as defined above, an instance of it at a given time is a set of $n$-tuples:
$\left\{<A_{1}: d_{1}, A_{2}: d_{2}, \ldots, A_{n}: d_{n}>\mid d_{1} \in \operatorname{Dom}_{1}, d_{2} \in \operatorname{Dom}_{2}, \ldots, d_{n} \in \operatorname{Dom}_{n}\right\}$
- Tabular structure of data where
- $R$ is the table heading
- attributes are table columns
- each tuple is a row


## Relation Schemes and Instances

## - Relational scheme

- A relation scheme is the definition; i.e., a set of attributes
- A relational database scheme is a set of relation schemes:
Ime i.e., a set of sets of attributes
- Relation instance (simply relation)
- An relation is an instance of a relation scheme
- a relation $\mathbf{r}$ over a relation scheme $R=\left\{A_{1}, \ldots, A_{n}\right\}$ is a subset of the Cartesian product of the domains of all attributes, i.e.,

$$
\mathbf{r} \subseteq \operatorname{Dom}_{1} \times \operatorname{Dom}_{2} \times \ldots \times \operatorname{Dom}_{n}
$$

## Domains

$\square$ A domain is a type in the programming language sense

- Name: String
- Salary: Real

■ Domain values is a set of acceptable values for a variable of a given type.

- Name: $\mathrm{CdnNames}=\{\ldots\}$,
- Salary: ProfSalary $=\{45,000-150,000\}$
- Simple/Composite domains
- Address $=$ Street name + street number + city + province + postal code


## Domain compatibility

- Binary operations (e.g., comparison to one another, addition, etc) can be performed on them.
- Full support for domains is not provided in many current relational DBMSs


## Relation Schemes

EMP(ENO, ENAME, TITLE)<br>PROJ (PNO, PNAME, BUDGET) WORKS(ENO,PNO, RESP, DUR)<br>PAY(TITLE, SALARY)

| EMP |
| :--- |
| ENO ENAME TITLE |
| PROJ   <br> PNO PNAME BUDGET |
| WORKS |
| ENO |

■ Underlined attributes are relation keys (tuple identifiers).

PAY
IITE SALARY

■ Tabular form

## Different Representation



## Example Relation Instances

EMP

| ENO | ENAME | TITLE |
| :--- | :--- | :--- |
| E1 | J. Doe | Elect. Eng. |
| E2 | M. Smith | Syst. Anal. |
| E3 | A. Lee | Mech. Eng. |
| E4 | J. Miller | Programmer |
| E5 | B. Casey | Syst. Anal. |
| E6 | L. Chu | Elect. Eng. |
| E7 | R. Davis | Mech. Eng. |
| E8 | J. Jones | Syst. Anal. |

WORKS

| ENO | PNO | RESP | DUR |
| :---: | :---: | :--- | :---: |
| E1 | P1 | Manager | 12 |
| E2 | P1 | Analyst | 24 |
| E2 | P2 | Analyst | 6 |
| E3 | P3 | Consultant | 10 |
| E3 | P4 | Engineer | 48 |
| E4 | P2 | Programmer | 18 |
| E5 | P2 | Manager | 24 |
| E6 | P4 | Manager | 48 |
| E7 | P3 | Engineer | 36 |
| E7 | P5 | Engineer | 23 |
| E8 | P3 | Manager | 40 |

PROJ

| PNO | PNAME | BUDGET |
| :---: | :--- | :---: |
| P1 | Instrumentation | 150000 |
| P2 | Database Develop. | 135000 |
| P3 | CAD/CAM | 250000 |
| P4 | Maintenance | 310000 |
| P5 | CAD/CAM | 500000 |

PAY

| TITLE | SALARY |
| :--- | :---: |
| Elect. Eng. | 55000 |
| Syst. Anal. | 70000 |
| Mech. Eng. | 45000 |
| Programmer | 60000 |

## Properties

- Based on finite set theory
- No ordering among attributes
(10) Sometimes we prefer to refer to them by their relative order
- No ordering among tuples
(10) Query results may be ordered, but two differently ordered relation instances are equivalent
- No duplicate tuples allowed
- Commercial systems allow duplicates (so bag semantics)
- Value-oriented: tuples are identified by the attributes values
- All attribute values are atomic
- no tuples, or sets, or other structures
- Degree or arity
- number of attributes
- Cardinality
- number of tuples


## Integrity Constraints

## - Key Constraints

- Key: a set of attributes that uniquely identifies tuples
- Candidate key: a minimum set of attributes that form a key
- Superkey: A set of one or more attributes, which, taken collectively, allow us to identify uniquely a tuple in a relation.
- Primary key: a designated candidate key

■ Data Constraints

- Functional dependency, multivalued dependency, ...
- Check constraints

■ Others

- Null constraints
- Referential constraints


## Views

Views can be defined

- on single relations PROJECT(PNO, PNAME)
- on multiple relations SAL(ENO,TITLE,SALARY)
- Relations from which they are derived are called base relations
- View relations can be
- virtual; never physically created
- updates to views is a problem
- materialized: physical relations exist
met propagation of base table updates to materialized view tables


## View Updates

■ Views that are derived from multiple tables may cause problems

| EMP |  |  |  | SAL |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ENO | ENAME | TITLE |  | ENO | TITLE | SALARY |
| E1 | J. Doe | Elect. Eng. |  | E1 | Elect. Eng. | 55000 |
| E2 | M. Smith | Syst. Anal. |  | E2 | Syst. Anal. | 70000 |
| E3 | A. Lee | Mech. Eng. |  | E3 | Mech. Eng. | 45000 |
| E4 | J. Miller | Programmer | $\sum$ | E4 | Programmer | 60000 |
| E5 | B. Casey | Syst. Anal. |  | E5 | Syst. Anal. | 70000 |
| E6 | L. Chu | Elect. Eng. |  | E6 | Elect. Eng. | 55000 |
| E7 | R. Davis | Mech. Eng. |  | E7 | Mech. Eng. | 45000 |
| E8 | J. Jones | Syst. Anal. |  | E8 | Syst. Anal. | 70000 |
| PAY |  |  |  |  |  |  |


| TITLE | SALARY |
| :--- | :---: |
| Elect. Eng. | 55000 |
| Syst. Anal. | 70000 |
| Mech. Eng. | 45000 |
| Programmer | 60000 |

How do you delete a tuple from SAL?

## Relational Algebra

Form


## Relational Algebra Operators

- Fundamental
- union
- set difference
- selection
- projection
- Cartesian product

Additional

- rename
- intersection
- join
- quotient (division)
- Union compatibility
- same degree
- corresponding attributes defined over the same domain


## Union

- Similar to set union

General form

$$
R \cup S=\{t \mid t \in R \text { or } t \in S\}
$$

where $R, S$ are relations, $t$ is a tuple variable

- Result contains tuples that are in $R$ or in $S$, but not both (duplicates removed)
- $R, S$ should be union-compatible


## Set Difference

■ General Form

$$
R-S=\{t \mid t \in R \text { and } t \notin S\}
$$

where $R$ and S are relations, $t$ is a tuple variable

- Result contains all tuples that are in $R$, but not in $S$.
- $R-S \neq S-R$
- $R, S$ union-compatible


## Selection

■ Produces a horizontal subset of the operand relation

- General form

$$
\sigma_{F}(R)=\{t \mid t \in R \text { and } F(t) \text { is true }\}
$$

where

- $R$ is a relation, $t$ is a tuple variable
- $F$ is a formula consisting of
n+ operands that are constants or attributes
- 1 ar arithmetic comparison operators

$$
<,>,=, \neq, \leq, \geq
$$

n
$\wedge, \vee, \neg$

## Selection Example

EMP

| ENO | ENAME | TITLE |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| E1 | J. Doe | Elect. Eng. |  |
| E2 | M. Smith | Syst. Anal. |  |
| E3 | A. Lee | Mech. Eng. |  |
| E4 | J. Miller | Programmer |  |
| E5 | B. Casey | Syst. Anal. |  |
| E6 | L. Chu | Elect. Eng. |  |
| E7 | R. Davis | Mech. Eng. |  |
| E8 | J. Jones | Syst. Anal. |  |

## Projection

■ Produces a vertical slice of a relation
■ General form

$$
\Pi_{A_{1}, \ldots, A_{n}}(R)=\left\{t\left[A_{1}, \ldots, A_{n}\right] \mid t \in R\right\}
$$

where

- $R$ is a relation, $t$ is a tuple variable
- $\left\{A_{1}, \ldots, A_{n}\right\}$ is a subset of the attributes of $R$ over which the projection will be performed
■ Note: projection can generate duplicate tuples.
Commercial systems (and SQL) allow this and provide
- Projection with duplicate elimination
- Projection without duplicate elimination


## Projection Example

PROJ

| PNO | PNAME | BUDGET |
| :---: | :--- | :---: |
| P1 | Instrumentation | 150000 |
| P2 | Database Develop. | 135000 |
| P3 | CAD/CAM | 250000 |
| P4 | Maintenance | 310000 |
| P5 | CAD/CAM | 500000 |

$\Pi_{\text {PNO,BUDGET }}$ (PROJ)

| PNO | BUDGET |
| :---: | :---: |
| P1 | 150000 |
| P2 | 135000 |
| P3 | 250000 |
| P4 | 310000 |
| P5 | 500000 |

## Cartesian (Cross) Product

- Given relations
- $R$ of degree $k_{1}$, cardinality $n_{1}$
- $S$ of degree $k_{2}$, cardinality $n_{2}$

■ Cartesian (cross) product:

$$
\begin{aligned}
R \times S= & \left\{t\left[A_{1}, \ldots, A_{k_{1}}, A_{k_{1}+1}, \ldots, A_{k_{1}+k_{2}}\right] \mid t\left[A_{1}, \ldots, A_{k_{1}}\right] \in R\right. \text { and } \\
& \left.t\left[A_{k_{1}+1}, \ldots, A_{k_{1}+k_{2}}\right] \in S\right\}
\end{aligned}
$$

The result of $R \times S$ is a relation of degree $\left(k_{1}+k_{2}\right)$ and consists of all $\left(n_{1} * n_{2}\right)$-tuples where each tuple is a concatenation of one tuple of $R$ with one tuple of $S$.

## Cartesian Product Example

EMP

| ENO | ENAME | TITLE |
| :---: | :--- | :--- |
| E1 | J. Doe | Elect. Eng |
| E2 | M. Smith | Syst. Anal. |
| E3 | A. Lee | Mech. Eng. |
| E4 | J. Miller | Programmer |
| E5 | B. Casey | Syst. Anal. |
| E6 | L. Chu | Elect. Eng. |
| E7 | R. Davis | Mech. Eng. |
| E8 | J. Jones | Syst. Anal. |

PAY

| TITLE | SALARY |
| :--- | ---: |
| Elect. Eng. | 55000 |
| Syst. Anal. | 70000 |
| Mech. Eng. | 45000 |
| Programmer | 60000 |

$E M P \times P A Y$

| ENO | ENAME | EMP.TITLE | PAY.TITLE | SALARY |
| :---: | :---: | :---: | :---: | :---: |
| E1 | J. Doe | Elect. Eng. | Elect. Eng. | 55000 |
| E1 | J. Doe | Elect. Eng. | Syst. Anal. | 70000 |
| E1 | J. Doe | Elect. Eng. | Mech. Eng. | 45000 |
| E1 | J. Doe | Elect. Eng. | Programmer | 60000 |
| E2 | M. Smith | Syst. Anal. | Elect. Eng. | 55000 |
| E2 | M. Smith | Syst. Anal. | Syst. Anal. | 70000 |
| E2 | M. Smith | Syst. Anal. | Mech. Eng. | 45000 |
| E2 | M. Smith | Syst. Anal. | Programmer | 60000 |
| E3 | A. Lee | Mech. Eng. | Elect. Eng. | 55000 |
| E3 | A. Lee | Mech. Eng. | Syst. Anal. | 70000 |
| E3 | A. Lee | Mech. Eng. | Mech. Eng. | 45000 |
| E3 | A. Lee | Mech. Eng. | Programmer | 60000 |
| E8 | J. Jones | Syst. Anal. | Elect. Eng. | 55000 |
| E8 | J. Jones | Syst. Anal. | Syst. Anal. | 70000 |
| E8 | J. Jones | Syst. Anal. | Mech. Eng. | 45000 |
| E8 | J. Jones | Syst. Anal. | Programmer | 60000 |

## Intersection

Typical set intersection

$$
\begin{aligned}
R \cap S & =\{t \mid t \in R \text { and } t \in S\} \\
& =R-(R-S)
\end{aligned}
$$

- $R, S$ union-compatible


## Join

## General form

$$
\begin{array}{rl}
R \bowtie_{F\left(R . A_{i} S . S B_{j}\right)} S & S \\
& \left\{t\left[A_{1}, \ldots, A_{n}, B_{1}, \ldots, B_{m}\right]\right. \\
& t\left[A_{1}, \ldots, A_{n}\right] \in R \text { and } t\left[B_{1}, \ldots, B_{m}\right] \in S \\
& \text { and } \left.F\left(R . A_{i}, S . B_{j}\right) \text { is true }\right\}
\end{array}
$$

- $R, S$ are relations, $t$ is a tuple variable
- $F\left(R . A_{i}, S . B_{j}\right)$ is a formula defined as that of selection.
- A derivative of Cartesian product
- $R \bowtie_{F} S=\sigma_{F}(R \times S)$

| EMP |  |  |  | Join Example EMP $\bowtie_{\text {EMP.ENO }}>$ WORKS.ENOWORKS |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ENO | ENA | ME | TITLE |  |  |  |  |  |  |  |  |
| E1 E2 | J. Doe |  | Elect. Eng |  | EMP. <br> ENO. | ENAME | TITLE | WORKS. ENO | PNO | RESP | DUR |
| E3 | A. Lee |  | Mech. Eng. |  | E2 | M. Smith | Elect. Eng. | E1 | P1 | Manager | 12 |
| E4 | J. Miller |  | Programmer |  | E3 | A. Lee | Syst. Anal. | E1 | P1 | Manager | 12 |
| E5 | B. Casey |  | Syst Anal |  | E3 | A. Lee | Syst. Anal. | E2 | P1 | Analyst | 24 |
| E6 | L. Chu |  | Syst. Anal. |  | E3 | A. Lee | Syst. Anal. | E2 | P2 | Analyst | 6 |
| E6 |  |  | Elect. Eng. |  | E4 | J. Miller | Programmer | E1 | P1 | Manager | 12 |
| E7 | R. Davis |  | Mech. Eng. |  | E4 | J. Miller | Programmer | E2 | P1 | Analyst | 24 |
| E8 | J. Jones |  | Syst. Anal. |  | E4 | J. Miller | Programmer | E2 | P2 | Analyst | 6 |
| WORKS |  |  |  |  | E4 | J. Miller | Programmer | E3 | P3 | Consultant | 10 |
|  |  |  |  |  | E4 | J. Miller | Programmer | E3 | P4 | Engineer | 48 |
| ENO | PNO | RESP |  | DUR | E5 | B. Casey | Syst. Anal. | E1 | P1 | Manager | 12 |
| ENO |  |  |  | E5 | B. Casey | Syst. Anal. | E2 | P1 | Analyst | 24 |
|  | P1 Manager |  |  |  |  | E5 | B. Casey | Syst. Anal. | E2 | P2 | Analyst | 6 |
| E1 |  |  |  | 12 | E5 | B. Casey | Syst. Anal. | E3 | P3 | Consultant | 10 |
| E2 | P1 Analyst |  |  | 24 | E5 | B. Casey | Syst. Anal. | E3 | P4 | Engineer | 48 |
| E2 | P2 | Analyst |  | 6 | E5 | B. Casey | Syst. Anal. | E4 | P2 | Programmer | 18 |
| E3 | P3 | Consultant |  | 10 | E6 | L. Chu | Elect. Eng. | E1 | P1 | Manager | 12 |
| E3 | P4 | Engineer |  | 48 | E6 | L. Chu | Elect. Eng. | E2 | P1 | Analyst | 24 |
| E4 | P2 | Programmer |  | 18 | E6 | L. Chu | Elect. Eng. | E2 | P2 | Analyst | 6 |
| E5 | P2 | Manager |  | 24 | E6 | L. Chu | Elect. Eng. | E3 | P3 | Consultant | 10 |
| E6 | P4 | Manager |  | 48 | E6 | L. Chu | Elect. Eng. | E3 | P4 | Engineer | 48 |
| E7 | P3 | Engineer |  | 36 | E6 | L. Chu | Elect. Eng. | E4 | P2 | Programmer | 18 |
| E7 | P5 | Engineer |  | 23 | E6 | L. Chu | Elect. Eng. | E5 | P2 | Manager | 24 |
| E8 | P3 Manager |  |  | 40 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

## Types of Join

- $\theta$-join
- The formula $F$ uses operator $\theta$
- Equi-join
- The formula $F$ only contains equality
- $R \bowtie_{R . A=S . B} S$

Natural join

- Equi-join of two relations $R$ and $S$ over an attribute (or attributes) common to both $R$ and $S$ and projecting out one copy of those attributes
- $R \bowtie S=\Pi_{R \cup S} \sigma_{F}(R \times S)$


## Natural Join Example

EMP

| ENO | ENAME | TITLE |
| ---: | :--- | :--- |
| E1 | J. Doe | Elect. Eng |
| E2 | M. Smith | Syst. Anal. |
| E3 | A. Lee | Mech. Eng. |
| E4 | J. Miller | Programmer |
| E5 | B. Casey | Syst. Anal. |
| E6 | L. Chu | Elect. Eng. |
| E7 | R. Davis | Mech. Eng. |
| E8 | J. Jones | Syst. Anal. |

PAY

| TITLE | SALARY |
| :---: | ---: |
| Elect. Eng. | 55000 |
| Syst. Anal. | 70000 |
| Mech. Eng. | 45000 |
| Programmer | 60000 |

EMP $\bowtie$ PAY

| ENO | ENAME | TITLE | SALARY |
| :--- | :--- | :--- | :--- |
| E1 | J. Doe | Elect. Eng. | 55000 |
| E2 | M. Smith | Analyst | 70000 |
| E3 | A. Lee | Mech. Eng. | 45000 |
| E4 | J. Miller | Programmer | 60000 |
| E5 | B. Casey | Syst. Anal. | 70000 |
| E6 | L. Chu | Elect. Eng. | 55000 |
| E7 | R. Davis | Mech. Eng. | 45000 |
| E8 | J. Jones | Syst. Anal. | 70000 |

Join is over the common attribute TITLE

## Types of Join

## - Outer-Join

- Ensures that tuples from one or both relations that do not satisfy the join condition still appear in the final result with other relation's attribute values set to NULL
- Left outer join $\lesssim \searrow$
- Right outer join $\bowtie$ -
- Full outer join $\triangle$


## Division (Quotient)

Given relations

- $R$ of degree $k_{1}\left(R=\left\{A_{1}, \ldots, A_{k_{1}}\right\}\right)$
- $S$ of degree $k_{2}\left(S=\left\{B_{1}, \ldots, B_{k_{2}}\right\}\right)$

Let $A=\left\{A_{1}, \ldots, A_{k_{1}}\right\}$ [i.e., $\left.R(A)\right]$ and $B=\left\{B_{1}, \ldots, B_{k_{2}}\right\}$
[i.e., $S(B)$ ] and $B \subseteq A$.
Then, $T=R \div S$ gives $T$ of degree $k_{1}-k_{2}$ [i.e., $T(Y)$ where $Y=A-B]$ such that for a tuple $t$ to appear in T, the values in $t$ must appear in $R$ in combination with every tuple in $S$.

## Division (cont'd)

| R |  | s |  |
| :---: | :---: | :---: | :---: |
| X | Y | X |  |
| x1 | y1 | $\times 1$ |  |
| x2 | y1 | x2 |  |
| x 3 | y1 | x3 | $\mathrm{T}_{1} \leftarrow \Pi_{\mathrm{Y}}(\mathrm{R})$ |
| x4 | y1 |  | $\mathrm{T}_{2} \leftarrow \Pi_{\mathrm{Y}}\left(\left(\mathrm{S} \times \mathrm{T}_{1}\right)-\mathrm{R}\right)$ |
| x1 | y2 |  | $\mathrm{T} \leftarrow \mathrm{T}_{1}-\mathrm{T}_{2}$ |
| x3 | y2 |  |  |
| x2 | y3 | $\stackrel{T}{4}^{\text {Y }}$ |  |
| x3 | y3 | $Y$ |  |
| x4 | y3 | y1 |  |
| x 1 | y4 | y4 |  |
| x2 | y4 |  |  |
| $\times 3$ | y 4 |  |  |

## Division - Formally

Given relations

- $R$ of degree $k_{1}\left(R=\left\{A_{1}, \ldots, A_{k_{1}}\right\}\right)$
- $S$ of degree $k_{2}\left(S=\left\{B_{1}, \ldots, B_{k_{2}}\right\}\right)$

Division of $R$ by $S$ (given , $\left\{B_{1}, \ldots, B_{k_{2}}\right\} \subseteq\left\{A_{1}, \ldots, A_{k_{1}}\right\}$ )

$$
\begin{aligned}
R \div S= & \left\{t\left[\left\{A_{1}, \ldots, A_{k_{1}}\right\}-\left\{B_{1}, \ldots, B_{k_{2}}\right\}\right] \mid\right. \\
& \forall u \in S \exists v \in R(v[S]=u \wedge v[R-S]=t)\}= \\
& \Pi_{R-S}(R)-\Pi_{R-S}\left(\left(\Pi_{R-S}(R) \times S\right)-R\right)
\end{aligned}
$$

$R \div S$ results in a relation of degree $\left(k_{1}-k_{2}\right)$ and consists of all ( $k_{1}-k_{2}$ )-tuples $t$ such that for all $k_{1}$-tuples $u$ in $S$, the tuple $t u$ is in $R$.

## Division Example

EMP

| ENO | PNO | PNAME | BUDGET |
| :--- | :--- | :--- | :--- |
| E1 | P1 | Instrumentation | 150000 |
| E2 | P1 | Instrumentation | 150000 |
| E2 | P2 | Database Develop. | 135000 |
| E3 | P1 | Instrumentation | 150000 |
| E3 | P4 | Maintenance | 310000 |
| E4 | P2 | Instrumentation | 150000 |
| E5 | P2 | Instrumentation | 150000 |
| E6 | P4 | Maintenance | 310000 |
| E7 | P3 | CAD/CAM | 250000 |
| E8 | P3 | CAD/CAM | 250000 |
| E3 | P2 | Database Develop. | 135000 |
| E3 | P3 | CAD/CAM | 250000 |

PROJ

| PNO | PNAME | BUDGET |
| :---: | :--- | :---: |
| P1 | Instrumentation | 150000 |
| P2 | Database Develop. | 135000 |
| P3 | CAD/CAM | 250000 |
| P4 | Maintenance | 310000 |


| ENO $\div$ PROJ |
| :---: | :---: |
| E3 |

## Example Queries

Emp (Eno, Ename, Title, City) (note we added City)
Project(Pno, Pname, Budget, City) (note we added City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

List names of all employees.

- $\Pi_{\text {Ename }}(\mathrm{Emp})$

■ List names of all projects together with their
budgets.

- $\Pi_{\text {Pname,Budget }}($ Project $)$


## Example Queries

Emp (Eno, Ename, Title, City) (note we added City)
Project(Pno, Pname, Budget, City) (note we added City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

■ Find all job titles to which at least one employee has been hired.

```
- \(\Pi_{\text {Title }}(\mathrm{Emp})\)
```

- Find the records of all employees who work in

Toronto.

- $\sigma_{\text {City='Toronto }}$ (Emp)


## Example Queries

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)
■ Find all cities where either an employee works or a project exists.

- $\Pi_{\text {City }}($ Emp $) \cup \Pi_{\text {City }}($ Project $)$

■ Find all cities that has a project but no employees who work there.

- $\Pi_{\text {City }}($ Project $)-\Pi_{\text {City }}($ Emp $)$


## Example Queries

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

- Find the names of all projects with budgets greater than $\$ 225,000$.
- $\Pi_{\text {Pname }}\left(\sigma_{\text {Budget }>225000}(\right.$ Project $\left.)\right)$
- List the names and budgets of projects on which employee E1 works.
- $\Pi_{\text {Pname, Budget }}\left(\right.$ Project $\bowtie\left(\sigma_{\text {Eno='E1 }},(\right.$ Works $\left.\left.)\right)\right)$
- $\Pi_{\text {Pname, Budget }}\left(\sigma_{\text {Emp.Eno=Works.Eno }}\left(\right.\right.$ Project $\times \sigma_{\text {Eno='El }}$, (Works)))


## Example Queries

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

- Find the name of all the employees who work in a city where no project is located.
- $\Pi_{\text {Ename }}\left(E m p \bowtie\left(\Pi_{\text {City }}(\right.\right.$ Emp $)-\Pi_{\text {City }}($ Project $\left.)\right)$

■ Find all the cities that have both employees and projects.

- $\Pi_{\text {City }}($ Emp $) \cap \Pi_{\text {City }}($ Project $)$
- Find all the employees who work on every project.
- $\Pi_{\text {Eno, Pno }}($ Works $) \div \Pi_{\text {Pno }}$ (Project)


# Example Queries 

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

Find the names and budgets of all projects who employ programmers.

- $\Pi_{\text {Pname,Budget }}\left(\right.$ Project $\bowtie W$ Works $\bowtie \sigma_{\text {Title= }}$ Programmer $($ Emp $\left.)\right)$

List the names of employees and projects that are co-located.

- $\Pi_{\text {Ename, Pname }}($ Emp $\bowtie$ Project $)$


## Relational Calculus

Instead of specifying how to obtain the result, specify what the result is, i.e., the relationships that is supposed to hold in the result.

- Based on first-order predicate logic.
- symbol alphabet
" $\quad$ + logic symbols (e.g., $\Rightarrow, \neg$ )
- a a set of constants
- a a set of variables
- a a set of n-ary predicates
** a set of n-ary functions
- $\quad$ parentheses
- expressions (called well formed formulae (wff)) built from this symbol alphabet.


# Types of Relational Calculus 

According to the primitive variable used in specifying the queries.

- tuple relational calculus
- domain relational calculus


## Tuple Relational Calculus

- The primitive variable is a tuple variable which specifies a tuple of a relation. In other words, it ranges over the tuples of a relation.
■ In tuple relational calculus queries are specified as

$$
\{t \mid F(t)\}
$$

where $t$ is a tuple variable and $F$ is a formula consisting of the atoms and operators. $F$ evaluates to True or False.
$t$ can be qualified for only some attributes: $t[A]$

## Tuple Relational Calculus

- The atoms are the following:
(1) Tuple variables

Nu*) If the relation over which the variable ranges is known, the variable may be qualified by the name of the relation as $R . t$ or $R(t)$.
(2) Conditions

N $s[A] \theta t[B]$, where $s$ and $t$ are tuple variables and $A$ and $B$ are components of $s$ and $t$, respectively; $\theta \in\{<,>,=, \neq, \leq, \geq\}$.
Specifies that component $A$ of $s$ stands in relation $\theta$ to the $B$ component of $t$ (e.g., $s$ [SALARY] $>t$ [SALARY]).
Nu* $S[A] \theta c$, where $s, A$ and $\theta$ are as defined above and $c$ is a constant. For example, $s[\mathrm{NAME}]=$ "Smith".

## Tuple Relational Calculus

- A formula $F$ is composed of
- atoms
- Boolean operators $\wedge, \vee, \neg$
- existential quantifier $\exists$
- universal quantifier $\forall$


## Formation rules:

- Each atom is a formula.
- If $F$ and $G$ are formulae, so are $F \wedge G, F \vee G, \neg F$, and $\neg G$.
- If $F$ is a formula, so is $(F)$.
- If $F$ is a formula and $t$ is a free variable in $F$, then $\exists t(F)$ and $\forall t(F)$ are also formulae. These can also be written as $\exists t F(t)$ and $\forall t F(t)$
- Nothing else is a formula.


## Safety of Calculus Expressions

## - Problem:

- the size of $\{t \mid F(t)\}$ must be finite.
- $\{t \mid \neg t \in R\}$ is not finite


## - Safety:

- A query is safe if, for all databases conforming to the schema, the query result can be computed using only constants appearing in the database or the query itself.
- Since database is finite, the set of constants appearing in it is finite as well as the constants in the query; therefore, the query result will be finite


## Example Queries

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

List names of all employees.
$\{t[$ Ename $] \mid t \in \mathrm{Emp}\}$
■ List names of all projects together with their
budgets.
$\{<t[$ Pname $], t[$ Budget $]>\mid t \in$ Project $]\}$

# Example Queries 

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)

■ Find all job titles to which at least one employee has been hired.
$\{t[$ Title $] \mid t \in \mathrm{Emp})\}$
■ Find the records of all employees who work in Toronto.
$\{t \mid t \in$ customer $\wedge t[$ City $]=$ 'Toronto' $\}$

## Example Queries

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)
Find all cities where either an employee works or a project exists. $\{t[$ City $] \mid t \in \operatorname{Emp} \vee \exists s(s \in$ Project $\wedge t[$ City $]=s[$ City $])\}$ $\{t[$ City $] \mid t \in$ Project $\vee \exists s(s \in \mathrm{Emp} \wedge t$ [City] $=s$ [City] $)\}$
■ Find all cities that has a project but no employees who work there. $\{t[$ City $] \mid t \in$ Project $\wedge \neg \exists s(s \in \operatorname{Emp} \wedge t[$ City $]=s[$ City $])\}$

# Example Queries 

Emp (Eno, Ename, Title, City)<br>Project(Pno, Pname, Budget, City)<br>Pay(Title, Salary)<br>Works(Eno, Pno, Resp, Dur)

■ Find the names of all projects with budgets greater than $\$ 225,000$.
$\{t[$ Pname $] \mid t \in$ Project $\wedge t[$ Budget $]>225000\}$

- List the names and budgets of projects in which employee E1 works.
$\{<t[$ Pname $], t[$ Budget $]>\mid$
$t \in$ Project $\wedge \exists s(s \in$ Works $\wedge t[$ Pname $]=s[$ Pname $] \wedge$ $s$ [Eno]='E1') $\}$


## Tuple Calculus and Relational Algebra

■ THEOREM: Safe relational calculus and algebra are equivalent in terms of their expressive power.

- This is called relational completeness.
- This does not mean all useful computations can be performed
( -4 Aggregation, counting, transitive closure not specified
- Basic Correspondence

Algebra Operation Calculus Operator

| $\Pi$ | $\exists$ |
| :---: | :---: |
| $\sigma$ | $x \theta$ constant |
| $\cup$ | $\vee$ |
| $\bowtie$ | $\wedge$ |
| - | $\neg$ |
| $\div$ | $\forall$ |

# Tuple Calculus and Relational Algebra 

$\Pi$ is like $\exists$ "there exists" ...

$\div$ is like $\forall$ "for all" ...
Expressing $\div$ using basic operators
Similar to $\begin{aligned} & R \div S=\Pi_{A}(R)-\Pi_{A}\left(\Pi_{A}(R) \bowtie S-R\right) \\ & \downarrow x F(x)=\neg \quad(\exists x \quad \neg F(x))\end{aligned}$

## Domain Relational Calculus

- The primitive variable is a domain variable which specifies a component of a tuple.
$\Rightarrow$ the range of a domain variable consists of the domains over which the relation is defined.
■ Other differences from tuple relational calculus:
- The atoms are the following :
m- Each domain is an atom.
- Conditions which can be defined as follows are atoms :
- $x \theta y$, where $x$ and $y$ are domain variables or constants;
$\uparrow\left\langle x_{1}, x_{2}, \ldots, x_{n}\right\rangle \in R$ where $R$ is a relation of degree $n$ and each $x_{i}$ is a domain variable or constant.
- Formulae are defined in exactly the same way as in tuple relational calculus, with the exception of using domain variables instead of tuple variables.


## Domain Relational Calculus

The queries are specified in the following form :

$$
\left\{<x_{1}, x_{2}, \ldots, x_{n}>\mid F\left(x_{1}, x_{2}, \ldots, x_{n}\right)\right\}
$$

where $F$ is a formula in which $x_{1}, \ldots, x_{n}$ are the free variables.

## Domain Relational Calculus

Emp (Eno, Ename, Title, City)
Project(Pno, Pname, Budget, City)
Pay(Title, Salary)
Works(Eno, Pno, Resp, Dur)
List the names and budgets of projects in which employee E1 works.
$\{<b, c>\mid \exists a, d(<a, b, c, d>\in \operatorname{Project} \wedge \exists e, f, g(<e, a f, f\rangle \in$ Works $\wedge e=$ ' E 1 ') ) $\}$

# QBE (Query-by-Example) Queries 

■ Find the names of all employees


## QBE Queries

■ Find the names of projects with budgets greater than $\$ 350,000$.

| Emp | Eno | Ename | Title | City | Pay | Title | Salary |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |


| Project | Pno | Pname | Budget | City |
| :---: | :---: | :---: | :---: | :---: |
|  |  | P. | $>350000$ |  |


| Works | Eno | Pno | Resp | Dur |
| :--- | :--- | :--- | :--- | :--- |

## QBE Queries

■ Find the name and cities of all employees who work on a project for more than 20 months.

| Emp | Eno | Ename | Title | City |
| :--- | :--- | :---: | :---: | :---: |
|  | X | P. |  | P. |
|  |  |  |  |  |


| Pay | Title | Salary |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |


| Project | Pno | Pname | Budget | City |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |


| Works | Eno | Pno | Resp | Dur |
| :---: | :---: | :---: | :---: | :---: |
|  | x |  |  | $>20$ |
|  |  |  |  |  |

