

# Relational Databases

- Basic concepts
  - Data model: organize data as tables
  - A relational database is a set of tables
- Advantages
  - Simple concepts
  - Solid mathematical foundation
    - set theory
  - Powerful query languages
  - Efficient query optimization strategies
  - Design theory
- Industry standard
  - Relational model
  - SQL language

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## Relational Model

- Relation
  - A relation  $R$  with attributes  $A = \{A_1, A_2, \dots, A_n\}$  defined over  $n$  domains  $D = \{D_1, D_2, \dots, D_n\}$  (not necessarily distinct) with values  $\{Dom_1, Dom_2, \dots, Dom_n\}$  is a finite, time varying set of  $n$ -tuples  $\langle d_1, d_2, \dots, d_n \rangle$  such that  $d_1 \in Dom_1, d_2 \in Dom_2, \dots, d_n \in Dom_n$  and  $A_1 \in D_1, A_2 \in D_2, \dots, A_n \in D_n$ .
  - Notation:  $R(A_1, A_2, \dots, A_n)$  or  $R(A_1: D_1, A_2: D_2, \dots, A_n: D_n)$
  - Alternatively, given  $R$  as defined above, an instance of it at a given time is a set of  $n$ -tuples:  
 $\{ \langle A_1: d_1, A_2: d_2, \dots, A_n: d_n \rangle \mid d_1 \in Dom_1, d_2 \in Dom_2, \dots, d_n \in Dom_n \}$
- Tabular structure of data where
  - $R$  is the table heading
  - attributes are table columns
  - each tuple is a row

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# Relation Schemes and Instances

- Relational scheme
  - A relation scheme is the definition; i.e., a set of attributes
  - A relational database scheme is a set of relation schemes:
    - i.e., a set of sets of attributes
- Relation instance (simply *relation*)
  - An relation is an instance of a relation scheme
  - a relation  $\mathbf{r}$  over a relation scheme  $R = \{A_1, \dots, A_n\}$  is a subset of the Cartesian product of the domains of all attributes, i.e.,
    - $$\mathbf{r} \subseteq Dom_1 \times Dom_2 \times \dots \times Dom_n$$

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## Domains

- A domain is a *type* in the programming language sense
  - Name: String
  - Salary: Real
- Domain values is a set of acceptable values for a variable of a given type.
  - Name: CdnNames = {...},
  - Salary: ProfSalary = {45,000 - 150,000}
  - Simple/Composite domains
    - Address = Street name+street number+city+province+ postal code
- Domain compatibility
  - Binary operations (e.g., comparison to one another, addition, etc) can be performed on them.
- Full support for domains is not provided in many current relational DBMSs

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# Relation Schemes

EMP(ENO, ENAME, TITLE)  
PROJ (PNO, PNAME, BUDGET)  
WORKS(ENO,PNO, RESP, DUR)  
PAY(TITLE, SALARY)

| EMP        |       |       |
|------------|-------|-------|
| <u>ENO</u> | ENAME | TITLE |

| PROJ       |       |        |
|------------|-------|--------|
| <u>PNO</u> | PNAME | BUDGET |

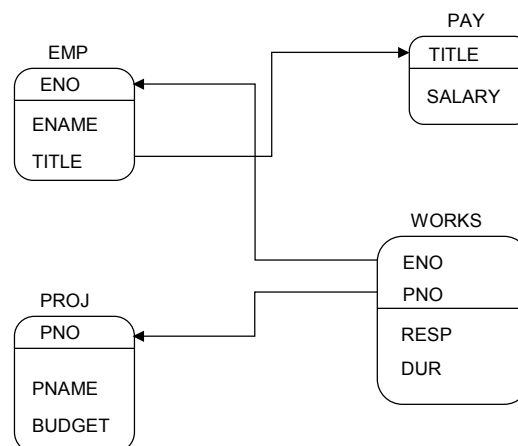
| WORKS      |            |      |     |
|------------|------------|------|-----|
| <u>ENO</u> | <u>PNO</u> | RESP | DUR |

| PAY          |        |
|--------------|--------|
| <u>TITLE</u> | SALARY |

- Underlined attributes are relation keys (tuple identifiers).
- Tabular form

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# Different Representation



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# Example Relation Instances

| ENO | ENAME     | TITLE       |
|-----|-----------|-------------|
| E1  | J. Doe    | Elect. Eng. |
| E2  | M. Smith  | Syst. Anal. |
| E3  | A. Lee    | Mech. Eng.  |
| E4  | J. Miller | Programmer  |
| E5  | B. Casey  | Syst. Anal. |
| E6  | L. Chu    | Elect. Eng. |
| E7  | R. Davis  | Mech. Eng.  |
| E8  | J. Jones  | Syst. Anal. |

| ENO | PNO | RESP       | DUR |
|-----|-----|------------|-----|
| E1  | P1  | Manager    | 12  |
| E2  | P1  | Analyst    | 24  |
| E2  | P2  | Analyst    | 6   |
| E3  | P3  | Consultant | 10  |
| E3  | P4  | Engineer   | 48  |
| E4  | P2  | Programmer | 18  |
| E5  | P2  | Manager    | 24  |
| E6  | P4  | Manager    | 48  |
| E7  | P3  | Engineer   | 36  |
| E7  | P5  | Engineer   | 23  |
| E8  | P3  | Manager    | 40  |

| PNO | PNAME             | BUDGET |
|-----|-------------------|--------|
| P1  | Instrumentation   | 150000 |
| P2  | Database Develop. | 135000 |
| P3  | CAD/CAM           | 250000 |
| P4  | Maintenance       | 310000 |
| P5  | CAD/CAM           | 500000 |

| TITLE       | SALARY |
|-------------|--------|
| Elect. Eng. | 55000  |
| Syst. Anal. | 70000  |
| Mech. Eng.  | 45000  |
| Programmer  | 60000  |

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# Properties

- Based on finite set theory
  - No ordering among attributes
    - Sometimes we prefer to refer to them by their relative order
  - No ordering among tuples
    - Query results may be ordered, but two differently ordered relation instances are equivalent
  - No duplicate tuples allowed
    - Commercial systems allow duplicates (so bag semantics)
  - Value-oriented: tuples are identified by the attributes values
- All attribute values are atomic
  - no tuples, or sets, or other structures
- Degree or arity
  - number of attributes
- Cardinality
  - number of tuples

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# Integrity Constraints

- Key Constraints
  - Key: a set of attributes that uniquely identifies tuples
  - Candidate key: a minimum set of attributes that form a key
  - Superkey: A set of one or more attributes, which, taken collectively, allow us to identify uniquely a tuple in a relation.
  - Primary key: a designated candidate key
- Data Constraints
  - Functional dependency, multivalued dependency, ...
  - Check constraints
- Others
  - Null constraints
  - Referential constraints

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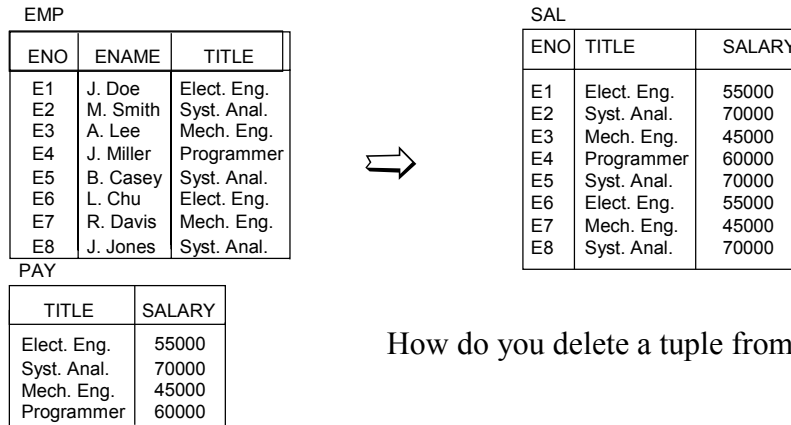
# Views

- Views can be defined
  - on single relations PROJECT(PNO, PNAME)
  - on multiple relations SAL(ENO, TITLE, SALARY)
- Relations from which they are derived are called base relations
- View relations can be
  - virtual; never physically created
    - updates to views is a problem
  - materialized: physical relations exist
    - propagation of base table updates to materialized view tables

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# View Updates

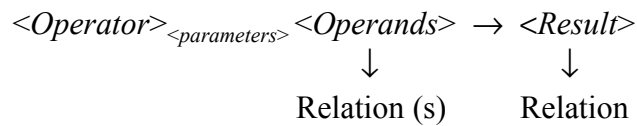
- Views that are derived from multiple tables may cause problems



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# Relational Algebra

Form



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# Relational Algebra Operators

- Fundamental
  - union
  - set difference
  - selection
  - projection
  - Cartesian product
- Additional
  - rename
  - intersection
  - join
  - quotient (division)
- Union compatibility
  - same degree
  - corresponding attributes defined over the same domain

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## Union

- Similar to set union
- General form

$$R \cup S = \{t \mid t \in R \text{ or } t \in S\}$$

where  $R, S$  are relations,  $t$  is a tuple variable

- Result contains tuples that are in  $R$  or in  $S$ , but not both (duplicates removed)
- $R, S$  should be union-compatible

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# Set Difference

## ■ General Form

$$R - S = \{t \mid t \in R \text{ and } t \notin S\}$$

where  $R$  and  $S$  are relations,  $t$  is a tuple variable

- Result contains all tuples that are in  $R$ , but not in  $S$ .
- $R - S \neq S - R$
- $R, S$  union-compatible

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# Selection

- Produces a horizontal subset of the operand relation
- General form

$$\sigma_F(R) = \{t \mid t \in R \text{ and } F(t) \text{ is true}\}$$

where

- $R$  is a relation,  $t$  is a tuple variable
- $F$  is a formula consisting of
  - ➔ operands that are constants or attributes
  - ➔ arithmetic comparison operators  
 $<, >, =, \neq, \leq, \geq$
  - ➔ logical operators  
 $\wedge, \vee, \neg$

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# Selection Example

EMP

| ENO | ENAME     | TITLE       |
|-----|-----------|-------------|
| E1  | J. Doe    | Elect. Eng. |
| E2  | M. Smith  | Syst. Anal. |
| E3  | A. Lee    | Mech. Eng.  |
| E4  | J. Miller | Programmer  |
| E5  | B. Casey  | Syst. Anal. |
| E6  | L. Chu    | Elect. Eng. |
| E7  | R. Davis  | Mech. Eng.  |
| E8  | J. Jones  | Syst. Anal. |

$\sigma_{\text{TITLE}='Elect. Eng.'}(\text{EMP})$

| ENO | ENAME  | TITLE       |
|-----|--------|-------------|
| E1  | J. Doe | Elect. Eng. |
| E6  | L. Chu | Elect. Eng. |

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# Projection

- Produces a vertical slice of a relation
- General form

$$\Pi_{A_1, \dots, A_n}(R) = \{t[A_1, \dots, A_n] \mid t \in R\}$$

where

- $R$  is a relation,  $t$  is a tuple variable
  - $\{A_1, \dots, A_n\}$  is a subset of the attributes of  $R$  over which the projection will be performed
- Note: projection can generate duplicate tuples. Commercial systems (and SQL) allow this and provide
    - Projection with duplicate elimination
    - Projection without duplicate elimination

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# Projection Example

PROJ

| PNO | PNAME             | BUDGET |
|-----|-------------------|--------|
| P1  | Instrumentation   | 150000 |
| P2  | Database Develop. | 135000 |
| P3  | CAD/CAM           | 250000 |
| P4  | Maintenance       | 310000 |
| P5  | CAD/CAM           | 500000 |

$\Pi_{\text{PNO,BUDGET}}(\text{PROJ})$

| PNO | BUDGET |
|-----|--------|
| P1  | 150000 |
| P2  | 135000 |
| P3  | 250000 |
| P4  | 310000 |
| P5  | 500000 |

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## Cartesian (Cross) Product

- Given relations
  - $R$  of degree  $k_1$ , cardinality  $n_1$
  - $S$  of degree  $k_2$ , cardinality  $n_2$

- Cartesian (cross) product:

$$R \times S = \{t [A_1, \dots, A_{k_1}, A_{k_1+1}, \dots, A_{k_1+k_2}] \mid t[A_1, \dots, A_{k_1}] \in R \text{ and } t[A_{k_1+1}, \dots, A_{k_1+k_2}] \in S\}$$

The result of  $R \times S$  is a relation of degree  $(k_1 + k_2)$  and consists of all  $(n_1 * n_2)$ -tuples where each tuple is a concatenation of one tuple of  $R$  with one tuple of  $S$ .

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# Cartesian Product Example

| ENO | ENAME     | TITLE       |
|-----|-----------|-------------|
| E1  | J. Doe    | Elect. Eng. |
| E2  | M. Smith  | Syst. Anal. |
| E3  | A. Lee    | Mech. Eng.  |
| E4  | J. Miller | Programmer  |
| E5  | B. Casey  | Syst. Anal. |
| E6  | L. Chu    | Elect. Eng. |
| E7  | R. Davis  | Mech. Eng.  |
| E8  | J. Jones  | Syst. Anal. |

| TITLE       | SALARY |
|-------------|--------|
| Elect. Eng. | 55000  |
| Syst. Anal. | 70000  |
| Mech. Eng.  | 45000  |
| Programmer  | 60000  |

| ENO | ENAME    | EMP.TITLE   | PAY.TITLE   | SALARY |
|-----|----------|-------------|-------------|--------|
| E1  | J. Doe   | Elect. Eng. | Elect. Eng. | 55000  |
| E1  | J. Doe   | Elect. Eng. | Syst. Anal. | 70000  |
| E1  | J. Doe   | Elect. Eng. | Mech. Eng.  | 45000  |
| E1  | J. Doe   | Elect. Eng. | Programmer  | 60000  |
| E2  | M. Smith | Syst. Anal. | Elect. Eng. | 55000  |
| E2  | M. Smith | Syst. Anal. | Syst. Anal. | 70000  |
| E2  | M. Smith | Syst. Anal. | Mech. Eng.  | 45000  |
| E2  | M. Smith | Syst. Anal. | Programmer  | 60000  |
| E3  | A. Lee   | Mech. Eng.  | Elect. Eng. | 55000  |
| E3  | A. Lee   | Mech. Eng.  | Syst. Anal. | 70000  |
| E3  | A. Lee   | Mech. Eng.  | Mech. Eng.  | 45000  |
| E3  | A. Lee   | Mech. Eng.  | Programmer  | 60000  |
| E8  | J. Jones | Syst. Anal. | Elect. Eng. | 55000  |
| E8  | J. Jones | Syst. Anal. | Syst. Anal. | 70000  |
| E8  | J. Jones | Syst. Anal. | Mech. Eng.  | 45000  |
| E8  | J. Jones | Syst. Anal. | Programmer  | 60000  |

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## Intersection

- Typical set intersection

$$R \cap S = \{t \mid t \in R \text{ and } t \in S\}$$

$$= R - (R - S)$$

- $R, S$  union-compatible

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# Join

## ■ General form

$$R \bowtie_{F(R.A_i, S.B_j)} S = \{t[A_1, \dots, A_n, B_1, \dots, B_m] \mid t[A_1, \dots, A_n] \in R \text{ and } t[B_1, \dots, B_m] \in S \text{ and } F(R.A_i, S.B_j) \text{ is true}\}$$

where

- $R, S$  are relations,  $t$  is a tuple variable
- $F(R.A_i, S.B_j)$  is a formula defined as that of selection.

## ■ A derivative of Cartesian product

- $R \bowtie_F S = \sigma_F(R \times S)$

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# Join Example

EMP

| ENO | ENAME     | TITLE       |
|-----|-----------|-------------|
| E1  | J. Doe    | Elect. Eng. |
| E2  | M. Smith  | Syst. Anal. |
| E3  | A. Lee    | Mech. Eng.  |
| E4  | J. Miller | Programmer  |
| E5  | B. Casey  | Syst. Anal. |
| E6  | L. Chu    | Elect. Eng. |
| E7  | R. Davis  | Mech. Eng.  |
| E8  | J. Jones  | Syst. Anal. |

WORKS

| ENO | PNO | RESP       | DUR |
|-----|-----|------------|-----|
| E1  | P1  | Manager    | 12  |
| E2  | P1  | Analyst    | 24  |
| E2  | P2  | Analyst    | 6   |
| E3  | P3  | Consultant | 10  |
| E3  | P4  | Engineer   | 48  |
| E4  | P2  | Programmer | 18  |
| E5  | P2  | Manager    | 24  |
| E6  | P4  | Manager    | 48  |
| E7  | P3  | Engineer   | 36  |
| E7  | P5  | Engineer   | 23  |
| E8  | P3  | Manager    | 40  |

EMP  $\bowtie_{EMP.ENO=WORKS.ENO}$  WORKS

| EMP. ENO. | ENAME     | TITLE       | WORKS. ENO | PNO | RESP       | DUR |
|-----------|-----------|-------------|------------|-----|------------|-----|
| E2        | M. Smith  | Elect. Eng. | E1         | P1  | Manager    | 12  |
| E3        | A. Lee    | Syst. Anal. | E1         | P1  | Manager    | 12  |
| E3        | A. Lee    | Syst. Anal. | E2         | P1  | Analyst    | 24  |
| E3        | A. Lee    | Syst. Anal. | E2         | P2  | Analyst    | 6   |
| E4        | J. Miller | Programmer  | E1         | P1  | Manager    | 12  |
| E4        | J. Miller | Programmer  | E2         | P1  | Analyst    | 24  |
| E4        | J. Miller | Programmer  | E2         | P2  | Analyst    | 6   |
| E4        | J. Miller | Programmer  | E3         | P3  | Consultant | 10  |
| E4        | J. Miller | Programmer  | E3         | P4  | Engineer   | 48  |
| E5        | B. Casey  | Syst. Anal. | E1         | P1  | Manager    | 12  |
| E5        | B. Casey  | Syst. Anal. | E2         | P1  | Analyst    | 24  |
| E5        | B. Casey  | Syst. Anal. | E2         | P2  | Analyst    | 6   |
| E5        | B. Casey  | Syst. Anal. | E3         | P3  | Consultant | 10  |
| E5        | B. Casey  | Syst. Anal. | E3         | P4  | Engineer   | 48  |
| E5        | B. Casey  | Syst. Anal. | E4         | P2  | Programmer | 18  |
| E6        | L. Chu    | Elect. Eng. | E1         | P1  | Manager    | 12  |
| E6        | L. Chu    | Elect. Eng. | E2         | P1  | Analyst    | 24  |
| E6        | L. Chu    | Elect. Eng. | E2         | P2  | Analyst    | 6   |
| E6        | L. Chu    | Elect. Eng. | E3         | P3  | Consultant | 10  |
| E6        | L. Chu    | Elect. Eng. | E3         | P4  | Engineer   | 48  |
| E6        | L. Chu    | Elect. Eng. | E4         | P2  | Programmer | 18  |
| E6        | L. Chu    | Elect. Eng. | E5         | P2  | Manager    | 24  |
| ...       | ...       | ...         | ...        | ... | ...        | ... |

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# Types of Join

- $\theta$ -join
  - The formula  $F$  uses operator  $\theta$
- Equi-join
  - The formula  $F$  only contains equality
  - $R \bowtie_{R.A=S.B} S$
- Natural join
  - Equi-join of two relations  $R$  and  $S$  over an attribute (or attributes) common to both  $R$  and  $S$  and projecting out one copy of those attributes
  - $R \bowtie S = \Pi_{R \cup S} \sigma_F(R \times S)$

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## Natural Join Example

EMP

| ENO | ENAME     | TITLE       |
|-----|-----------|-------------|
| E1  | J. Doe    | Elect. Eng. |
| E2  | M. Smith  | Syst. Anal. |
| E3  | A. Lee    | Mech. Eng.  |
| E4  | J. Miller | Programmer  |
| E5  | B. Casey  | Syst. Anal. |
| E6  | L. Chu    | Elect. Eng. |
| E7  | R. Davis  | Mech. Eng.  |
| E8  | J. Jones  | Syst. Anal. |

PAY

| TITLE       | SALARY |
|-------------|--------|
| Elect. Eng. | 55000  |
| Syst. Anal. | 70000  |
| Mech. Eng.  | 45000  |
| Programmer  | 60000  |

EMP  $\bowtie$  PAY

| ENO | ENAME     | TITLE       | SALARY |
|-----|-----------|-------------|--------|
| E1  | J. Doe    | Elect. Eng. | 55000  |
| E2  | M. Smith  | Analyst     | 70000  |
| E3  | A. Lee    | Mech. Eng.  | 45000  |
| E4  | J. Miller | Programmer  | 60000  |
| E5  | B. Casey  | Syst. Anal. | 70000  |
| E6  | L. Chu    | Elect. Eng. | 55000  |
| E7  | R. Davis  | Mech. Eng.  | 45000  |
| E8  | J. Jones  | Syst. Anal. | 70000  |

Join is over the common attribute TITLE

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# Types of Join

## ■ Outer-Join

- Ensures that tuples from one or both relations that do not satisfy the join condition still appear in the final result with other relation's attribute values set to NULL
- Left outer join  $\bowtie\leftarrow$
- Right outer join  $\rightarrow\bowtie$
- Full outer join  $\bowtie\rightleftarrows$

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# Division (Quotient)

Given relations

- $R$  of degree  $k_1$  ( $R = \{A_1, \dots, A_{k_1}\}$ )
- $S$  of degree  $k_2$  ( $S = \{B_1, \dots, B_{k_2}\}$ )

Let  $A = \{A_1, \dots, A_{k_1}\}$  [i.e.,  $R(A)$ ] and  $B = \{B_1, \dots, B_{k_2}\}$  [i.e.,  $S(B)$ ] and  $B \subseteq A$ .

Then,  $T = R \div S$  gives  $T$  of degree  $k_1 - k_2$  [i.e.,  $T(Y)$  where  $Y = A - B$ ] such that for a tuple  $t$  to appear in  $T$ , the values in  $t$  must appear in  $R$  in combination with *every* tuple in  $S$ .

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## Division (cont'd)

| R  |    |
|----|----|
| X  | Y  |
| x1 | y1 |
| x2 | y1 |
| x3 | y1 |
| x4 | y1 |
| x1 | y2 |
| x3 | y2 |
| x2 | y3 |
| x3 | y3 |
| x4 | y3 |
| x1 | y4 |
| x2 | y4 |
| x3 | y4 |

| S  |
|----|
| X  |
| x1 |
| x2 |
| x3 |

$$T_1 \leftarrow \Pi_Y(R)$$

$$T_2 \leftarrow \Pi_Y((S \times T_1) - R)$$

$$T \leftarrow T_1 - T_2$$

| T  |
|----|
| Y  |
| y1 |
| y4 |

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## Division - Formally

Given relations

- $R$  of degree  $k_1$  ( $R = \{A_1, \dots, A_{k_1}\}$ )
- $S$  of degree  $k_2$  ( $S = \{B_1, \dots, B_{k_2}\}$ )

Division of  $R$  by  $S$  (given,  $\{B_1, \dots, B_{k_2}\} \subseteq \{A_1, \dots, A_{k_1}\}$ )

$$R \div S = \{t[\{A_1, \dots, A_{k_1}\} - \{B_1, \dots, B_{k_2}\}] \mid \forall u \in S \exists v \in R (v[S] = u \wedge v[R - S] = t)\} = \Pi_{R-S}(R) - \Pi_{R-S}((\Pi_{R-S}(R) \times S) - R)$$

$R \div S$  results in a relation of degree  $(k_1 - k_2)$  and consists of all  $(k_1 - k_2)$ -tuples  $t$  such that for all  $k_1$ -tuples  $u$  in  $S$ , the tuple  $tu$  is in  $R$ .

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# Division Example

| EMP |     |                   |        | PROJ |                   |        |
|-----|-----|-------------------|--------|------|-------------------|--------|
| ENO | PNO | PNAME             | BUDGET | PNO  | PNAME             | BUDGET |
| E1  | P1  | Instrumentation   | 150000 | P1   | Instrumentation   | 150000 |
| E2  | P1  | Instrumentation   | 150000 | P2   | Database Develop. | 135000 |
| E2  | P2  | Database Develop. | 135000 | P3   | CAD/CAM           | 250000 |
| E3  | P1  | Instrumentation   | 150000 | P4   | Maintenance       | 310000 |
| E3  | P4  | Maintenance       | 310000 |      |                   |        |
| E4  | P2  | Instrumentation   | 150000 |      |                   |        |
| E5  | P2  | Instrumentation   | 150000 |      |                   |        |
| E6  | P4  | Maintenance       | 310000 |      |                   |        |
| E7  | P3  | CAD/CAM           | 250000 |      |                   |        |
| E8  | P3  | CAD/CAM           | 250000 |      |                   |        |
| E3  | P2  | Database Develop. | 135000 |      |                   |        |
| E3  | P3  | CAD/CAM           | 250000 |      |                   |        |

| EMP+PROJ |  |
|----------|--|
| ENO      |  |
| E3       |  |

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## Example Queries

**Emp** (Eno, Ename, Title, City) (note we added City)

**Project**(Pno, Pname, Budget, City) (note we added City)

**Pay**(Title, Salary)

**Works**(Eno, Pno, Resp, Dur)

- List names of all employees.
  - $\Pi_{\text{Ename}}(\text{Emp})$
- List names of all projects together with their budgets.
  - $\Pi_{\text{Pname, Budget}}(\text{Project})$

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## Example Queries

**Emp** (Eno, Ename, Title, City) (note we added City)  
**Project**(Pno, Pname, Budget, City) (note we added City)  
**Pay**(Title, Salary)  
**Works**(Eno, Pno, Resp, Dur)

- Find all job titles to which at least one employee has been hired.
  - $\Pi_{\text{Title}}(\text{Emp})$
- Find the records of all employees who work in Toronto.
  - $\sigma_{\text{City}='Toronto'}(\text{Emp})$

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## Example Queries

**Emp** (Eno, Ename, Title, City)  
**Project**(Pno, Pname, Budget, City)  
**Pay**(Title, Salary)  
**Works**(Eno, Pno, Resp, Dur)

- Find all cities where either an employee works or a project exists.
  - $\Pi_{\text{City}}(\text{Emp}) \cup \Pi_{\text{City}}(\text{Project})$
- Find all cities that has a project but no employees who work there.
  - $\Pi_{\text{City}}(\text{Project}) - \Pi_{\text{City}}(\text{Emp})$

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## Example Queries

**Emp** (Eno, Ename, Title, City)

**Project**(Pno, Pname, Budget, City)

**Pay**(Title, Salary)

**Works**(Eno, Pno, Resp, Dur)

- Find the names of all projects with budgets greater than \$225,000.
  - $\Pi_{\text{Pname}}(\sigma_{\text{Budget} > 225000}(\text{Project}))$
- List the names and budgets of projects on which employee E1 works.
  - $\Pi_{\text{Pname}, \text{Budget}}(\text{Project} \bowtie (\sigma_{\text{Eno}='E1'}(\text{Works})))$
  - $\Pi_{\text{Pname}, \text{Budget}}(\sigma_{\text{Emp.Eno}=\text{Works.Eno}}(\text{Project} \times \sigma_{\text{Eno}='E1'}(\text{Works})))$

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## Example Queries

**Emp** (Eno, Ename, Title, City)

**Project**(Pno, Pname, Budget, City)

**Pay**(Title, Salary)

**Works**(Eno, Pno, Resp, Dur)

- Find the name of all the employees who work in a city where no project is located.
  - $\Pi_{\text{Ename}}(\text{Emp} \bowtie (\Pi_{\text{City}}(\text{Emp}) - \Pi_{\text{City}}(\text{Project})))$
- Find all the cities that have both employees and projects.
  - $\Pi_{\text{City}}(\text{Emp}) \cap \Pi_{\text{City}}(\text{Project})$
- Find all the employees who work on every project.
  - $\Pi_{\text{Eno}, \text{Pno}}(\text{Works}) \div \Pi_{\text{Pno}}(\text{Project})$

2-36

## Example Queries

**Emp** (Eno, Ename, Title, City)

**Project**(Pno, Pname, Budget, City)

**Pay**(Title, Salary)

**Works**(Eno, Pno, Resp, Dur)

- Find the names and budgets of all projects who employ programmers.
  - $\Pi_{\text{Pname, Budget}}(\text{Project} \bowtie \text{Works} \bowtie \sigma_{\text{Title}='Programmer'}(\text{Emp}))$
- List the names of employees and projects that are co-located.
  - $\Pi_{\text{Ename, Pname}}(\text{Emp} \bowtie \text{Project})$

2-37

## Relational Calculus

- Instead of specifying how to obtain the result, specify what the result is, i.e., the relationships that is supposed to hold in the result.
- Based on first-order predicate logic.
  - symbol alphabet
    - logic symbols (e.g.,  $\Rightarrow$ ,  $\neg$ )
    - a set of constants
    - a set of variables
    - a set of n-ary predicates
    - a set of n-ary functions
    - parentheses
  - expressions (called well formed formulae (wff)) built from this symbol alphabet.

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# Types of Relational Calculus

- According to the primitive variable used in specifying the queries.
  - tuple relational calculus
  - domain relational calculus

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## Tuple Relational Calculus

- The primitive variable is a tuple variable which specifies a tuple of a relation. In other words, it ranges over the tuples of a relation.
- In tuple relational calculus queries are specified as

$$\{t \mid F(t)\}$$

where  $t$  is a tuple variable and  $F$  is a formula consisting of the atoms and operators.  $F$  evaluates to True or False.

$t$  can be qualified for only some attributes:  $t[A]$

2-40

# Tuple Relational Calculus

## ■ The atoms are the following:

### ① Tuple variables

- If the relation over which the variable ranges is known, the variable may be qualified by the name of the relation as  $R.t$  or  $R(t)$ .

### ② Conditions

- $s[A] \theta t[B]$ , where  $s$  and  $t$  are tuple variables and  $A$  and  $B$  are components of  $s$  and  $t$ , respectively;  
 $\theta \in \{<, >, =, \neq, \leq, \geq\}$ .  
Specifies that component  $A$  of  $s$  stands in relation  $\theta$  to the  $B$  component of  $t$  (e.g.,  $s[\text{SALARY}] > t[\text{SALARY}]$ ).
- $s[A] \theta c$ , where  $s$ ,  $A$  and  $\theta$  are as defined above and  $c$  is a constant. For example,  $s[\text{NAME}] = \text{“Smith”}$ .

2-41

# Tuple Relational Calculus

## ■ A formula $F$ is composed of

- atoms
- Boolean operators  $\wedge, \vee, \neg$
- existential quantifier  $\exists$
- universal quantifier  $\forall$

## ■ Formation rules:

- Each atom is a formula.
- If  $F$  and  $G$  are formulae, so are  $F \wedge G$ ,  $F \vee G$ ,  $\neg F$ , and  $\neg G$ .
- If  $F$  is a formula, so is  $(F)$ .
- If  $F$  is a formula and  $t$  is a free variable in  $F$ , then  $\exists t(F)$  and  $\forall t(F)$  are also formulae. These can also be written as  $\exists tF(t)$  and  $\forall tF(t)$
- Nothing else is a formula.

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# Safety of Calculus Expressions

## ■ Problem:

- the size of  $\{t \mid F(t)\}$  must be finite.
- $\{t \mid \neg t \in R\}$  is not finite

## ■ Safety:

- A query is safe if, for all databases conforming to the schema, the query result can be computed using only constants appearing in the database or the query itself.
- Since database is finite, the set of constants appearing in it is finite as well as the constants in the query; therefore, the query result will be finite

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# Example Queries

**Emp** (Eno, Ename, Title, City)

**Project**(Pno, Pname, Budget, City)

**Pay**(Title, Salary)

**Works**(Eno, Pno, Resp, Dur)

## ■ List names of all employees.

$\{t[\text{Ename}] \mid t \in \text{Emp}\}$

## ■ List names of all projects together with their budgets.

$\{\langle t[\text{Pname}], t[\text{Budget}] \rangle \mid t \in \text{Project}\}$

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## Example Queries

**Emp** (Eno, Ename, Title, City)  
**Project**(Pno, Pname, Budget, City)  
**Pay**(Title, Salary)  
**Works**(Eno, Pno, Resp, Dur)

- Find all job titles to which at least one employee has been hired.  
 $\{t[\text{Title}] \mid t \in \text{Emp}\}$
- Find the records of all employees who work in Toronto.  
 $\{t \mid t \in \text{customer} \wedge t[\text{City}] = \text{'Toronto'}\}$

2-45

## Example Queries

**Emp** (Eno, Ename, Title, City)  
**Project**(Pno, Pname, Budget, City)  
**Pay**(Title, Salary)  
**Works**(Eno, Pno, Resp, Dur)

- Find all cities where either an employee works or a project exists.  
 $\{t[\text{City}] \mid t \in \text{Emp} \vee \exists s(s \in \text{Project} \wedge t[\text{City}] = s[\text{City}])\}$   
 $\{t[\text{City}] \mid t \in \text{Project} \vee \exists s(s \in \text{Emp} \wedge t[\text{City}] = s[\text{City}])\}$
- Find all cities that has a project but no employees who work there.  
 $\{t[\text{City}] \mid t \in \text{Project} \wedge \neg \exists s(s \in \text{Emp} \wedge t[\text{City}] = s[\text{City}])\}$

2-46

## Example Queries

**Emp** (Eno, Ename, Title, City)  
**Project**(Pno, Pname, Budget, City)  
**Pay**(Title, Salary)  
**Works**(Eno, Pno, Resp, Dur)

- Find the names of all projects with budgets greater than \$225,000.

$$\{t[\text{Pname}] \mid t \in \text{Project} \wedge t[\text{Budget}] > 225000\}$$

- List the names and budgets of projects in which employee E1 works.

$$\{ \langle t[\text{Pname}], t[\text{Budget}] \rangle \mid t \in \text{Project} \wedge \exists s (s \in \text{Works} \wedge t[\text{Pname}] = s[\text{Pname}] \wedge s[\text{Eno}] = \text{'E1'}) \}$$

2-47

## Tuple Calculus and Relational Algebra

- THEOREM: Safe relational calculus and algebra are equivalent in terms of their expressive power.
  - This is called relational completeness.
  - This does not mean all useful computations can be performed
    - Aggregation, counting, transitive closure not specified
- Basic Correspondence

| Algebra Operation | Calculus Operator           |
|-------------------|-----------------------------|
| $\Pi$             | $\exists$                   |
| $\sigma$          | $x \theta \text{ constant}$ |
| $\cup$            | $\vee$                      |
| $\bowtie$         | $\wedge$                    |
| $-$               | $\neg$                      |
| $\div$            | $\forall$                   |

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# Tuple Calculus and Relational Algebra

$\Pi$  is like  $\exists$  “there exists” ...

$\div$  is like  $\forall$  “for all” ...

Expressing  $\div$  using basic operators

$$R \div S = \frac{\Pi_A(R) - \Pi_A(\Pi_A(R) \bowtie S - R)}{\quad}$$

Similar to

$$\forall x F(x) = \neg (\exists x \neg F(x))$$

2-49

## Domain Relational Calculus

- The primitive variable is a domain variable which specifies a component of a tuple.
  - $\Rightarrow$  the range of a domain variable consists of the domains over which the relation is defined.
- Other differences from tuple relational calculus:
  - The atoms are the following :
    - $\rightarrow$  Each domain is an atom.
    - $\rightarrow$  Conditions which can be defined as follows are atoms :
      - ◆  $x \theta y$ , where  $x$  and  $y$  are domain variables or constants;
      - ◆  $\langle x_1, x_2, \dots, x_n \rangle \in R$  where  $R$  is a relation of degree  $n$  and each  $x_i$  is a domain variable or constant.
  - Formulae are defined in exactly the same way as in tuple relational calculus, with the exception of using domain variables instead of tuple variables.

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## Domain Relational Calculus

The queries are specified in the following form :

$$\{ \langle x_1, x_2, \dots, x_n \rangle \mid F(x_1, x_2, \dots, x_n) \}$$

where  $F$  is a formula in which  $x_1, \dots, x_n$  are the free variables.

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## Domain Relational Calculus

**Emp** (Eno, Ename, Title, City)

**Project**(Pno, Pname, Budget, City)

**Pay**(Title, Salary)

**Works**(Eno, Pno, Resp, Dur)

List the names and budgets of projects in which employee E1 works.

$$\{ \langle b, c \rangle \mid \exists a, d (\langle a, b, c, d \rangle \in \text{Project} \wedge \exists e, f, g (\langle e, a, f, g \rangle \in \text{Works} \wedge e = \text{'E1'})) \}$$

2-52

# QBE (Query-by-Example) Queries

- Find the names of all employees

| Emp | Eno | Ename | Title | City | Pay | Title | Salary |
|-----|-----|-------|-------|------|-----|-------|--------|
|     |     | P.    |       |      |     |       |        |

| Project | Pno | Pname | Budget | City |
|---------|-----|-------|--------|------|
|         |     |       |        |      |

| Works | Eno | Pno | Resp | Dur |
|-------|-----|-----|------|-----|
|       |     |     |      |     |

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# QBE Queries

- Find the names of projects with budgets greater than \$350,000.

| Emp | Eno | Ename | Title | City | Pay | Title | Salary |
|-----|-----|-------|-------|------|-----|-------|--------|
|     |     |       |       |      |     |       |        |

| Project | Pno | Pname | Budget  | City |
|---------|-----|-------|---------|------|
|         |     | P.    | >350000 |      |

| Works | Eno | Pno | Resp | Dur |
|-------|-----|-----|------|-----|
|       |     |     |      |     |

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# QBE Queries

- Find the name and cities of all employees who work on a project for more than 20 months.

| <b>Emp</b> | Eno | Ename | Title | City | <b>Pay</b> | Title | Salary |
|------------|-----|-------|-------|------|------------|-------|--------|
|            | _X  | P.    |       | P.   |            |       |        |

| <b>Project</b> | Pno | Pname | Budget | City |
|----------------|-----|-------|--------|------|
|                |     |       |        |      |

| <b>Works</b> | Eno | Pno | Resp | Dur |
|--------------|-----|-----|------|-----|
|              | _X  |     |      | >20 |