SIGGRAPH ASIA 2013

Path Space Regularization Framework

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Motivation

Why Photon Mapping / Vertex Merging is useful?
- Caustics/reflected caustics
- Helps sampling difficult transport paths

Why is it not the ultimate technique?
- Slower convergence on diffuse
- High memory and b/w requirements
- Cache efficiency is unpredictable

Can we have all pros and no cons?
- Handling difficult illumination
- No memory pressure
- Converges as fast as possible

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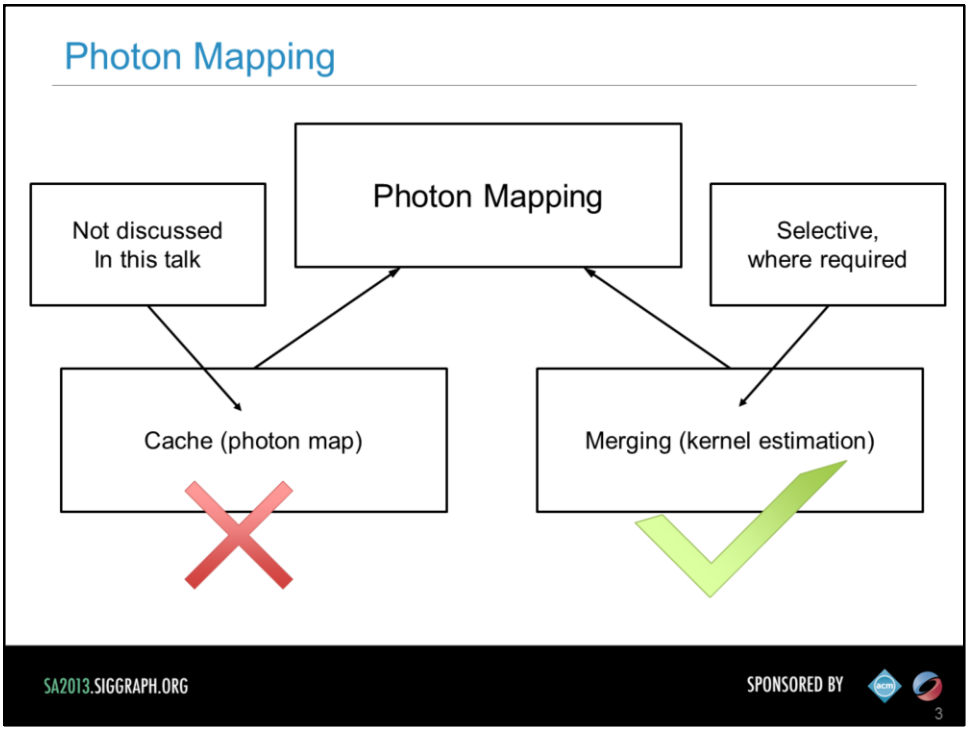
As it was thoroughly discussed by the previous course speakers, photon mapping methods are good at handling such difficult light transport methods, as caustics and reflected caustics.

However, on the other hand, it is usually used in production as a supplementary method.
There are multiple reasons for that. First of all, it might be not the best choice for diffuse surfaces due to "blotchy" appearance.
Besides, it might require quite some memory and bandwidth in practice, especially if additional amount of information has to be stored per photon.
Lastly, the efficiency of photon map can be unpredictable due to high scene occlusion or low usage of photons.

In this part of the course we will discuss a mathematical framework, which makes photon mapping so efficient in sampling difficult paths. We will also discuss how it can be efficiently combined with more advanced light transport methods, while avoiding the typical disadvantages of photon mapping class methods.



Photon mapping has two underlying key components that make it so efficient.
The first one is the large cache of light subpaths, called a photon map.
The second is the vertex merging technique for path construction, that Iliyan Georgiev has explained in his part of the course.

In this part, I will mostly talk about the vertex merging technique (a.k.a. kernel estimation).

Motivation

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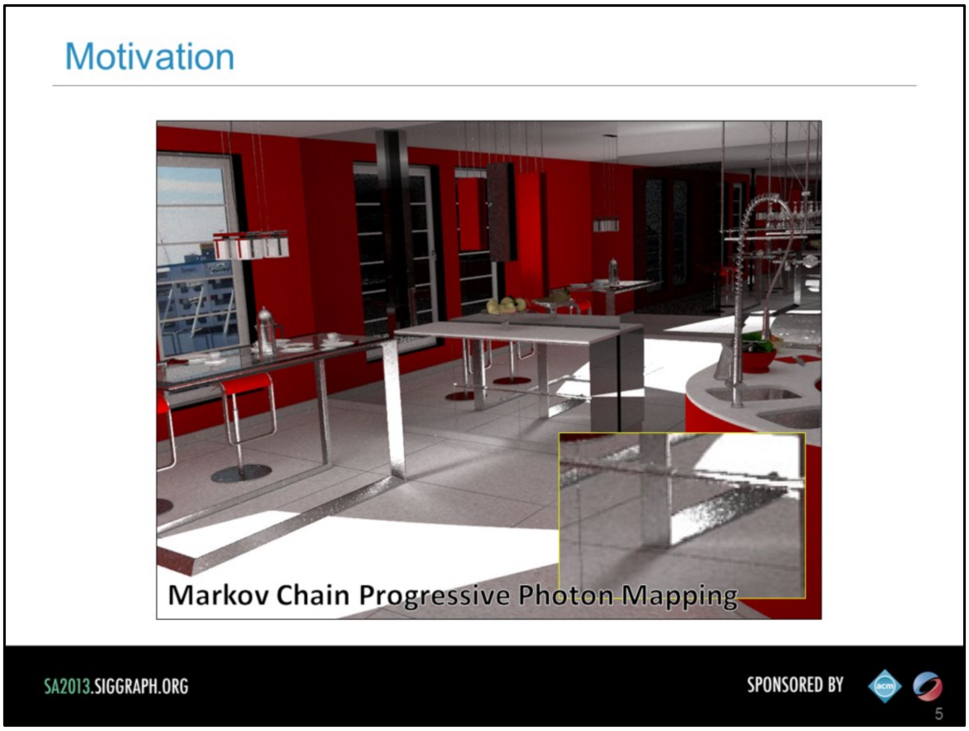
Even the most recent existing methods are either not good **at** or not capable **of** handling complex illumination, such as reflected caustics.
For example, in this scene, light comes through a window, and VCM still has some noise after an hour of rendering. This is due to inefficiency of light path sampling and caching.



Recent Markov chain PPM from Toshiya Hachisuka addresses the problem of light path sampling at the price of caching camera subpaths. This can be observed as an image noise on glossy surfaces, which require extensive resampling of camera subpaths.

Motivation

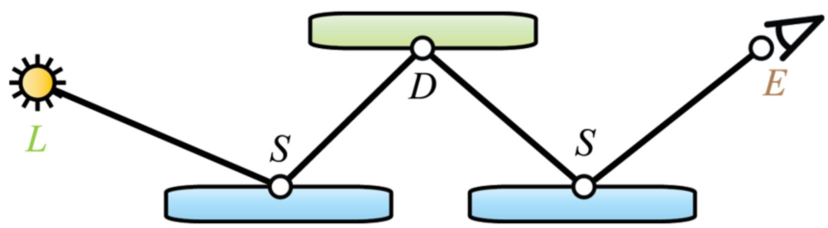
On the other hand, Metropolis light transport with recent manifold exploration mutation handles this scene well in the same time budget.
However, it lacks reflected caustics completely, because it cannot find them within the given rendering time.
In this part we will show how to combine the **strengths** of the aforementioned methods together.

Non-Sampleable Paths

What if all edges of the path contain singularities?

- No place to perform a connection
- Impossible to sample with any unbiased sampling [Veach97]

Example: Reflected caustics from point light (LSDSE)

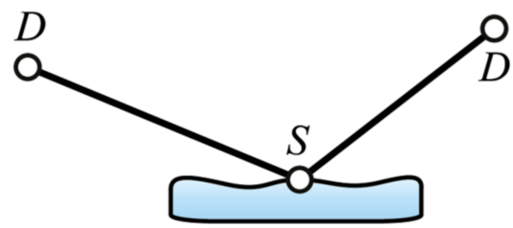


Even such advanced methods as BDPT can **fail** constructing a path in some cases. One of these cases is the case of **reflected caustic**.
The reason is that **every edge** adjoin a singularity at one of its ends (specular reflection is a Dirac delta distribution), making all connections fail.
Such paths with zero connectible edges are showed to be **non-sampleable with local unbiased methods** by Eric Veach.
And there is a more fundamental reason for that. Let’s take a brief **excursus**.

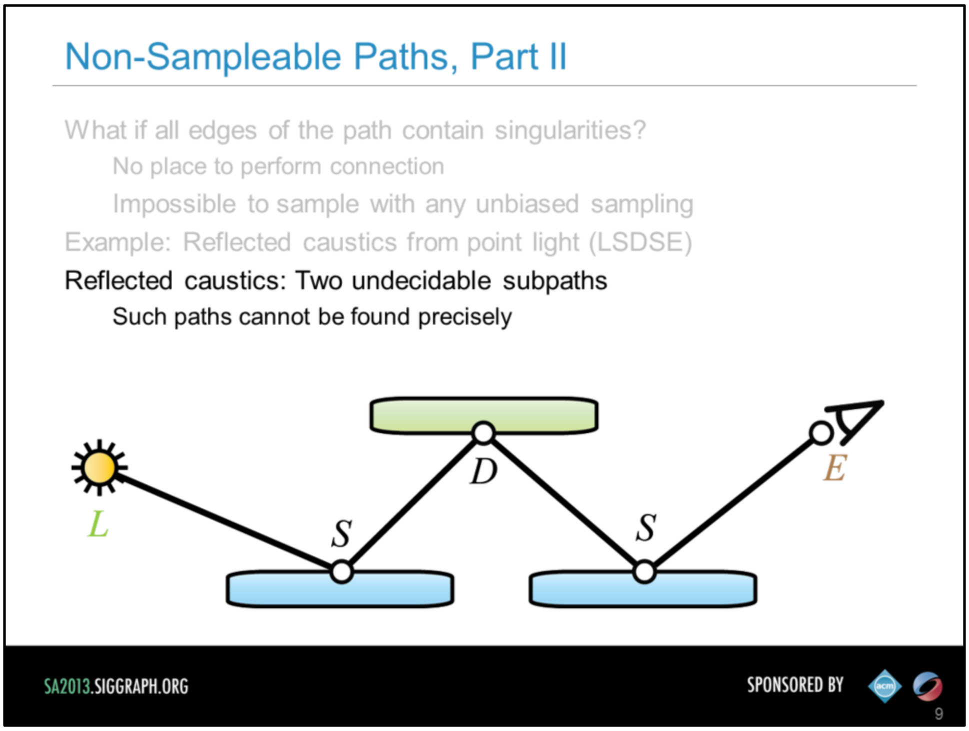
Excursus: Undecidability of Ray Tracing

Pure specular path tracing is *undecidable* [Reif et al. 1994]

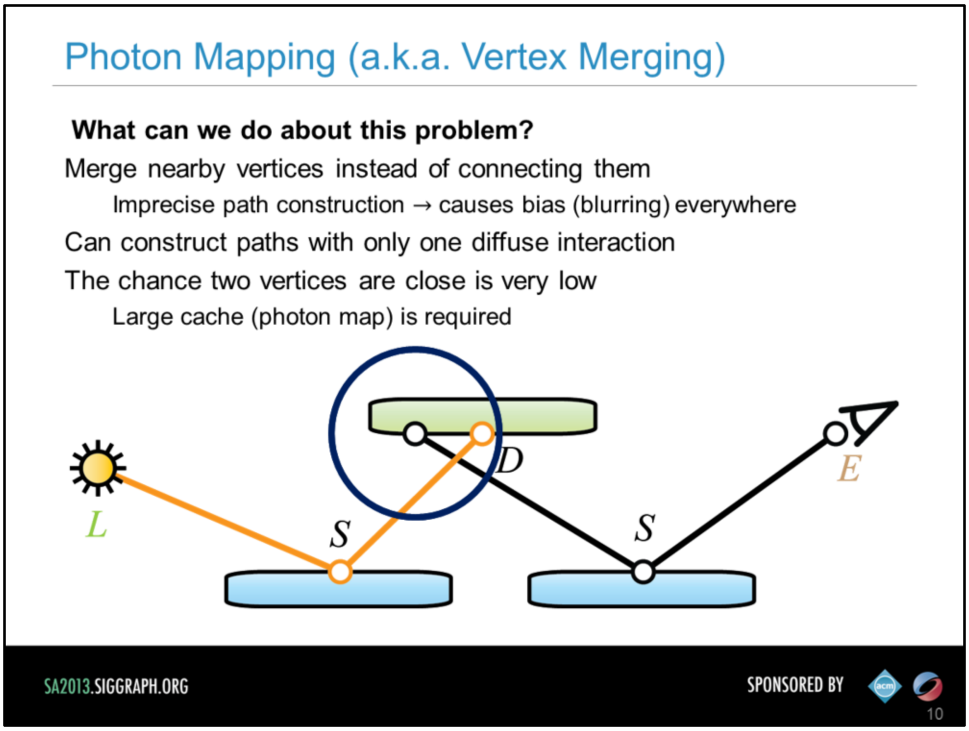
- Task: Find all specular paths from one fixed point to another ($DS^{+}D$)
- Impossible to find all such paths on a Turing machine
- Given arbitrary specular geometry



Interestingly, it is **not possible** to find all specular paths from one fixed point to another on a Turing machine, given an arbitrary configuration of specular surfaces.



Coming back to an example of reflected caustics, we can see that no matter from which end we trace this path, it will always pose such an undecidable problem at some point during the construction.



As was pointed out by Toshiya Hachisuka in his part of the course, photon mapping **can** sample paths that are considered non-sampleable by local unbiased methods. The reason is photon mapping constructs the full path by **merging** proximate vertices of different subpaths.
This is a **biased way**, causing blurring of image features. However, this way **difficult paths**, such as reflected caustics, can be sampled.

Note that the chance that two vertices happened to be located nearby, is **very low**. Thus photon mapping requires a **large cache** of the light subpaths, called a photon map.

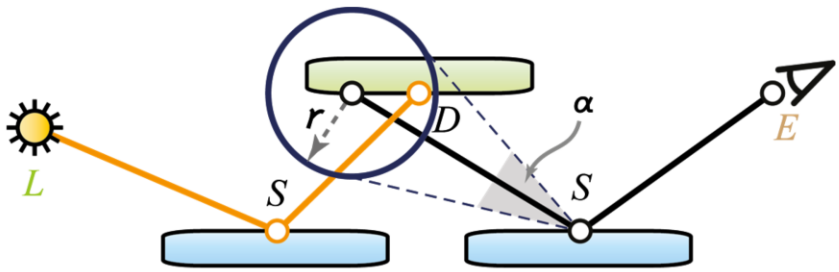
Photon Mapping: Under the Hood

Photon mapping essentially regularizes specular interactions

Regularization angle depends on the connection distance

- On-surface radius is fixed for a given point

It is a known mathematical procedure…



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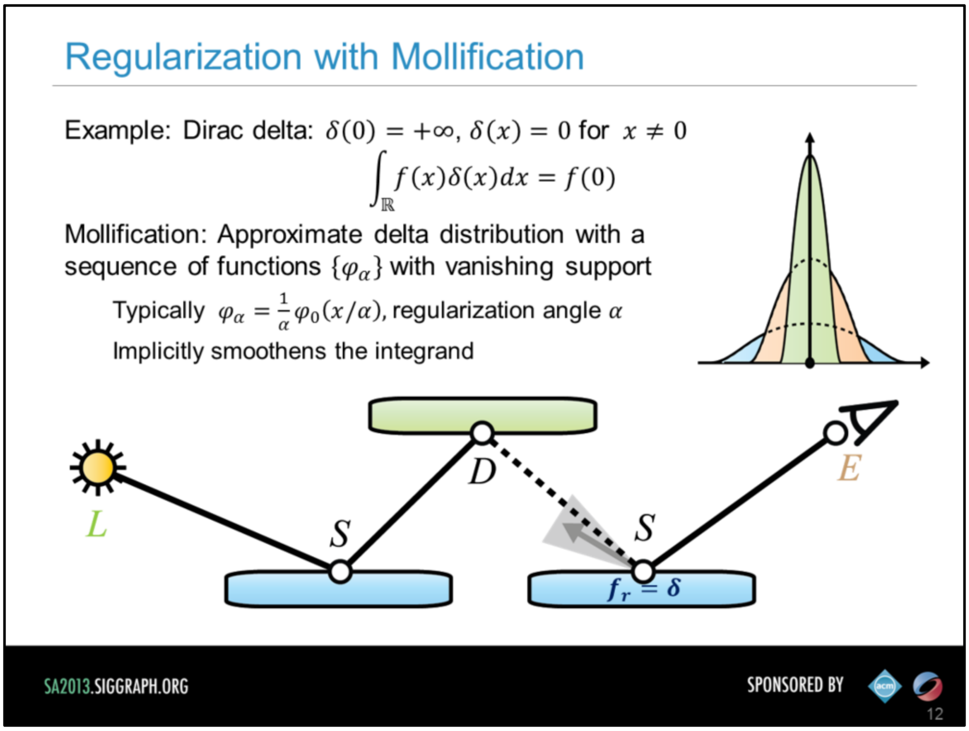
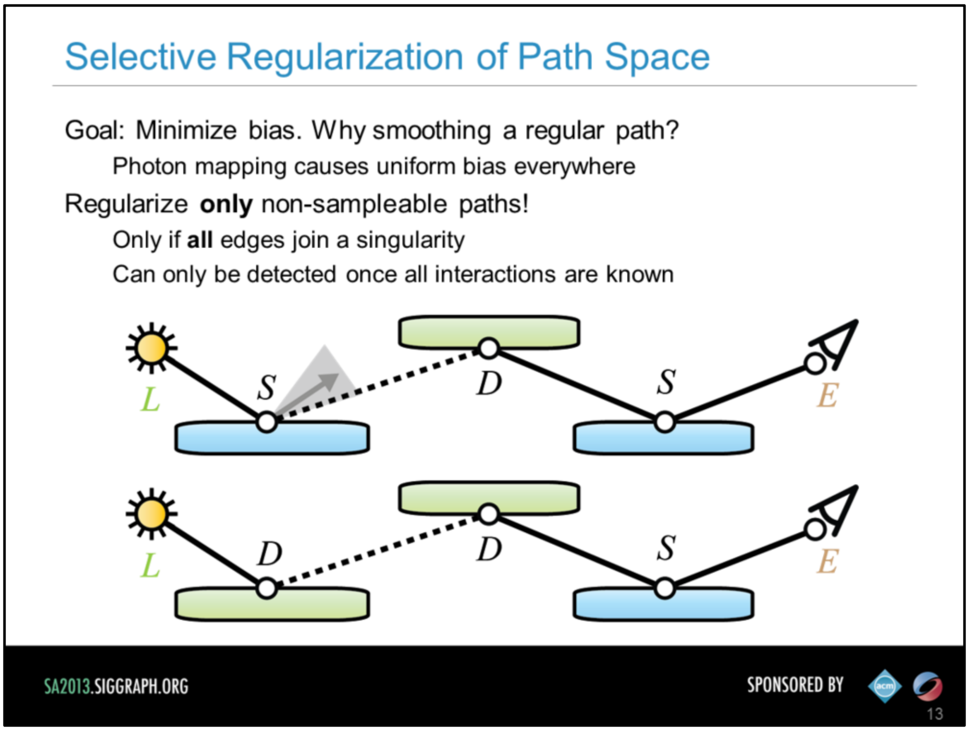
What photon mapping performs during kernel estimation is also known as *regularization* in mathematics.
Photon mapping **regularizes the interaction** by merging the path at the next vertex.
The **regularization angle**, *alpha*, can be derived from the fixed photon mapping radius *r* and the distance to the estimation center.
It appears to be a standard mathematical procedure.



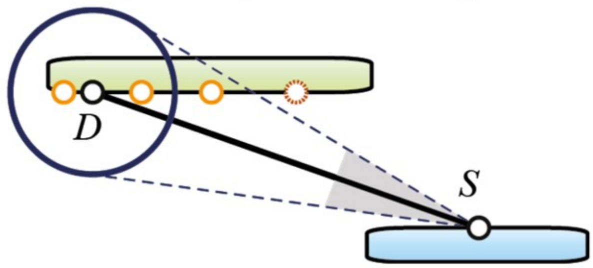
This procedure is called **mollification**.
Given a singularity caused by a **delta distribution**, like the one at the right specular vertex.
We construct a **sequence** of integrable smooth functions, that approach delta distribution in the limit.
During the integration, we shrink the regularization angle, gradually making the integrand less and less smooth.



So, now that we formalized the process, we can selectively regularize only when **necessary**.
Instead of regularizing all paths, including regular ones, we regularize **only non-sampleable paths (irregular for the sampling method)**, thus **minimizing** the amount of bias.
In case of BDPT, non-sampleable paths are detected if **all edges** of the path adjoin at least one singularity.
This situation can be recognized only after **all subpaths are traced** and all interactions are known.

Regularization Bandwidth

- Fix on-surface radius, as in photon mapping
  - Convenient to control perceived bias
  - Shoot a reflection ray to detect the distance
- Optimal bandwidth depends on light field around the path
  - Requeres bias estimation
- Optimal bandwidth might depend on the goal / application
  - E.g., regularize to sample all image features within given time

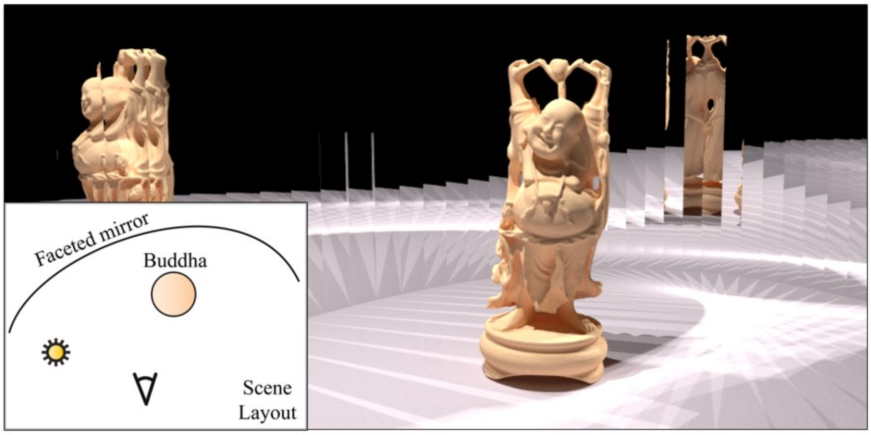


How to select the regularization angle? This is, generally, not a solved problem yet. However in practice, we have found that just linking it to on-surface radius, in spirit of photon mapping, allows to conveniently control the amount of bias.
One thing that might be not well described in the original path space regularization paper, is that in this case the distance should be measured to the intersection point along the exact, non-mollified outgoing direction (as shown in this figure).

Selecting the optimal bandwidth requires estimating the amount of bias introduced by mollification. This is a not easy task to do, as we have seen from the adaptive progressive photon mapping part of the talk.
Generally, selection of regularization bandwidth can also depend on the goal of the algorithm. For example, a goal of the interactive preview renderer might sound like "sample all features given fixed rendering budget". In this case, the bandwidth can be adjusted to sample every path "almost surely", that is, with probability one, within given sample budget regardless of amount of bias introduced.

Comparison of Different Methods

Reference



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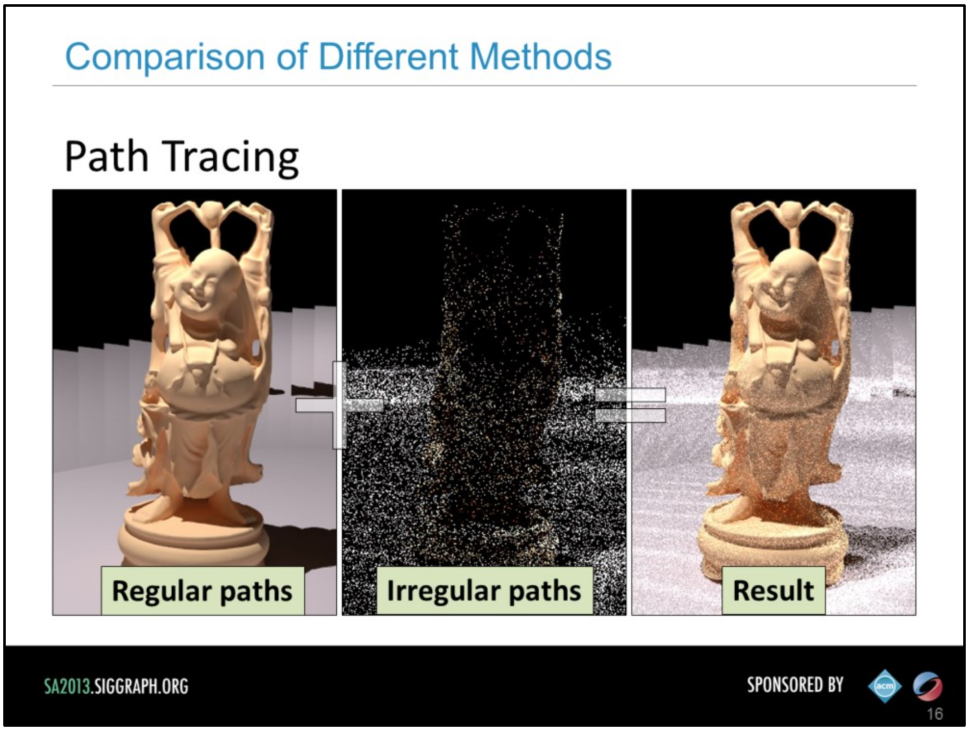
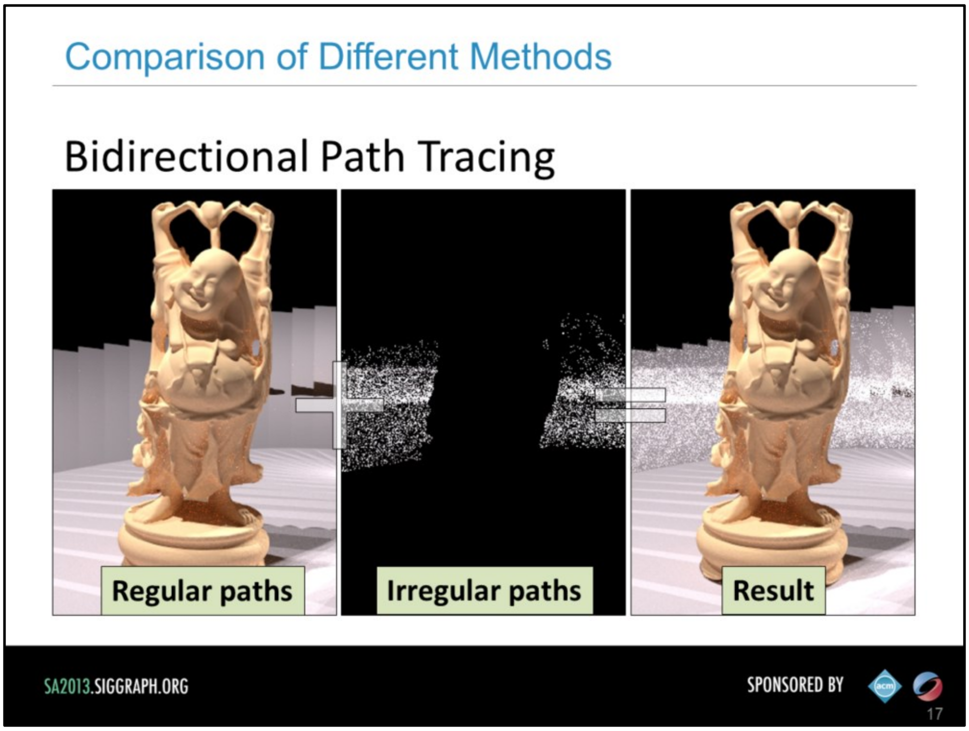
We compare different common methods with and without regularization.
This is a simple scene with caustics and reflected caustics.



Original PT is not capable of sampling **both caustics and reflected caustics**, thus requiring high amount of regularization.



BDPT only requires regularization when the path cannot be constructed with **any** edge connection, thus requiring the regularization of only **reflected caustics**. Note that the amount of noise caused by irregular paths is always **high**, because there is no cache and the regularization angle is small.

Noisy without cache?

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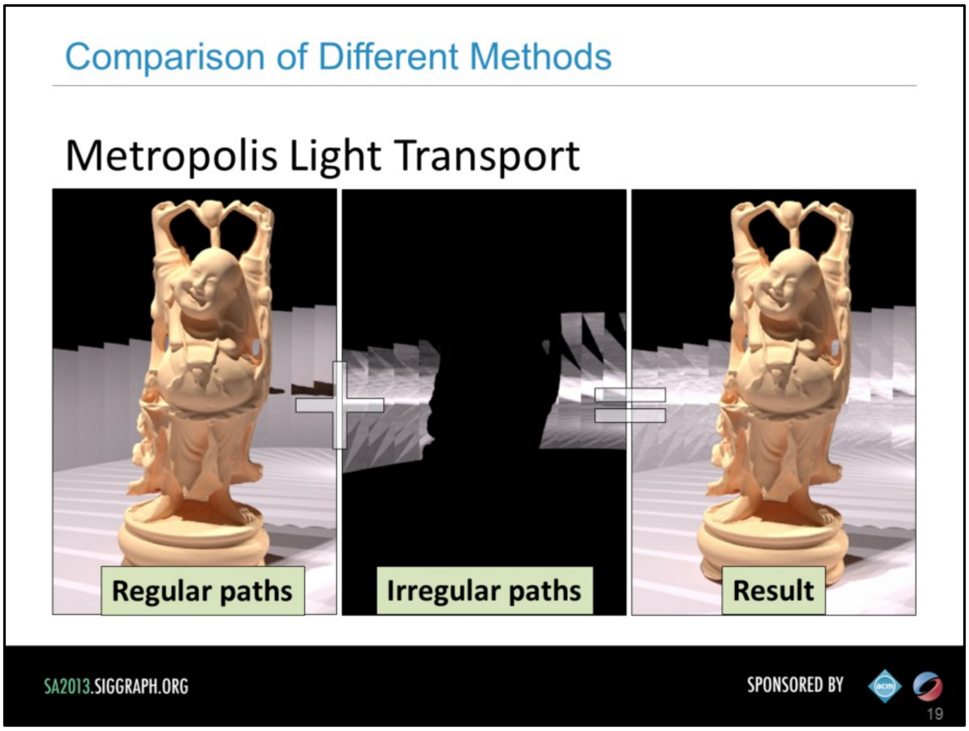
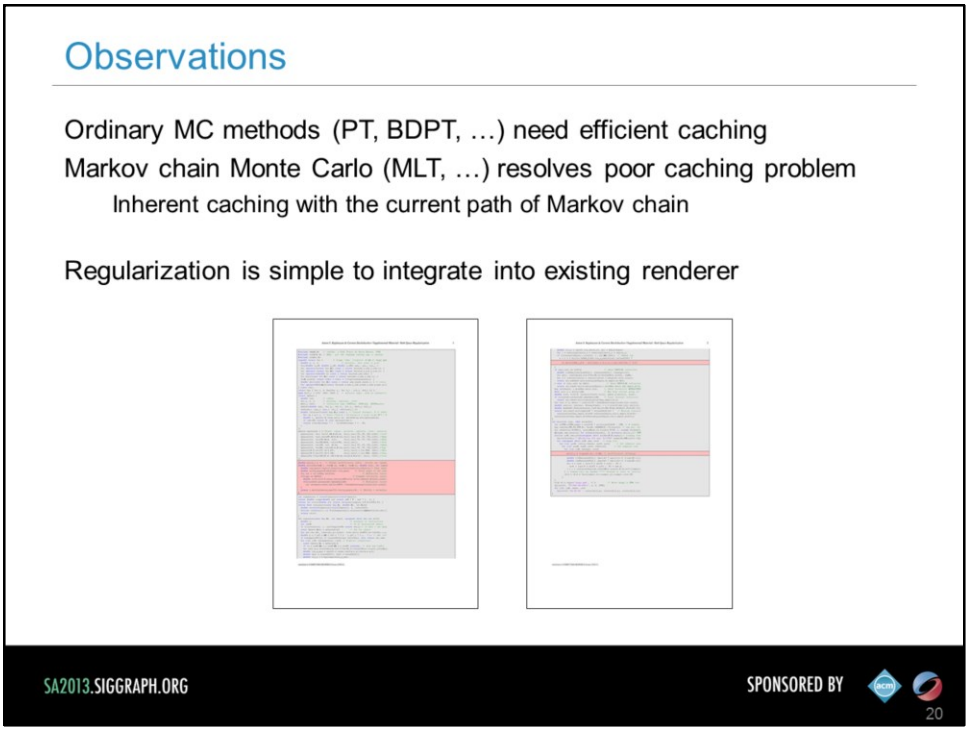
Sampling such irregular paths with regularization might be **much** noisier comparing to photon mapping class methods, because it essentially performs photon mapping without a photon map! However, more interestingly, the regularization framework can be easily applied to **any** integration method, for example, to Metropolis light transport (MLT).

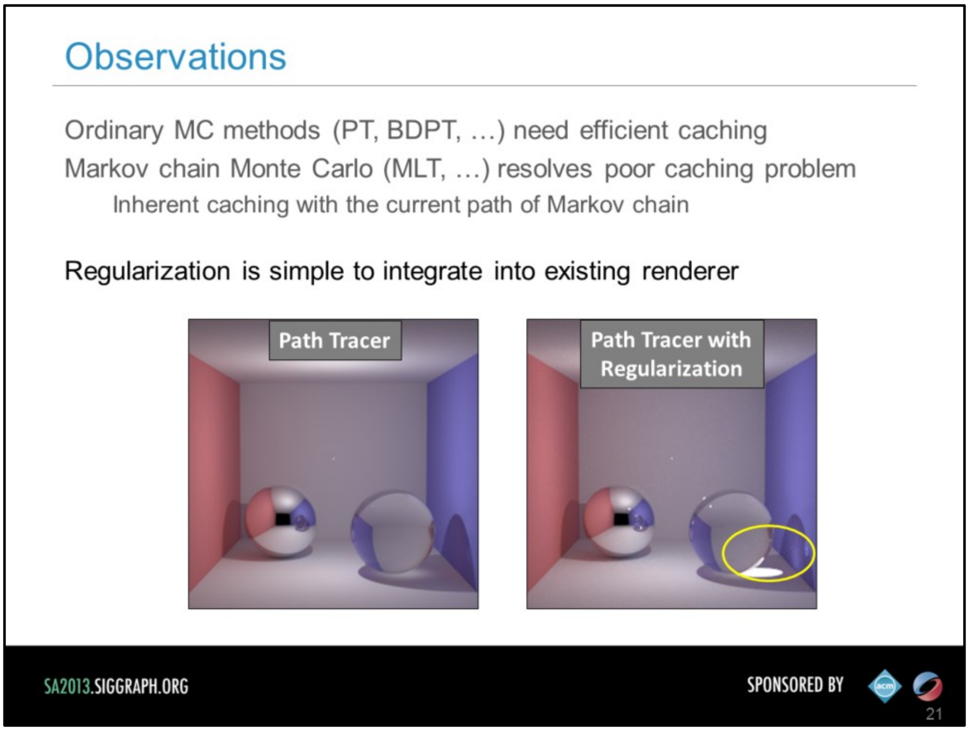


MLT solves the noise problem by implicitly **caching** the path, that it has already found before, as a current state of Markov chain. Using this method, we can naturally avoid storing a large cache of light subpaths.

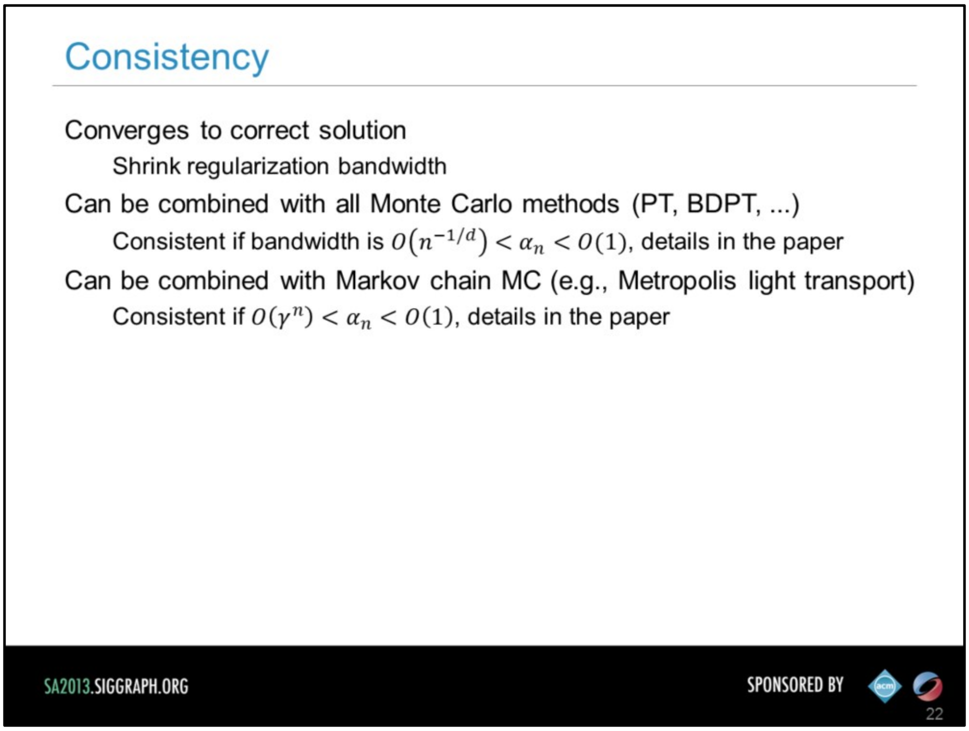


As we have seen, the ordinary Monte Carlo methods might suffer from noise. Markov chain Monte Carlo based methods, such as Metropolis light transport, naturally **resolve** this issue by caching the last valid path as a current state of Markov chain.

On the other hand, regularization can be useful even for very **simple** scenarios, like sampling caustics with path tracing. It is also **simple** to implement. Here is a code of a minimalistic path tracing. Regularization requires some changes only to the **evaluation routine** of specular BRDF. The required additions are marked in **red**.



And you can see, with these small changes, path tracing can **handle caustics** of **all kinds** from point lights. And no bias is introduced for **regular** paths. It appears to be a simple, yet practical addition to a part tracer.



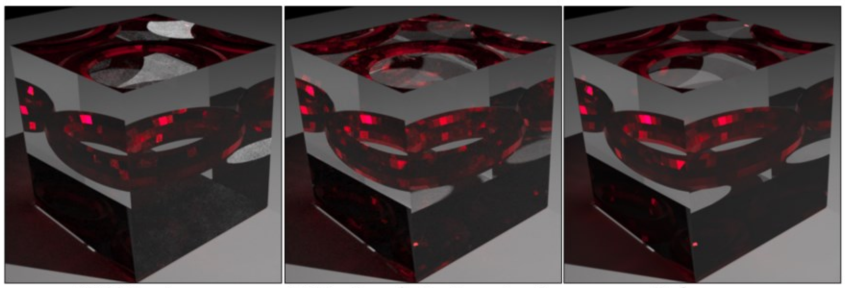
In order to make sure the regularization converges to **correct solution** in the limit, we need to **decrease** the regularization angle throughout the integration.
The Monte Carlo methods have the **same shrinkage conditions** as progressive photon mapping, which is not surprising.
However the MCMC methods, such as MLT, require slightly **different rate**. Please see the **details** in the original paper.

Practical Convergence of MLT

Powerful tool to temper the integrand

“Improving Robustness of Monte-Carlo Global Illumination with Directional Regularization” brief today by G. Bouchard et al.

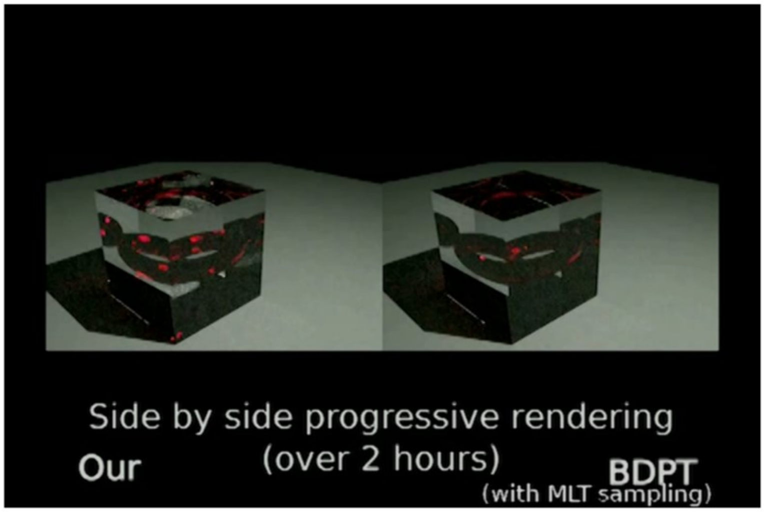
Can achieve more uniform convergence with MLT



MLT, 1 min | MLT + Regularization, 1 min | Reference

One example of applying regularization framework is demonstrated as one of the technical briefs at SIGGRAPH Asia 2013.
This work applies the regularization to **all** paths and combines with unbiased techniques using MIS in MLT framework.
You can observe more uniform convergence with regularization, which demonstrates that tempering the integrand can help finding and exploring image features.

Practical Convergence of MLT

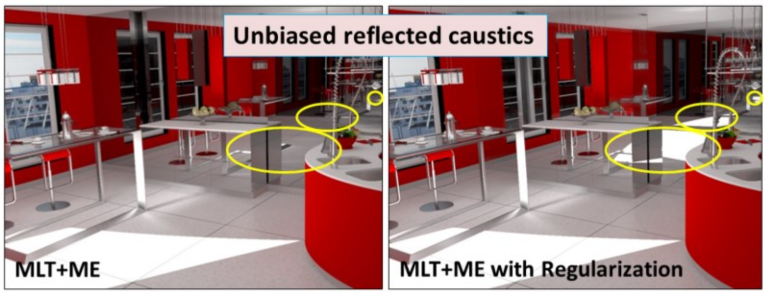


This is a video demonstrating the uniformness of MLT convergence, when applying regularization to make the integrand less spiky.

Combination with Manifold Exploration

Enables unbiased sampling of non-sampleable paths!
- Regularization provides local parameterization
- Manifold exploration explores the surrounding paths

Avoids the undecidability
- By finding a local parameterization
- Undecidable paths are sampled “almost surely”



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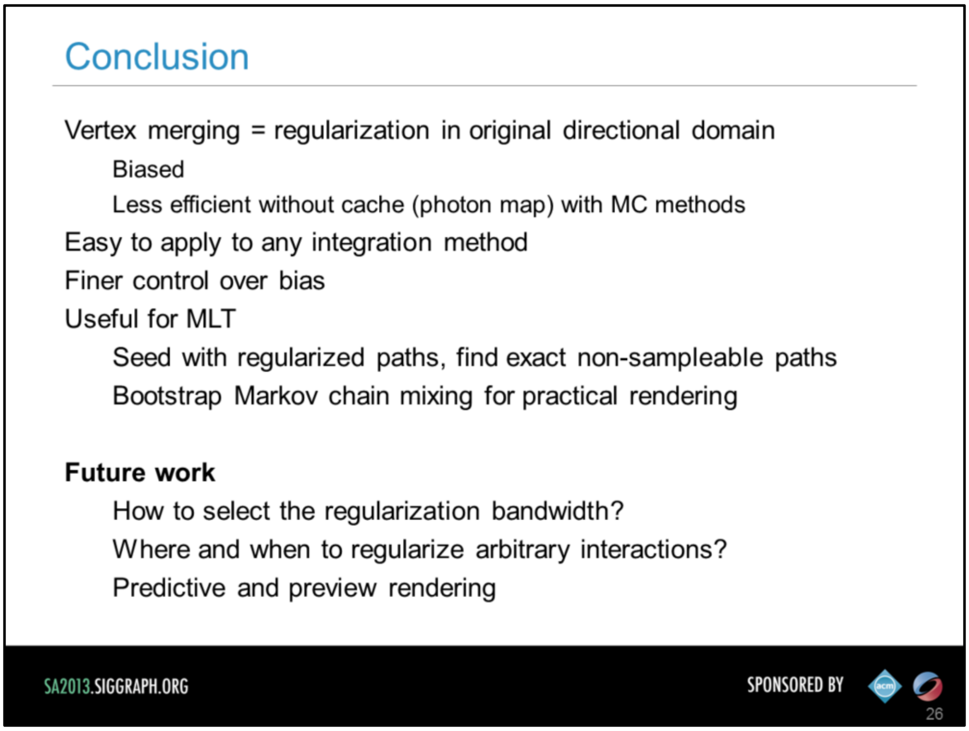
Recent manifold exploration mutation can connect two vertices through a chain of specular interactions, but only given a **valid local parameterization** for the connection.
Regularized BDPT can provide the local parameterization for non-samplable paths **almost surely** (i.e., with probability one), making the sampling of such paths **practical**.
This way manifold exploration **fixes** the regularized biased path it was seeded with; and can explore the whole feature in an unbiased manner.
Please note that we do not solve the undecidability this way; we avoid **stating** undecidable problems.



To conclude my talk, vertex merging can be interpreted as regularization in original directional domain.
It introduces bias, and thus it is less efficient than PPM and VCM, when combined with ordinary MC methods. However, even just adding regularization to a simple path tracer can be very practical for rendering difficult paths, like caustics, more efficiently. It is also simple to integrate into existing renderers.
We have showed that thinking about vertex merging as a regularization helps explaining why it can sample some difficult paths. Moreover, this formulation allows to introduce the regularization selectively, only where it is necessary.
It can be also of immediate practical use for MLT-class methods.
It can be used as a seeding technique for MLT, allowing advanced mutation strategies to employ it, for finding the exact path with machine precision. On the other hand, it allows Markov chain to find and explore different subspaces of the path space more easily, leading to more uniform image convergence.

However, it also opens many opportunities for potential research in light transport. Here are just some of the possible future improvements we might expect to be investigated soon.

Thank you for your attention