At the Boundary Between Light Transport Simulation and Computational Statistics

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MCQMC2014

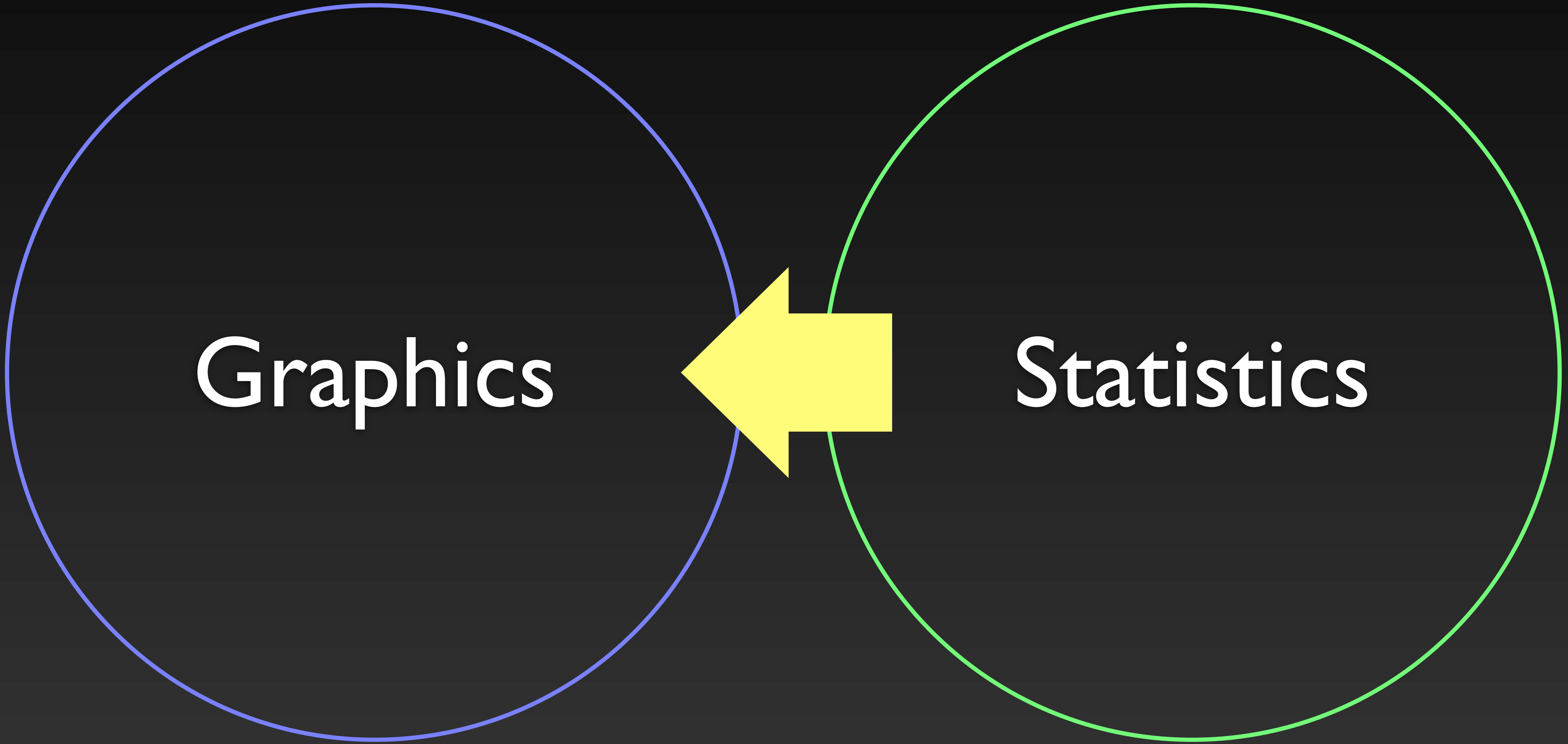
At the boundary

Me

$$I_j = \mathrm{E}\left[\frac{f_j(X)}{p(X)}\right]$$

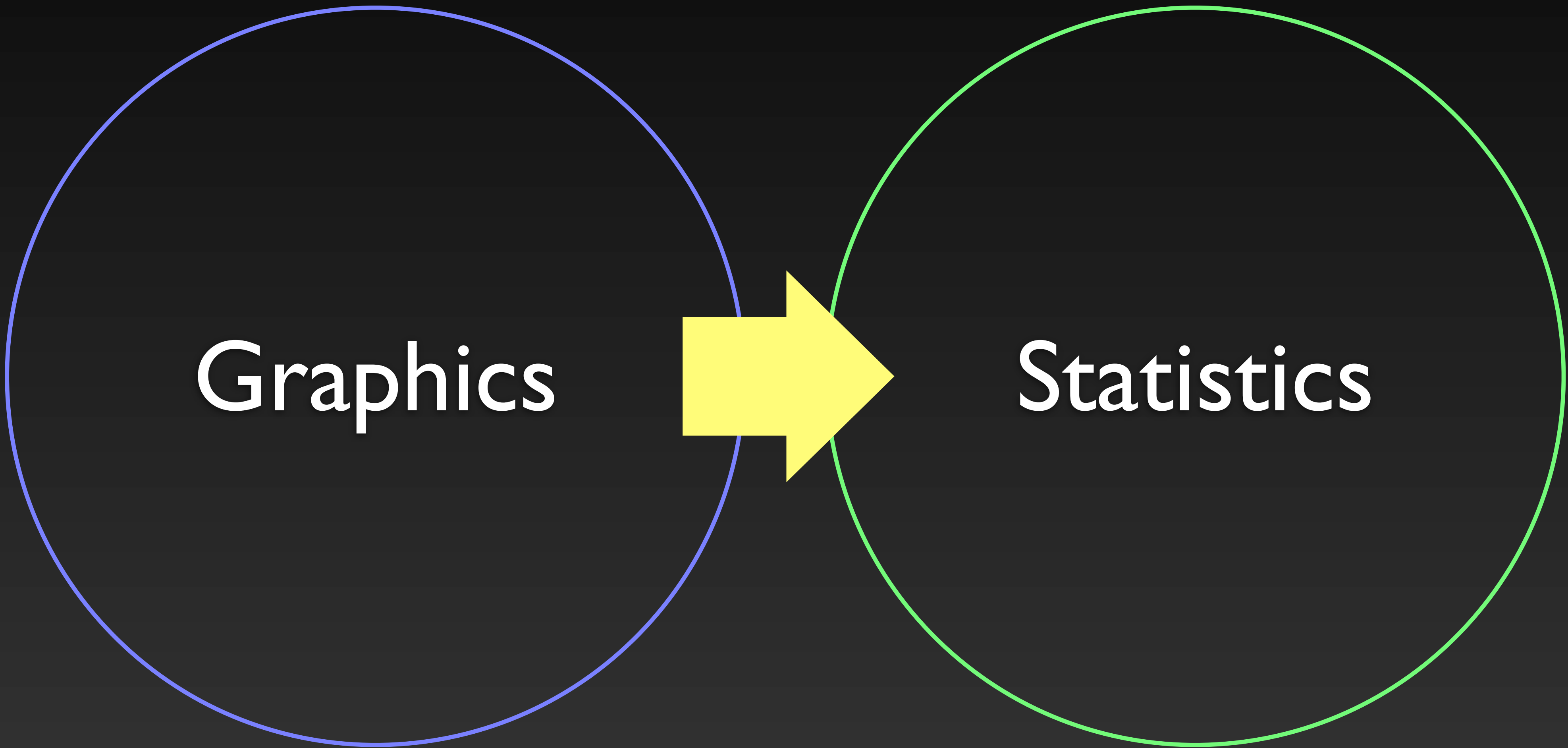
What's often considered

- Graphics communities "just use" statistics



What I hope to see

- Statistics communities take something from graphics



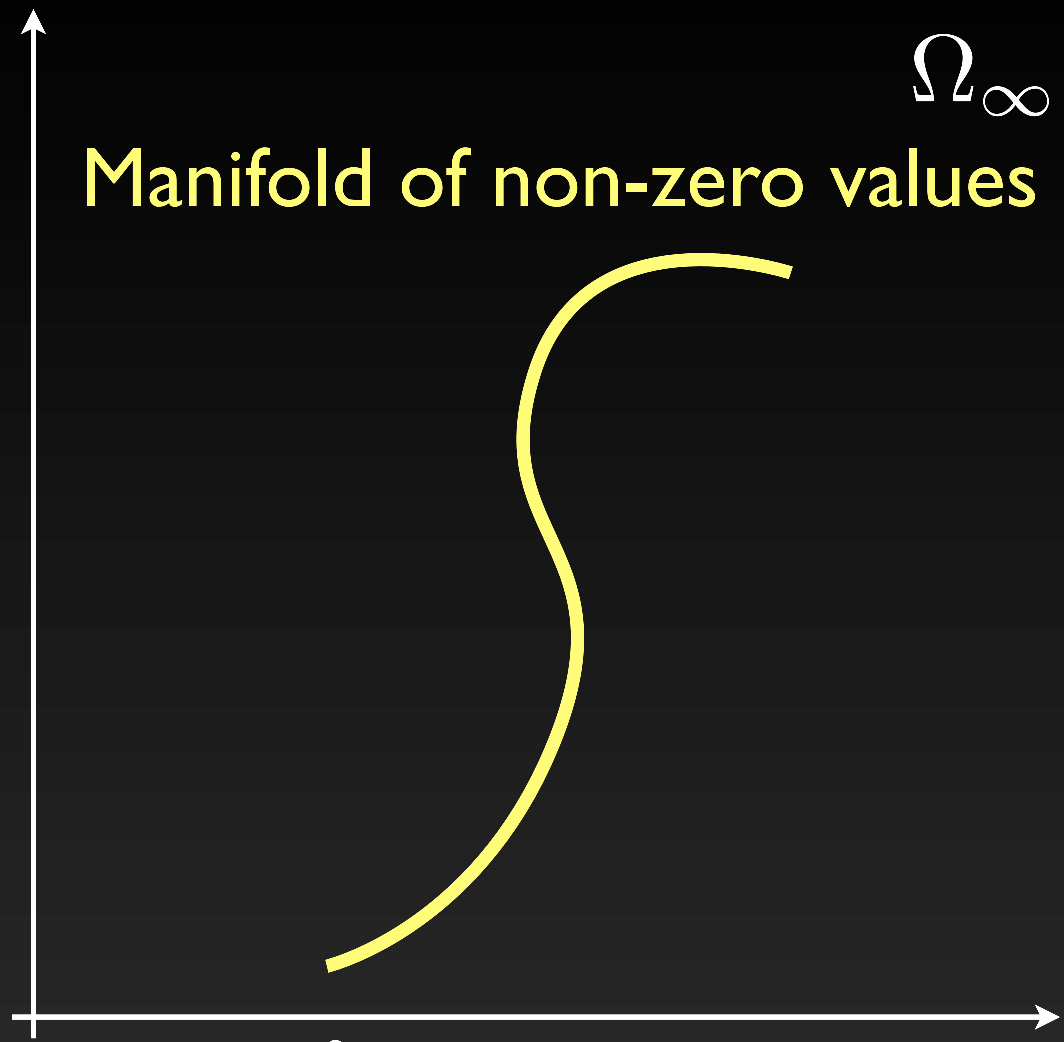
Aim of this talk

- Motivate you to facilitate cross-pollination
  - Both ways (graphics ↔ statistics)
  - Using two examples
    - Progressive density estimation
    - Primary space serial tempering

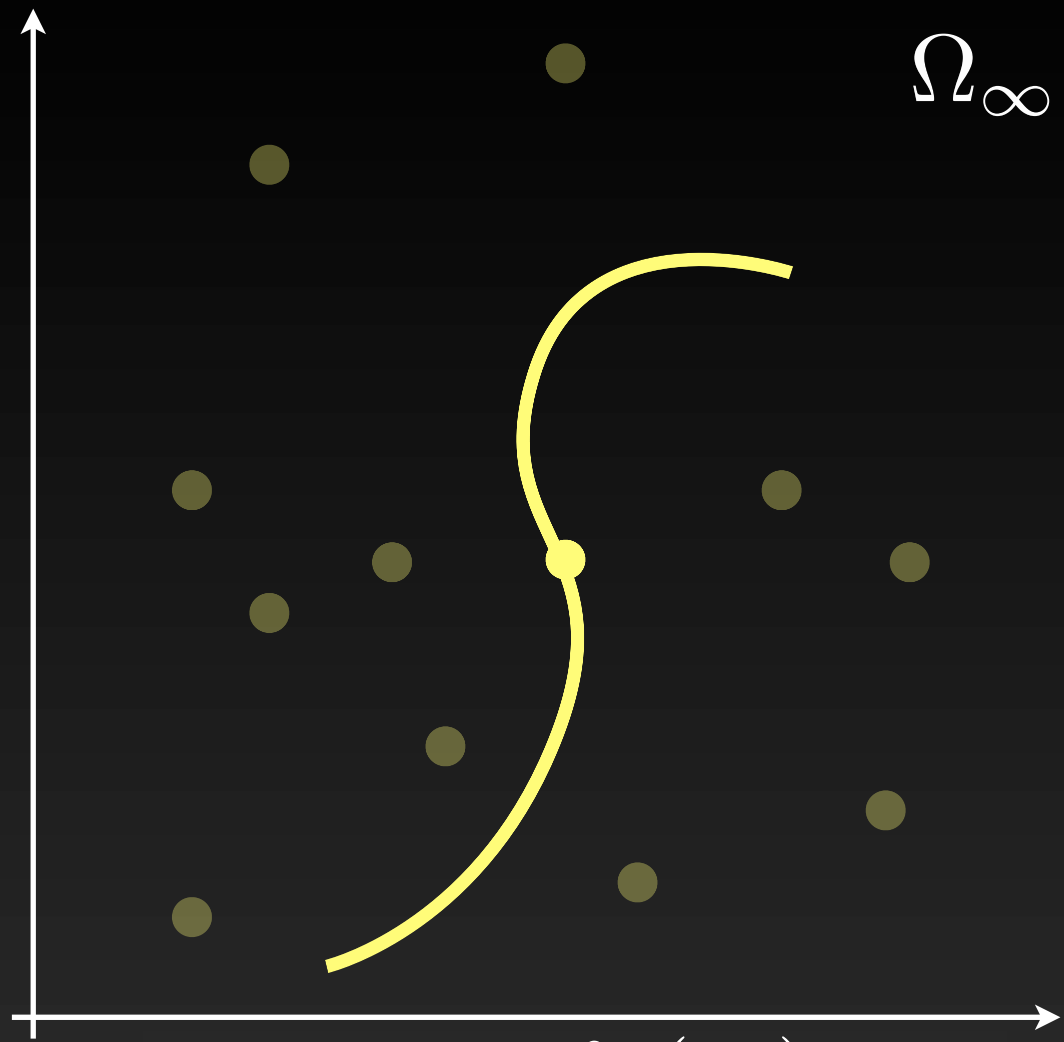
“Progressive Photon Mapping” Hachisuka et al. ACM SIGGRAPH Asia 2008

“Multiplexed Metropolis Light Transport” Hachisuka et al. ACM SIGGRAPH 2014 (conditionally accepted)

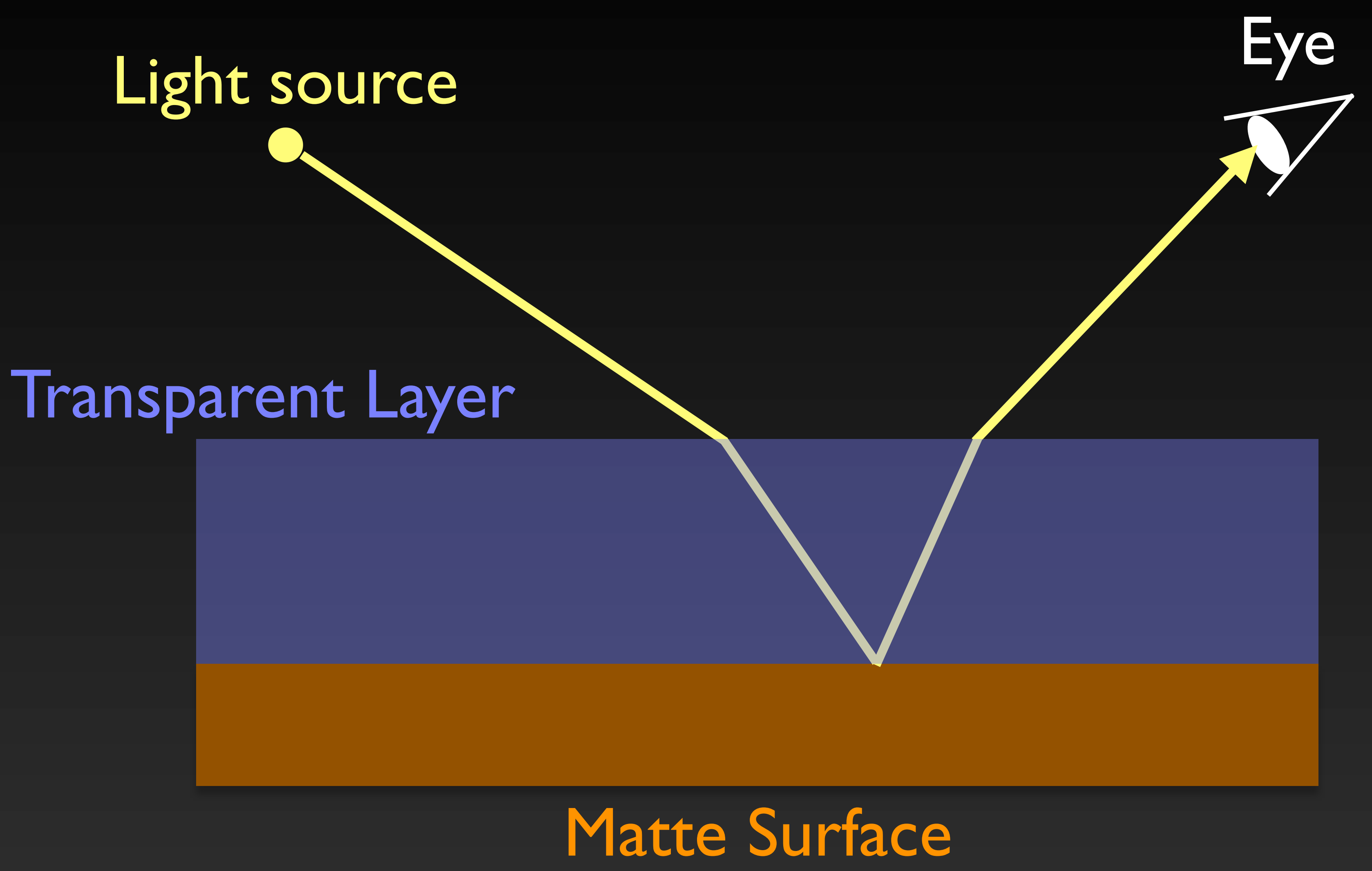
Progressive Density Estimation

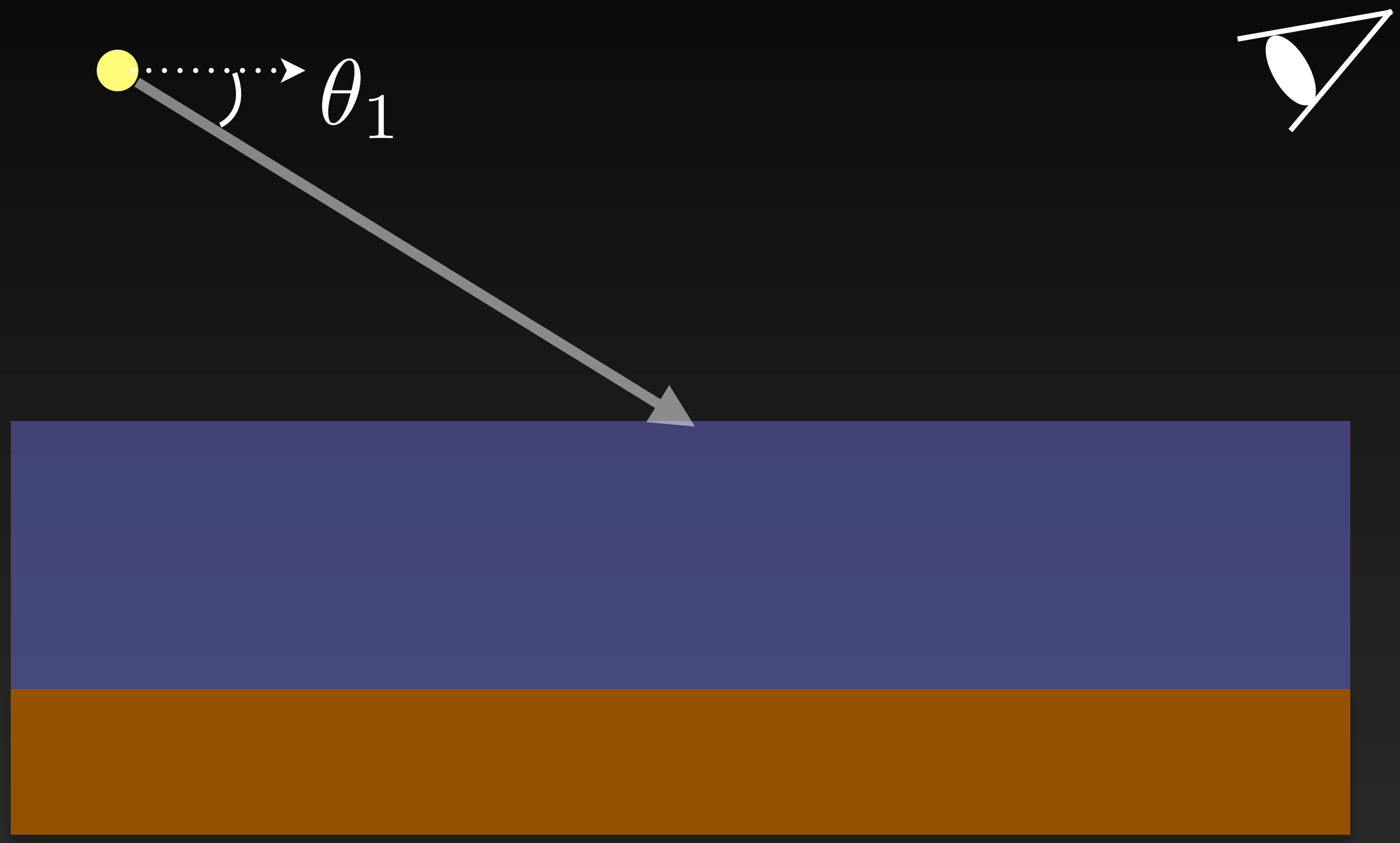


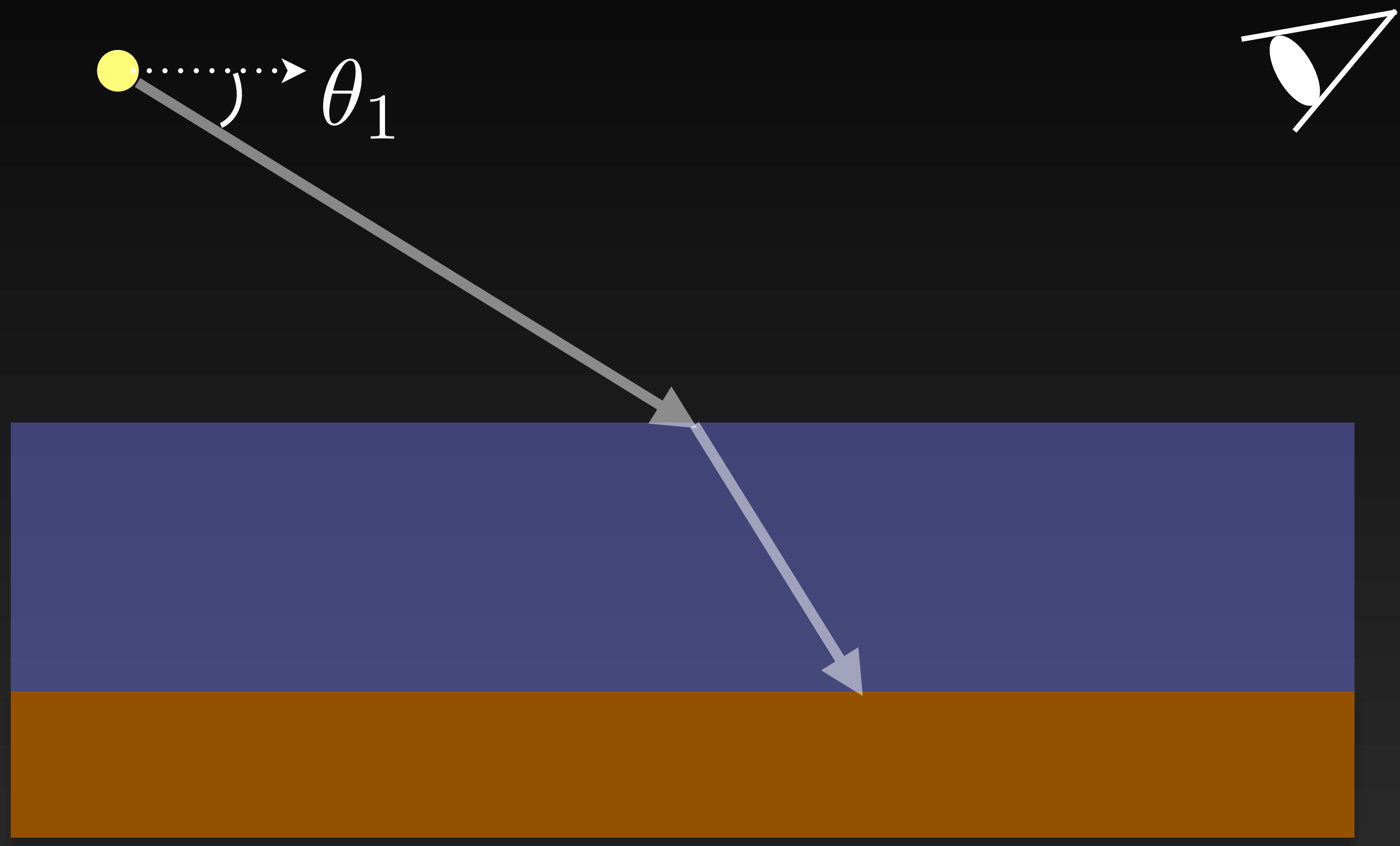
$$L = \int_{\Omega_\infty} f_j\left(\bar{X}\right) d\mu(\bar{X})$$

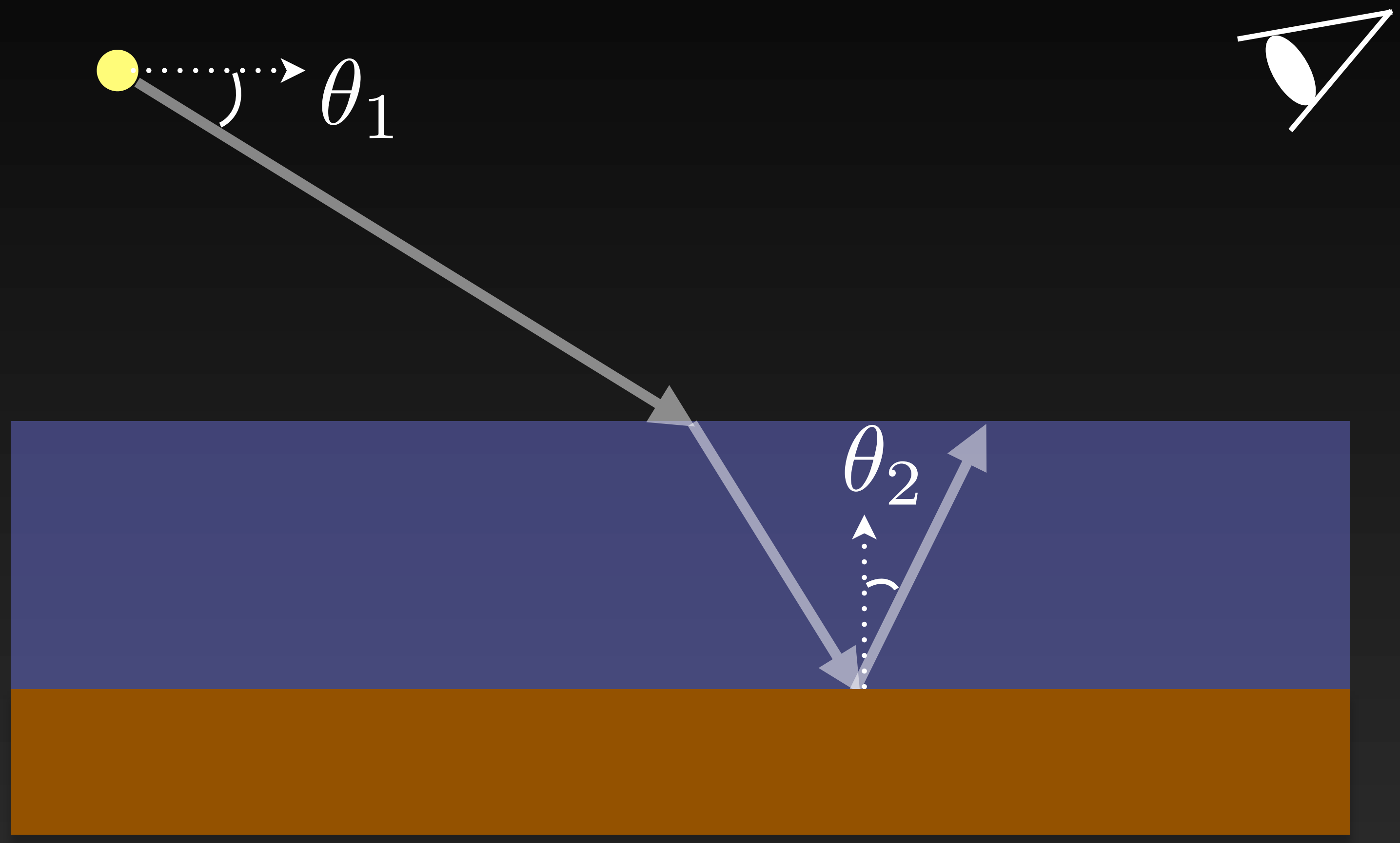


$$L \approx \frac{1}{N} \sum_i \frac{f_j\left(\bar{x}_i\right)}{p(\bar{x}_i)} \qquad \bar{x}_i \sim p(\bar{x}_i)$$

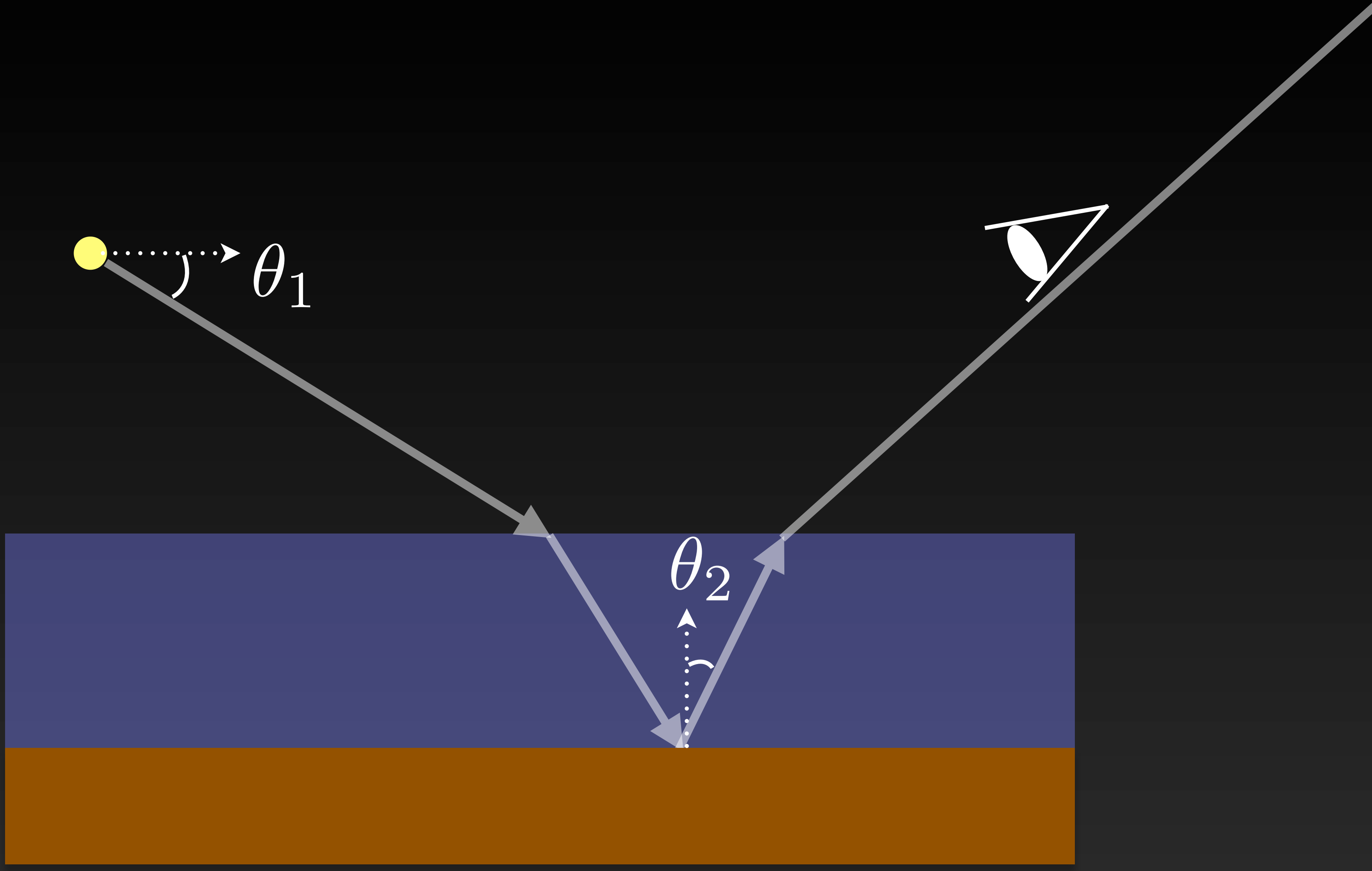

Light source
Eye
Transparent Layer
Matte Surface


θ_1

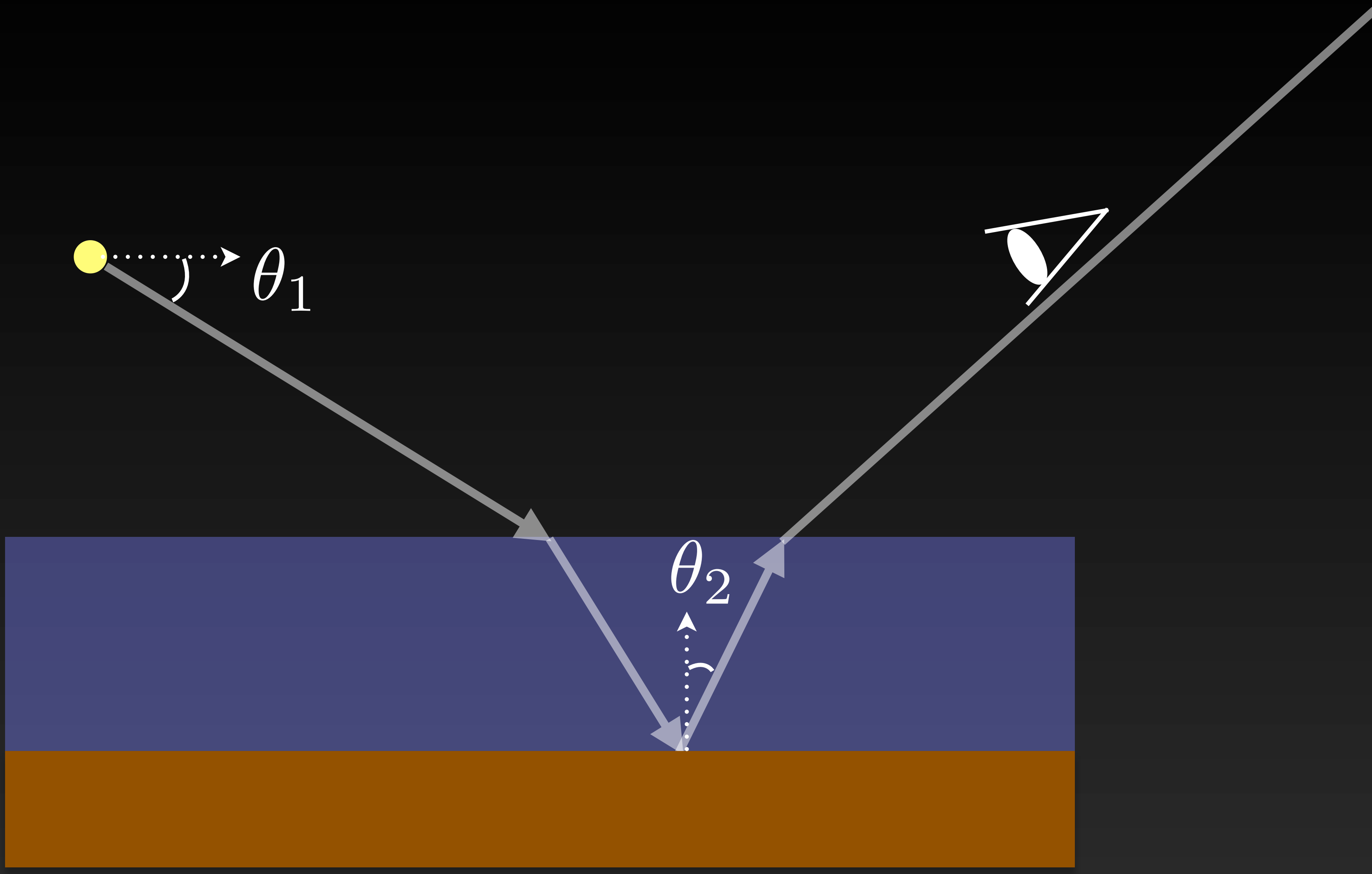

θ_1



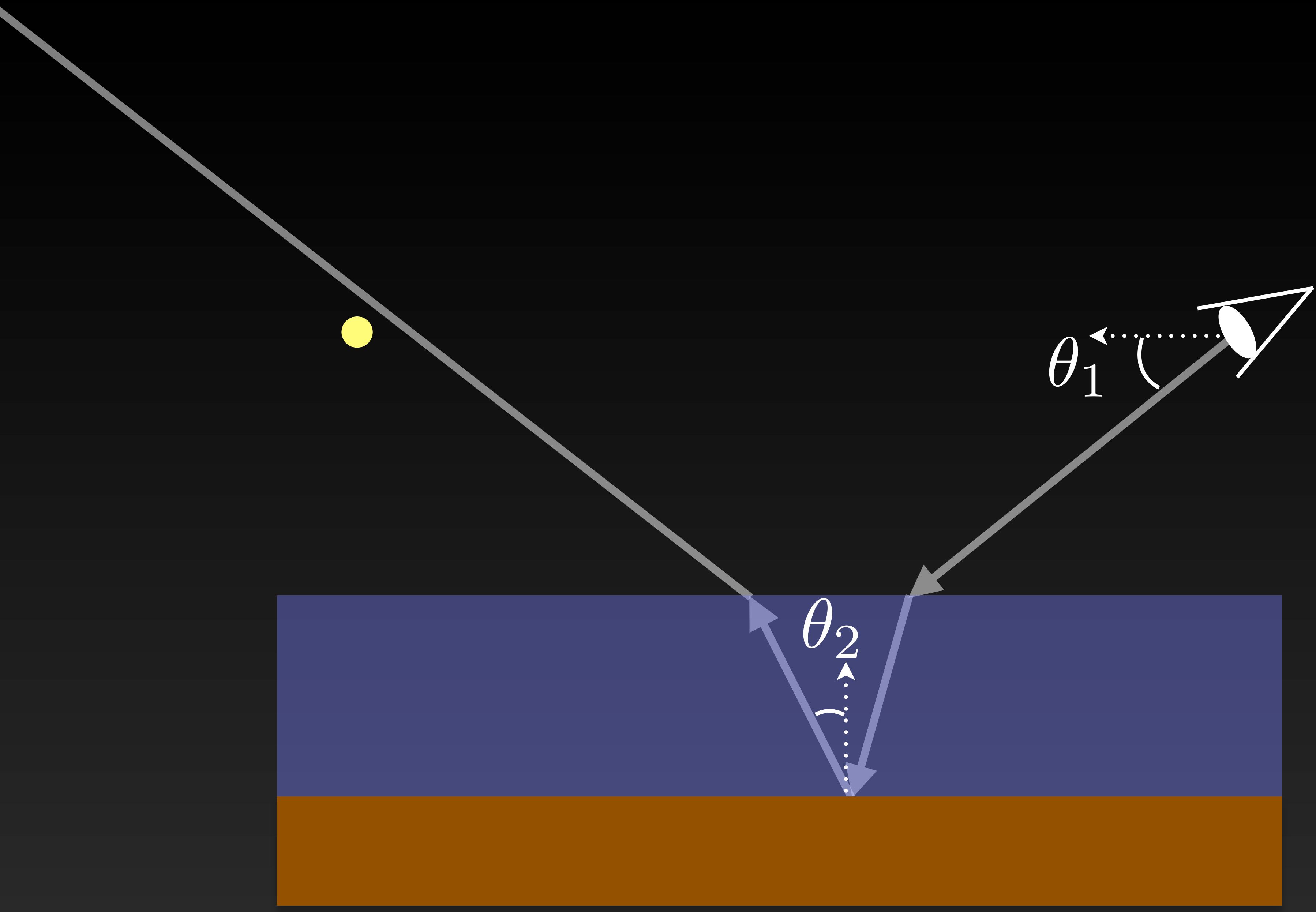
θ_1
θ_2



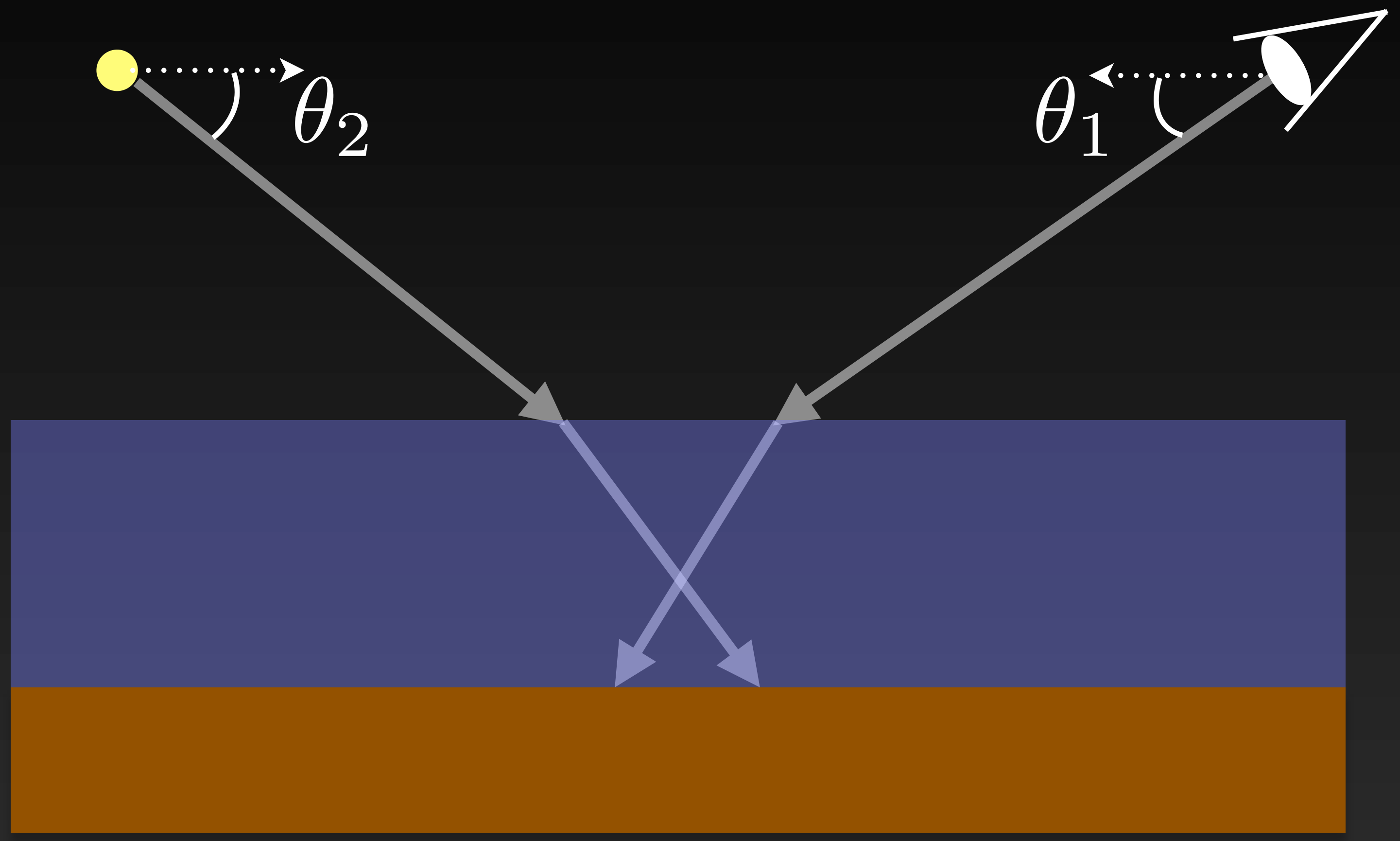
θ_1
θ_2



Probability of sampling this path is nearly zero

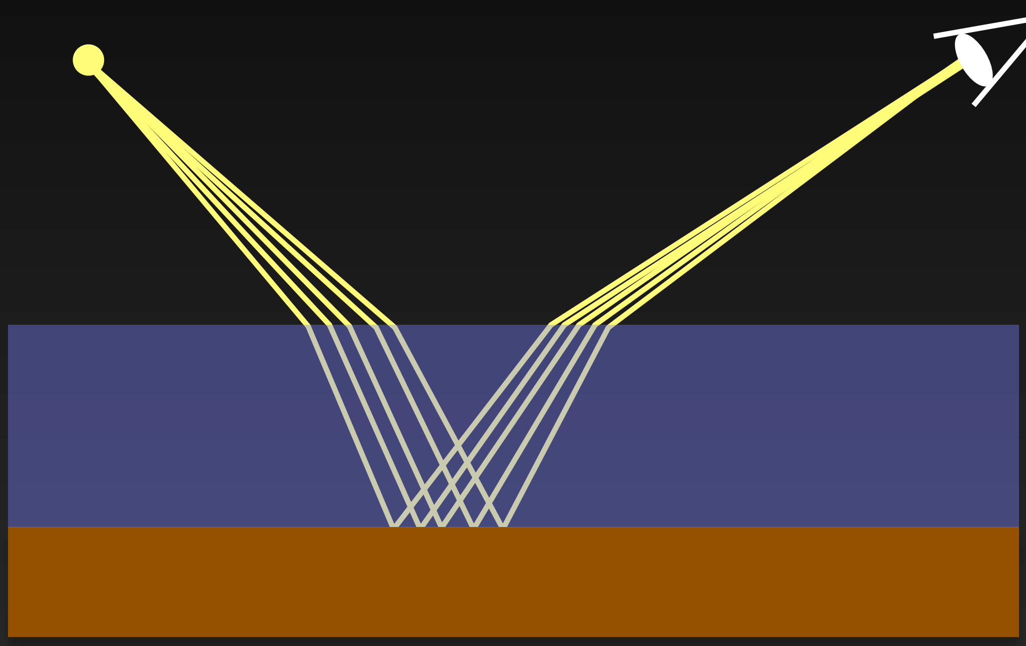


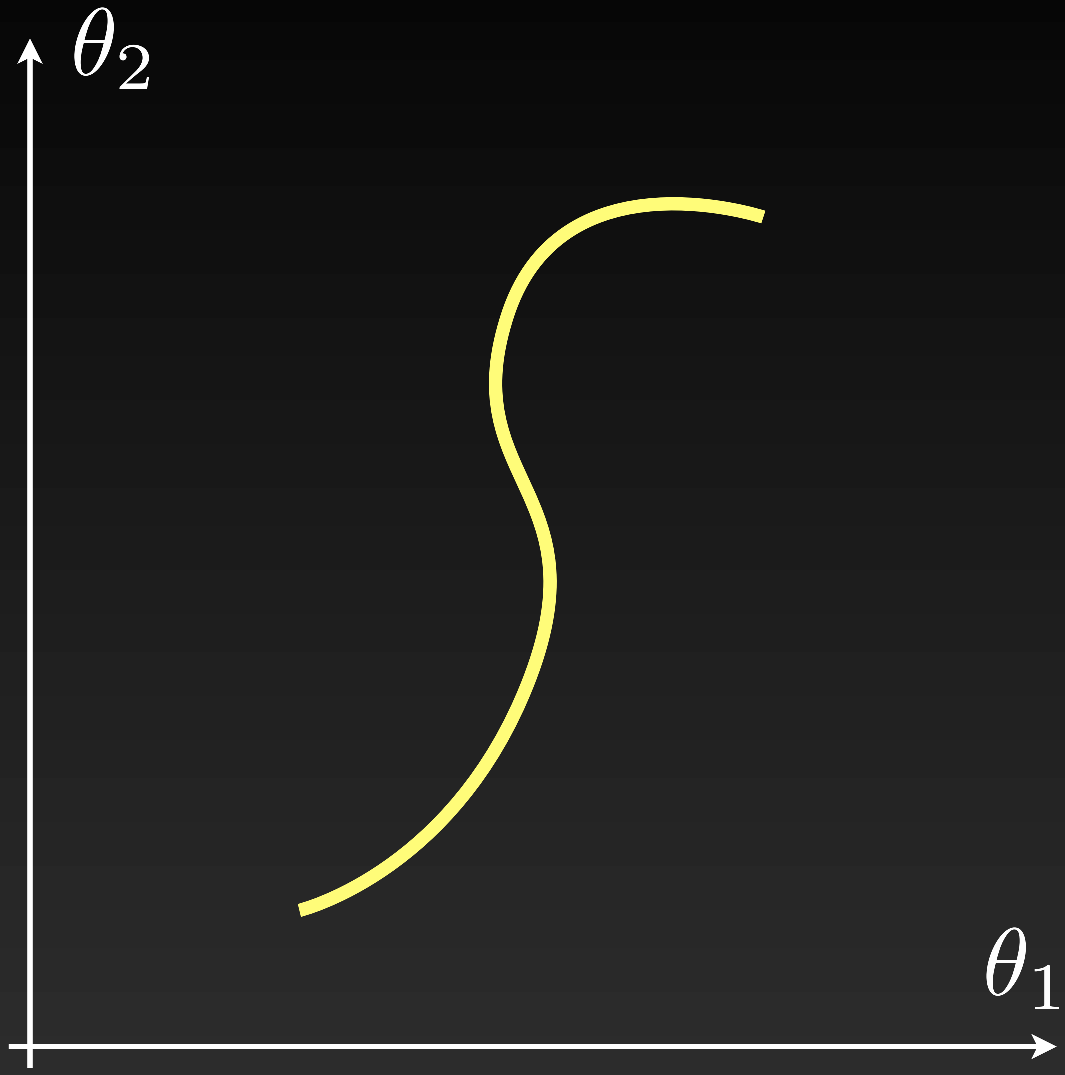
Probability of sampling this path is nearly zero

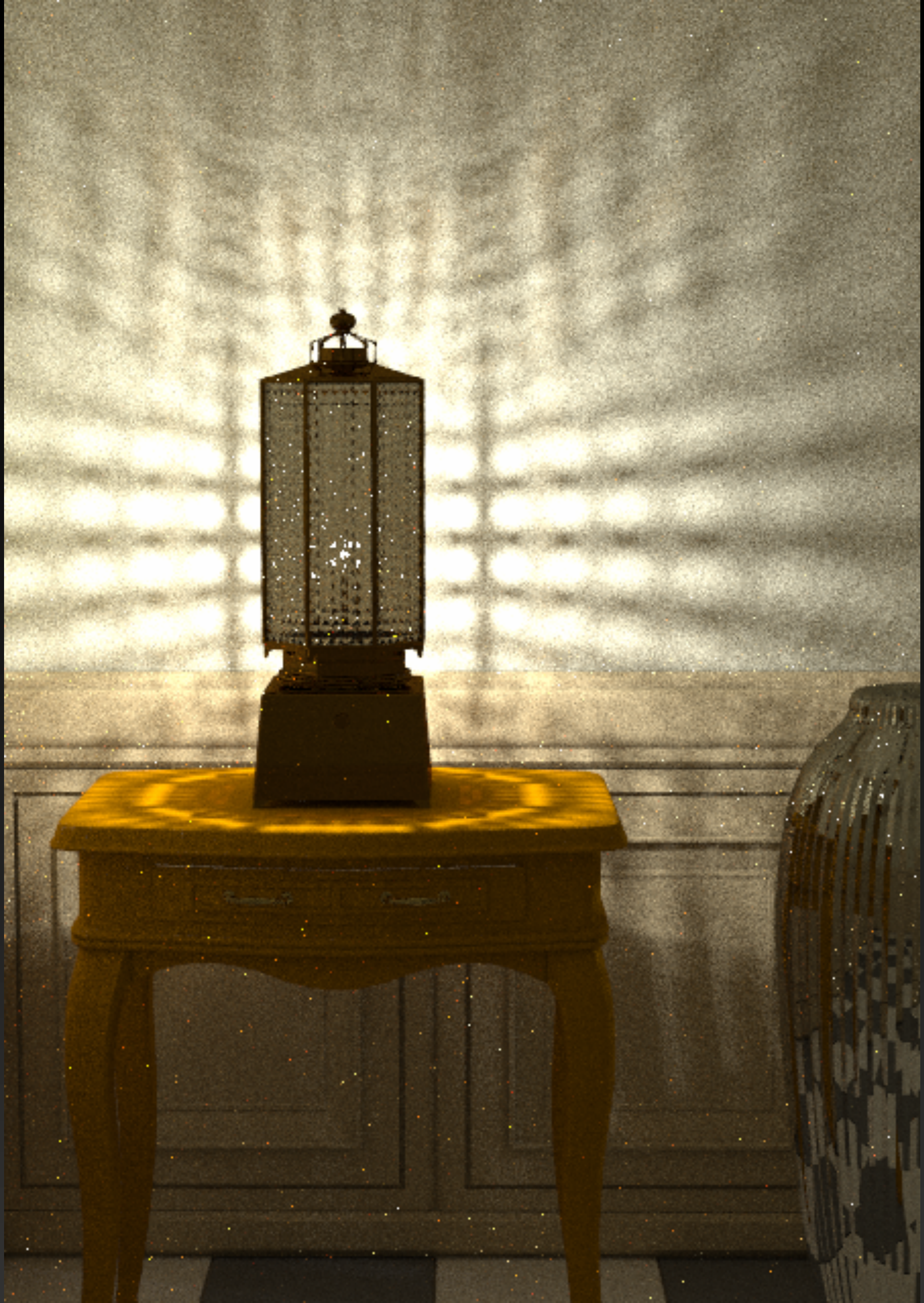


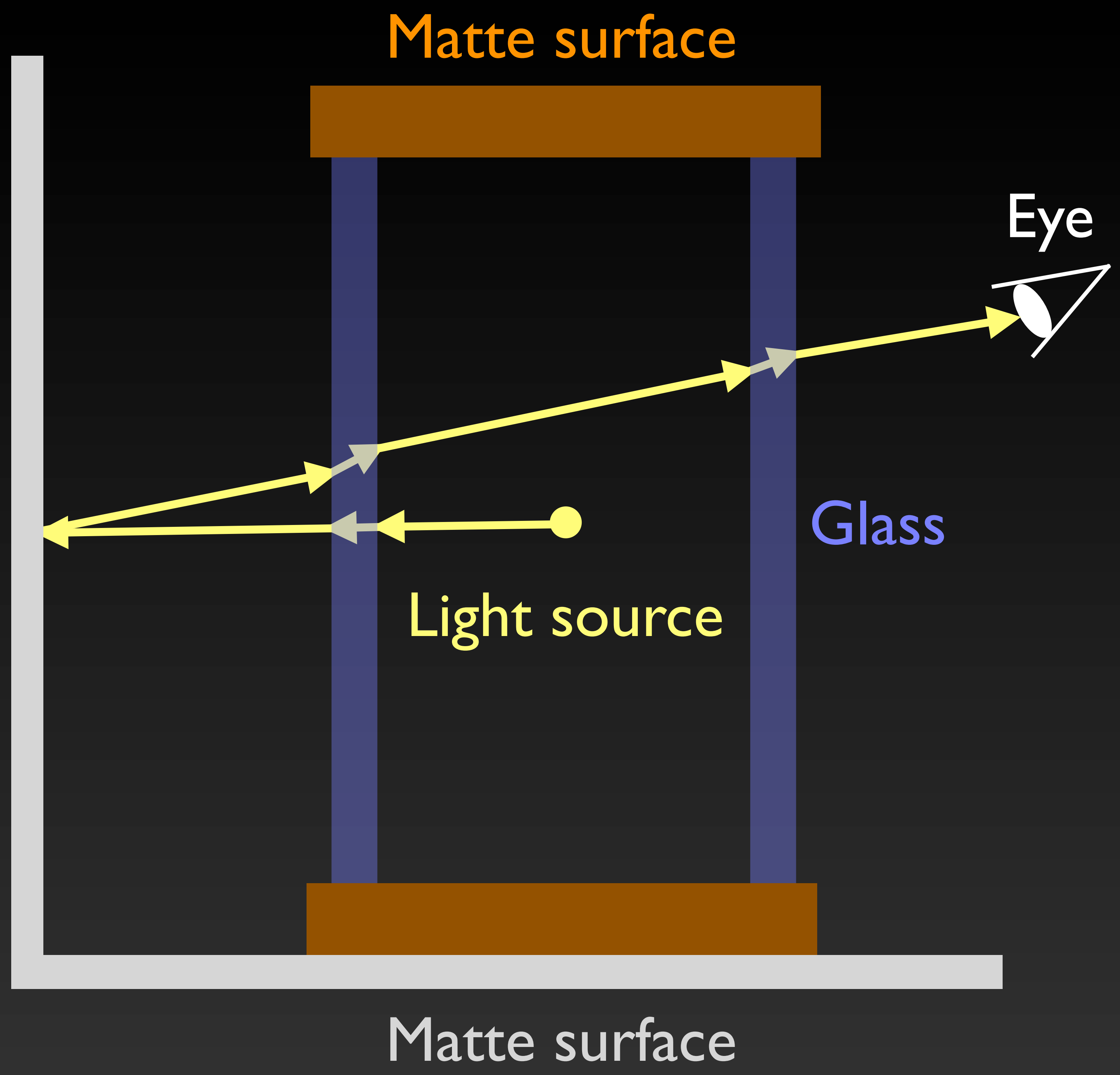
Probability of hitting at the same point is zero

Manifold of non-samplable paths



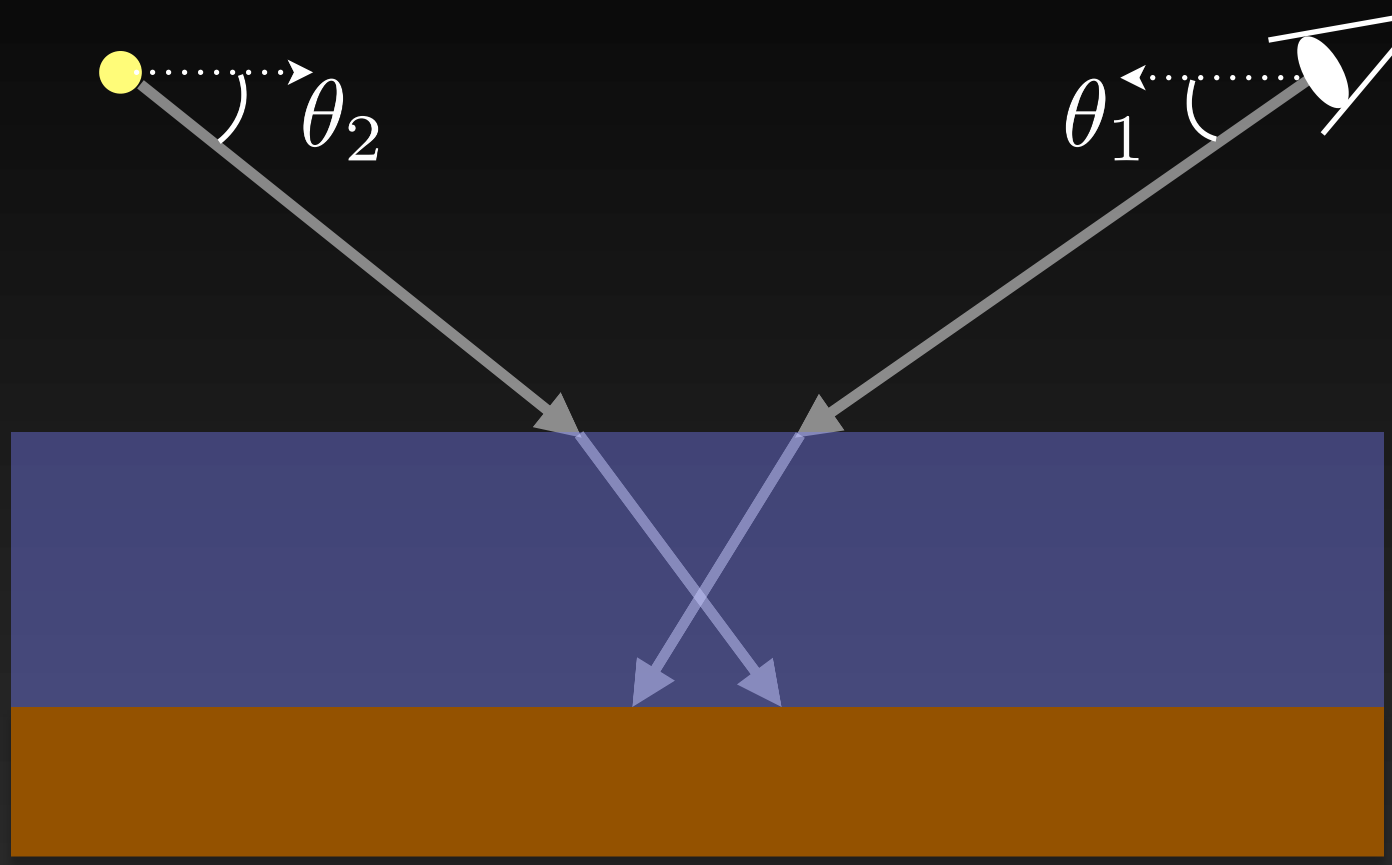


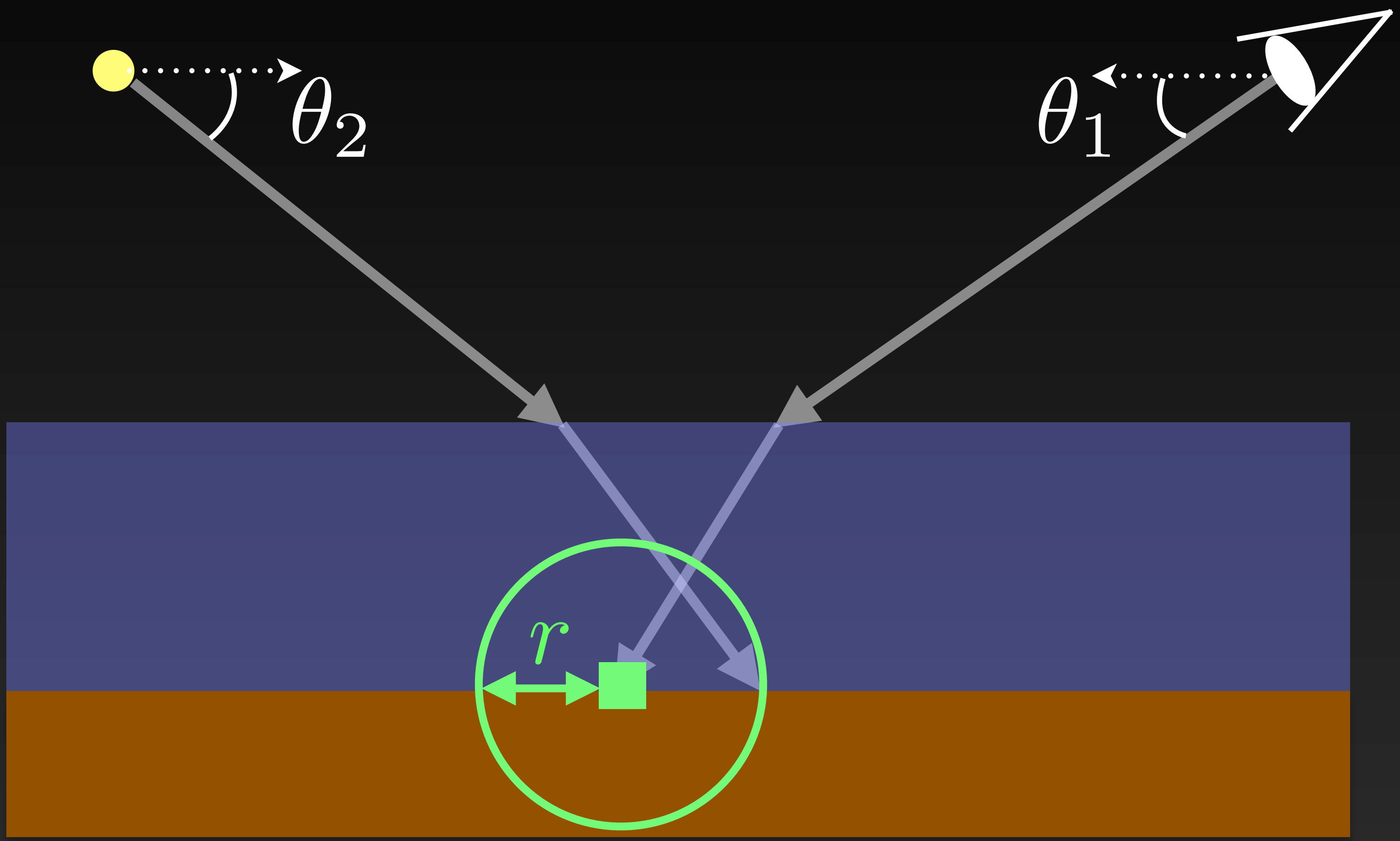



Matte surface
Eye
Glass
Light source
Matte surface

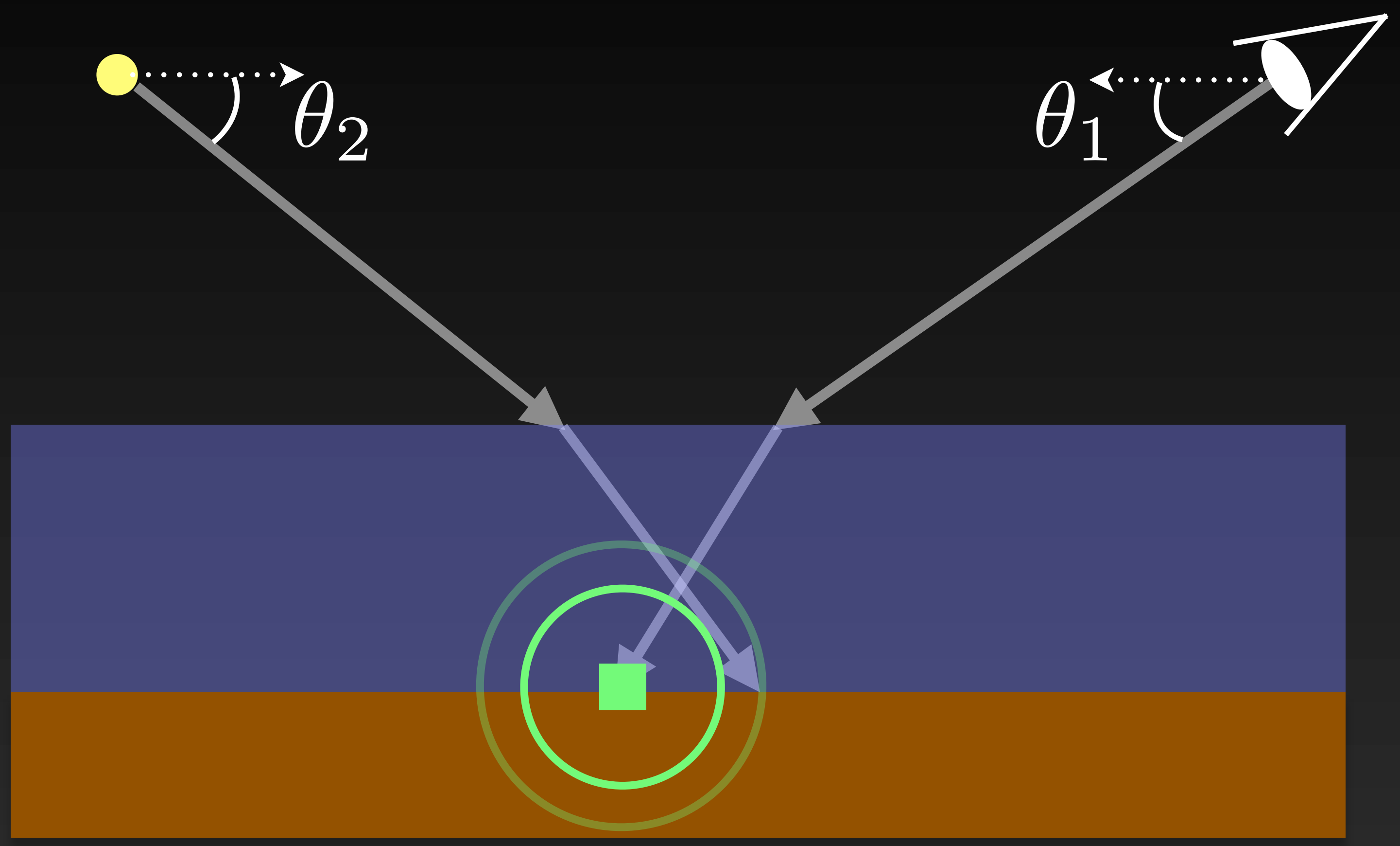
How can we capture non-samplable paths?

Progressive Density Estimation


θ_2
θ_1



Allow approximate connection within distance r



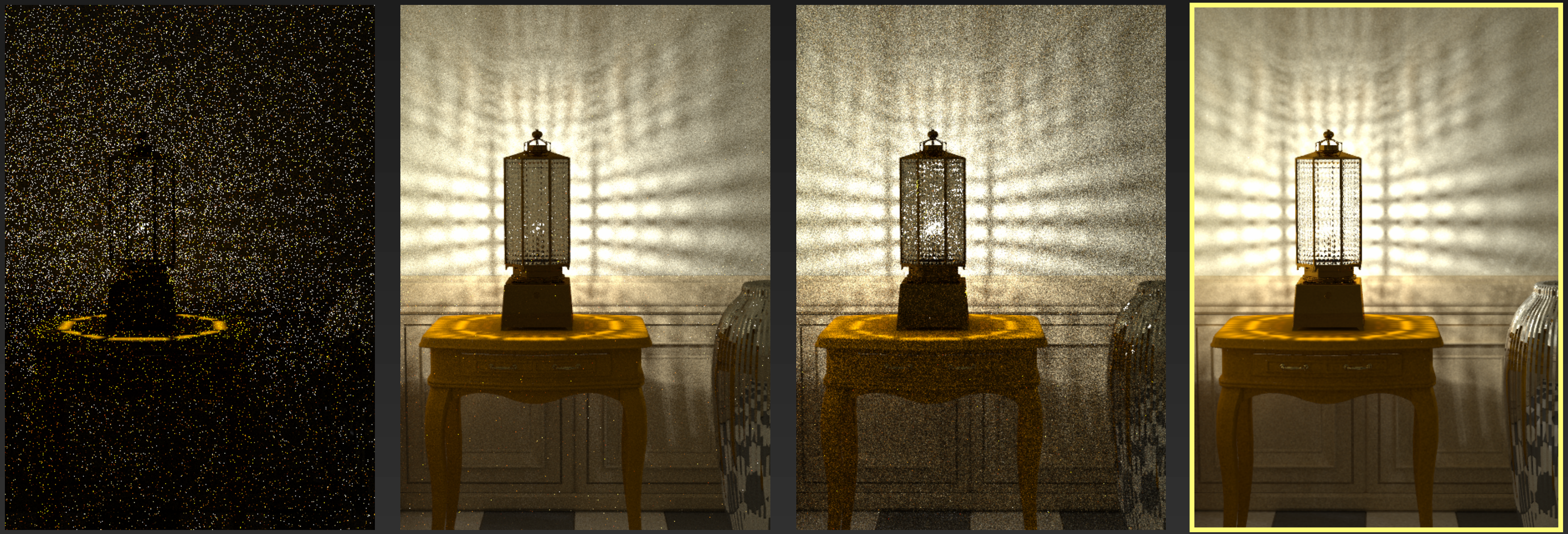
Allow approximate connection within distance r

... and progressively reduce r

Progressive density estimation

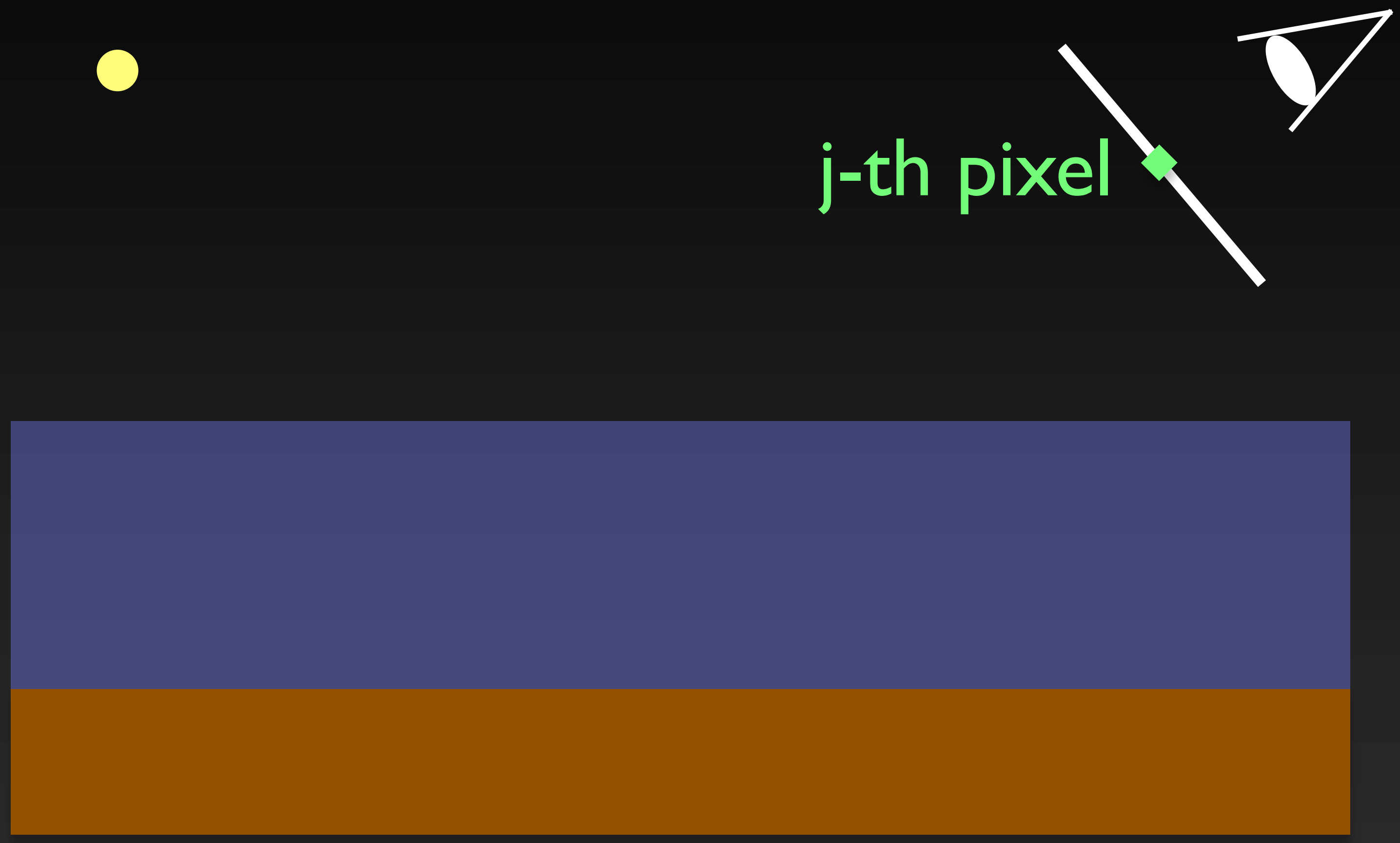
[Hachisuka et al. 2008]

- Algorithmically convergent density estimation
  - Add gradually reducing bias to relax the problem
  - First to solve the issue of non-samplable paths

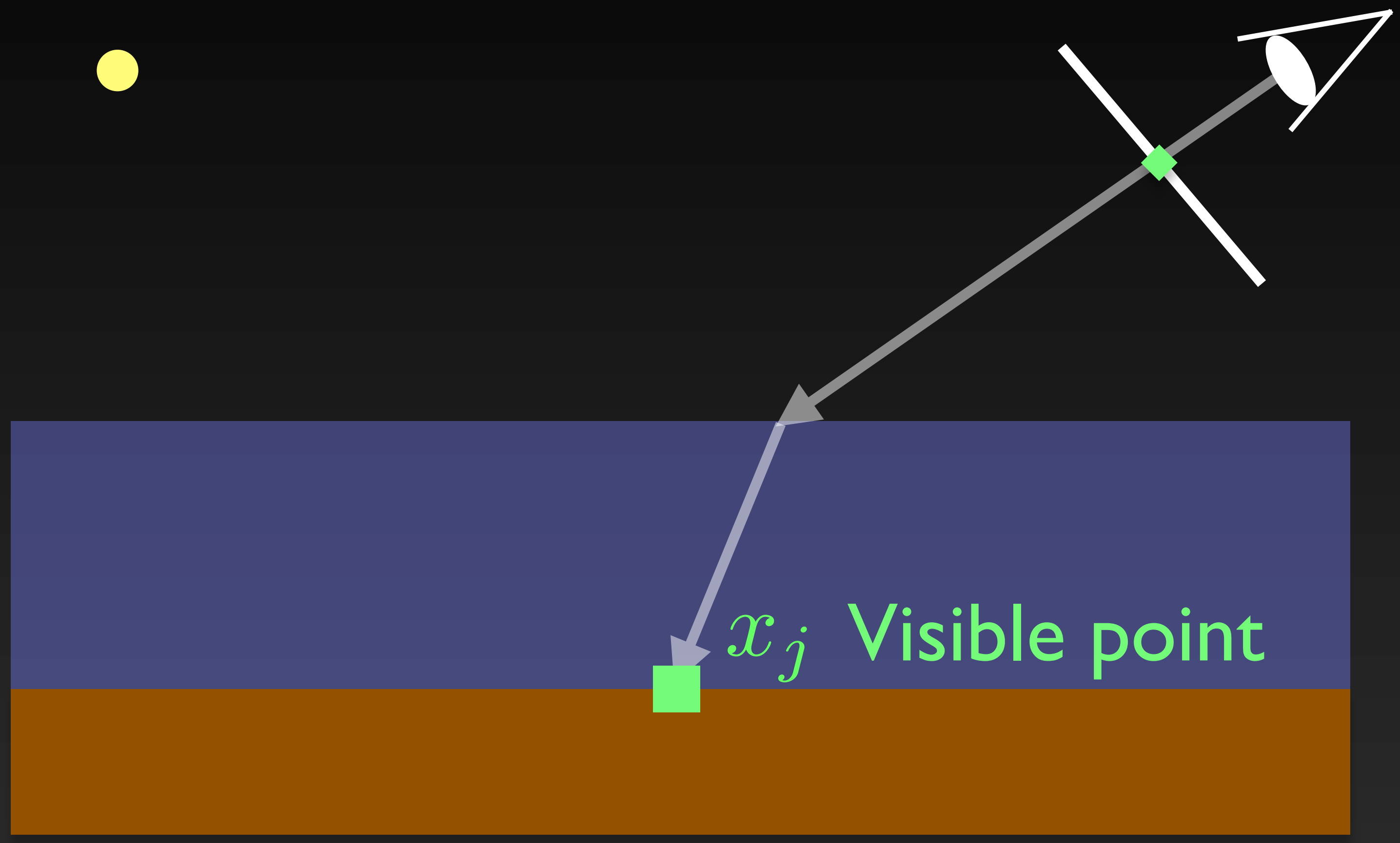


Initial pass

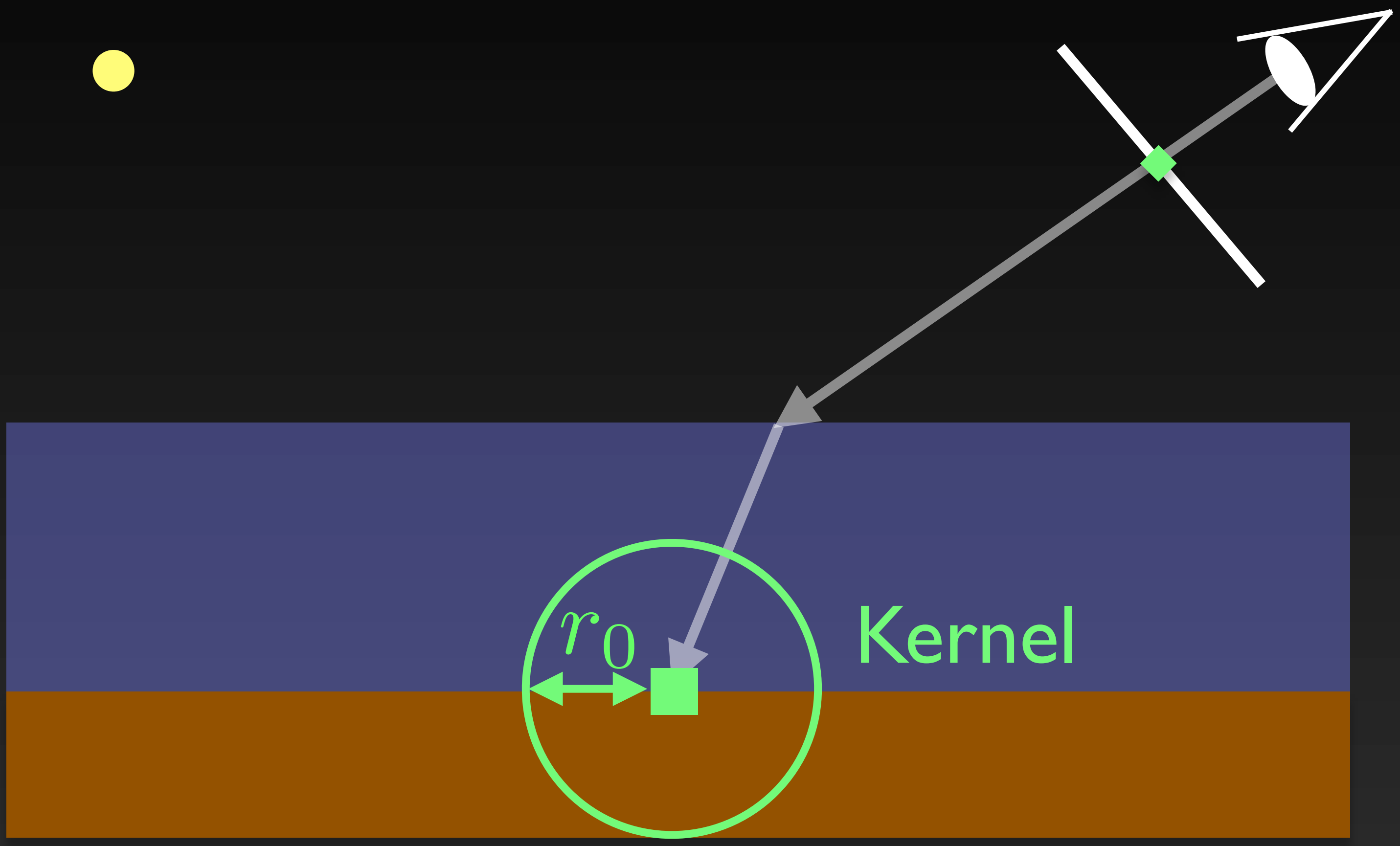
Initial pass



Initial pass



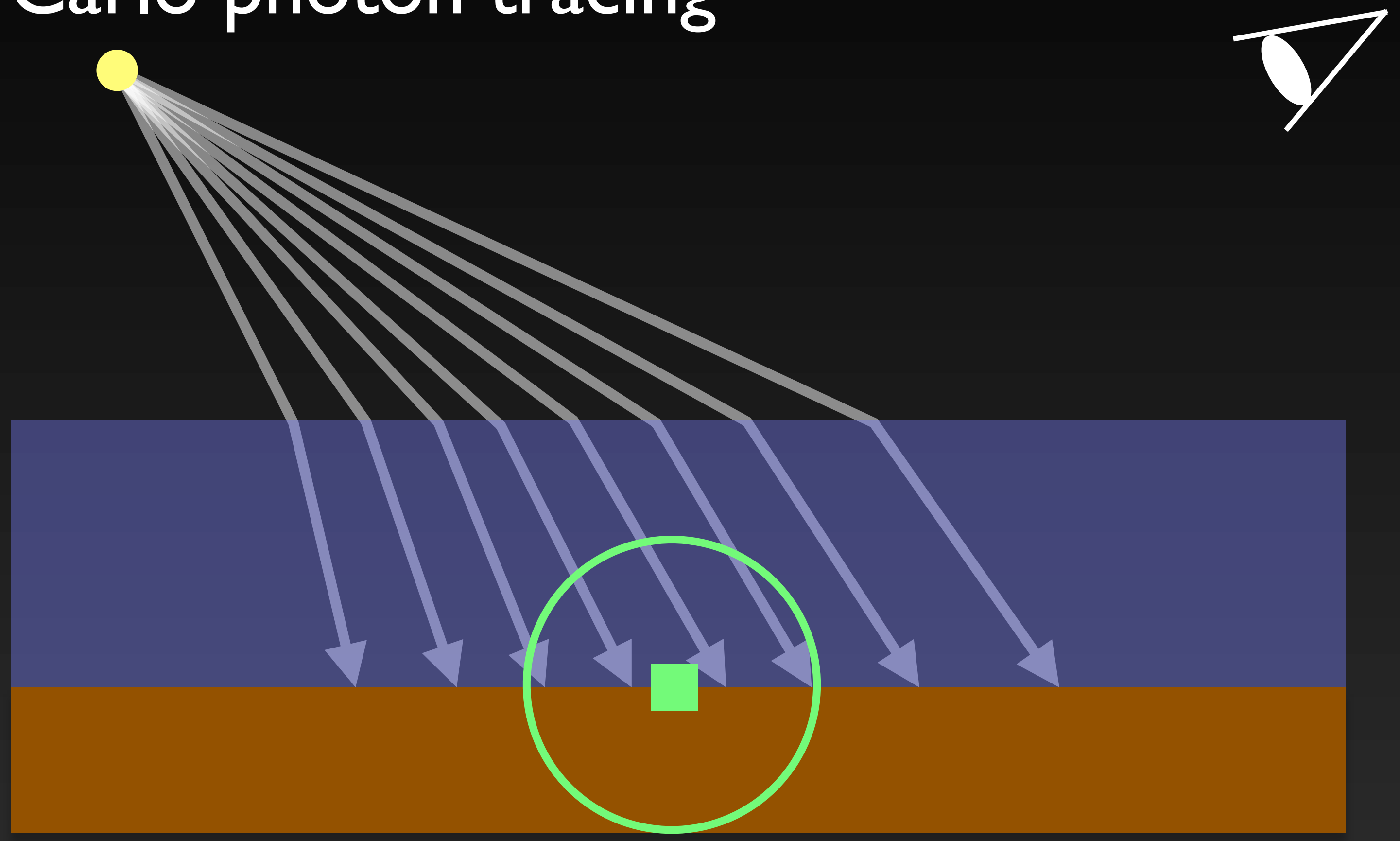
Initial pass



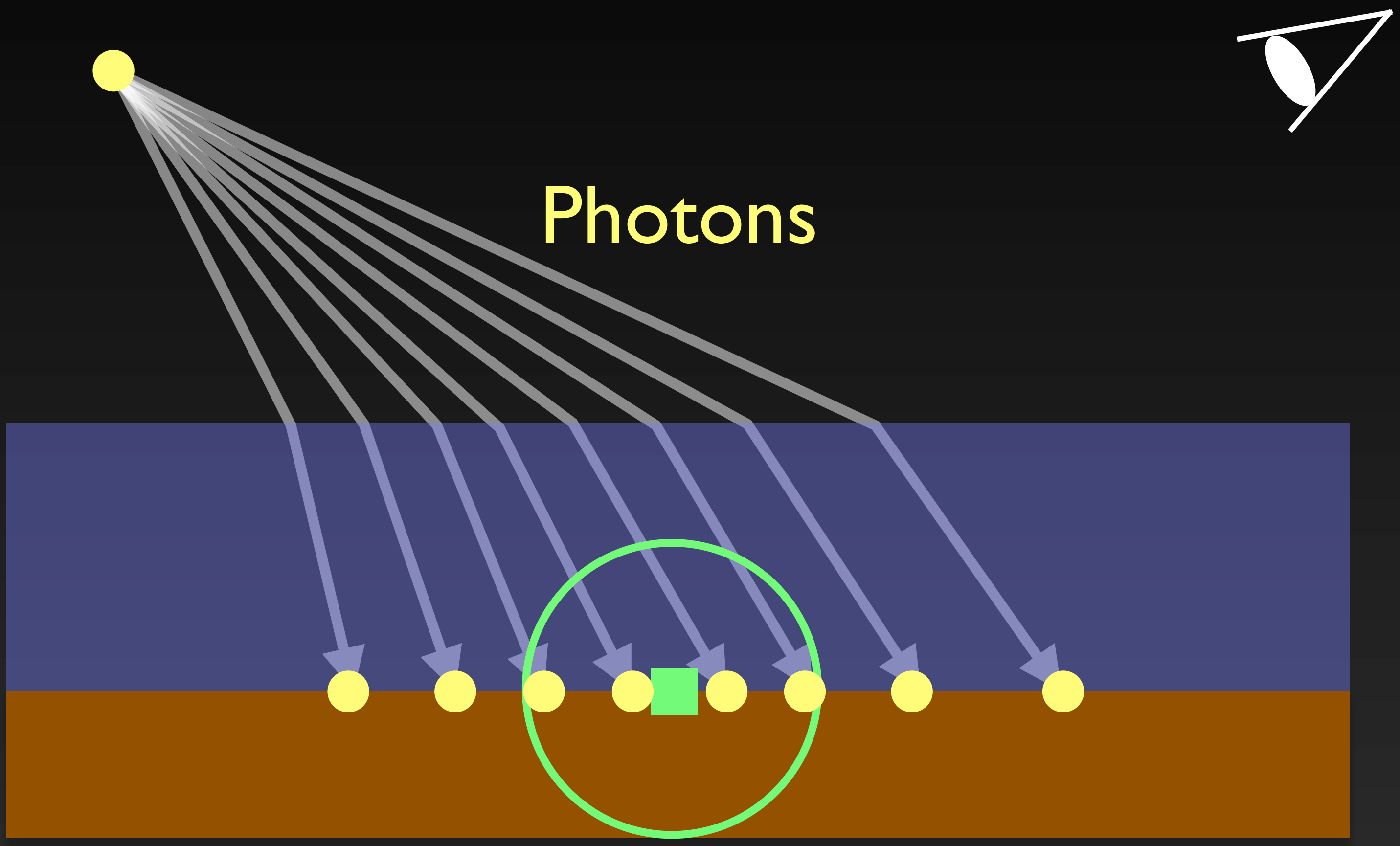
Initial kernel bandwidth: r_0

1st pass

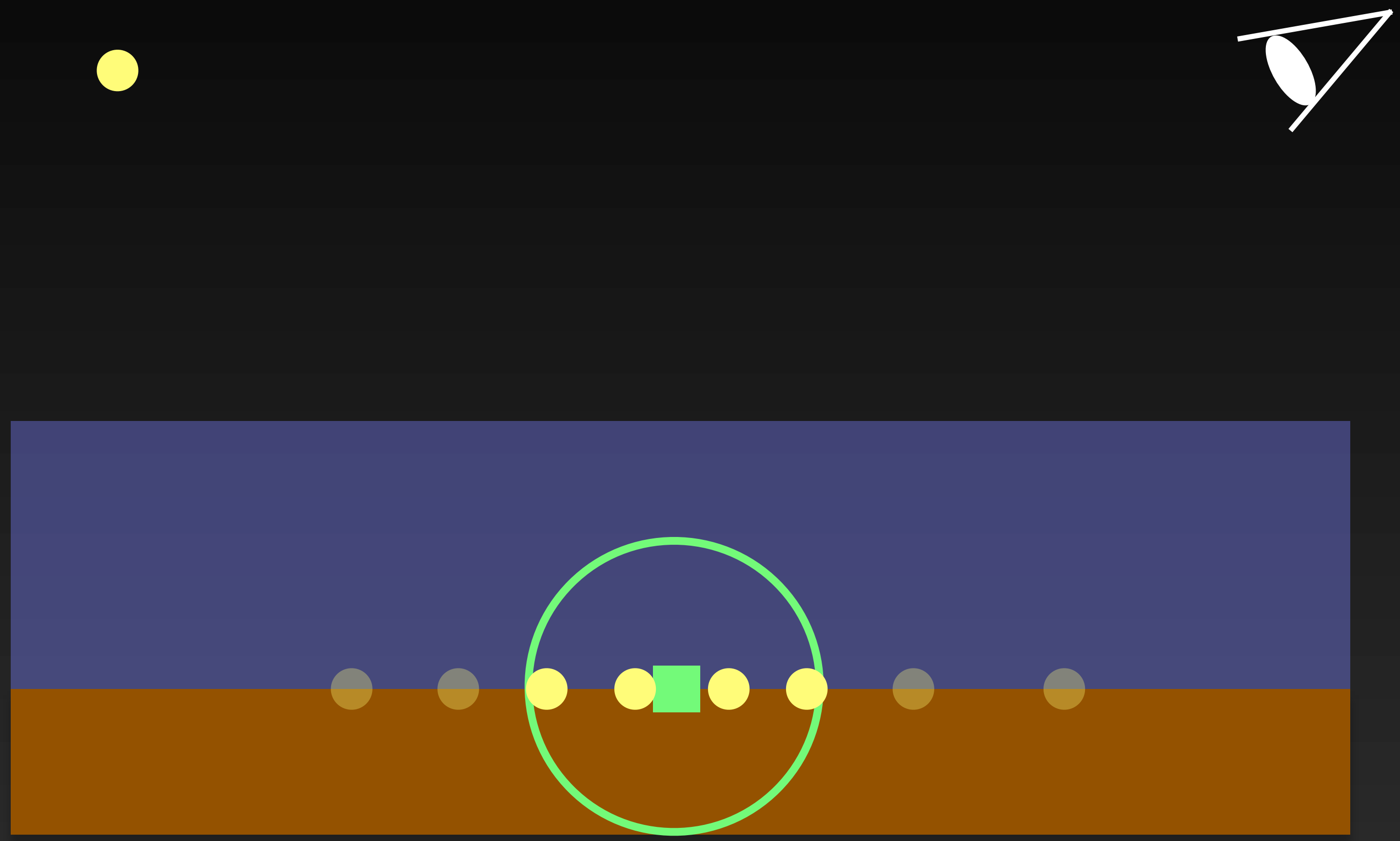
Monte Carlo photon tracing



1st pass

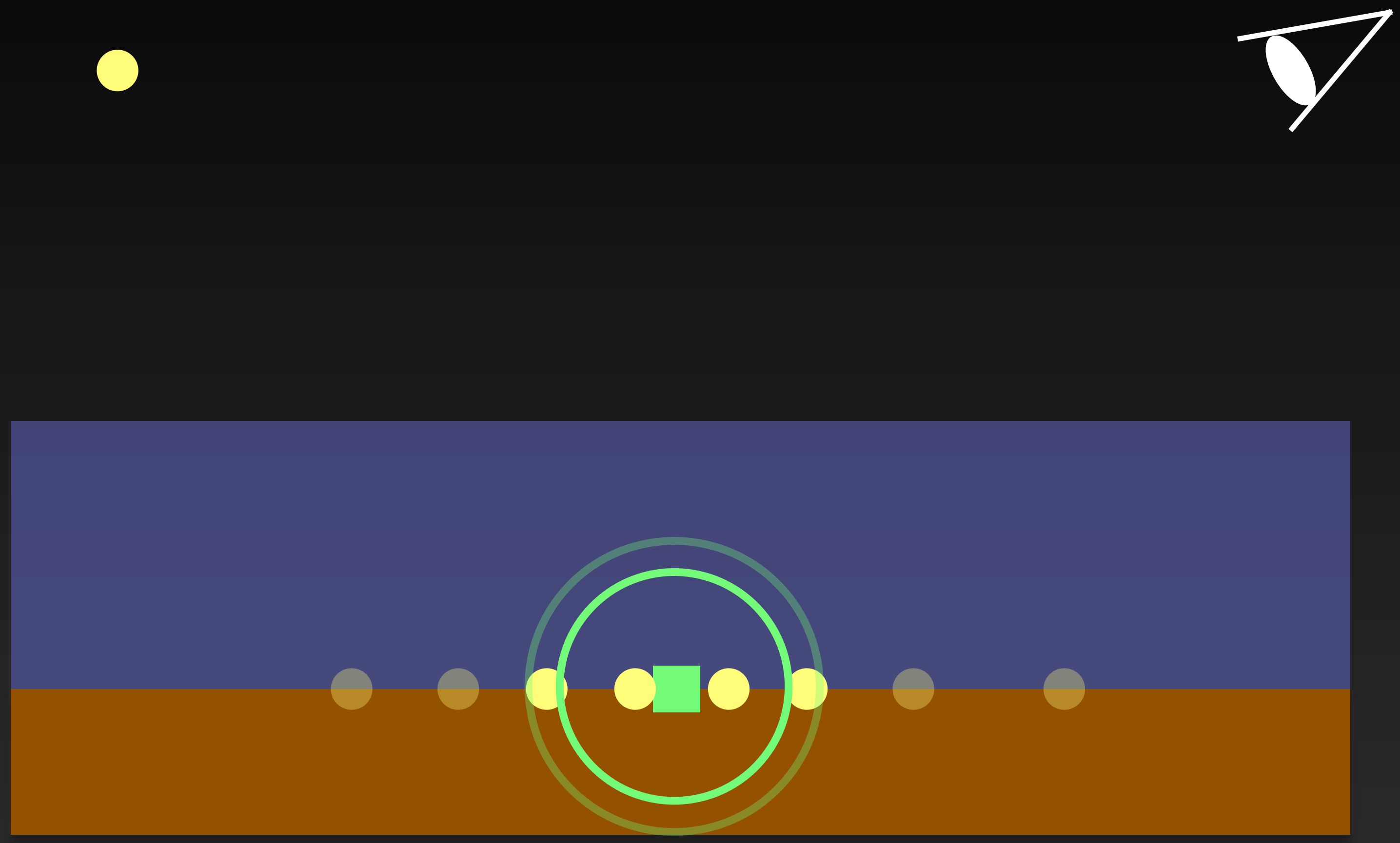


1st pass



Range query with r_0

1st pass



Reduce radius into r_1

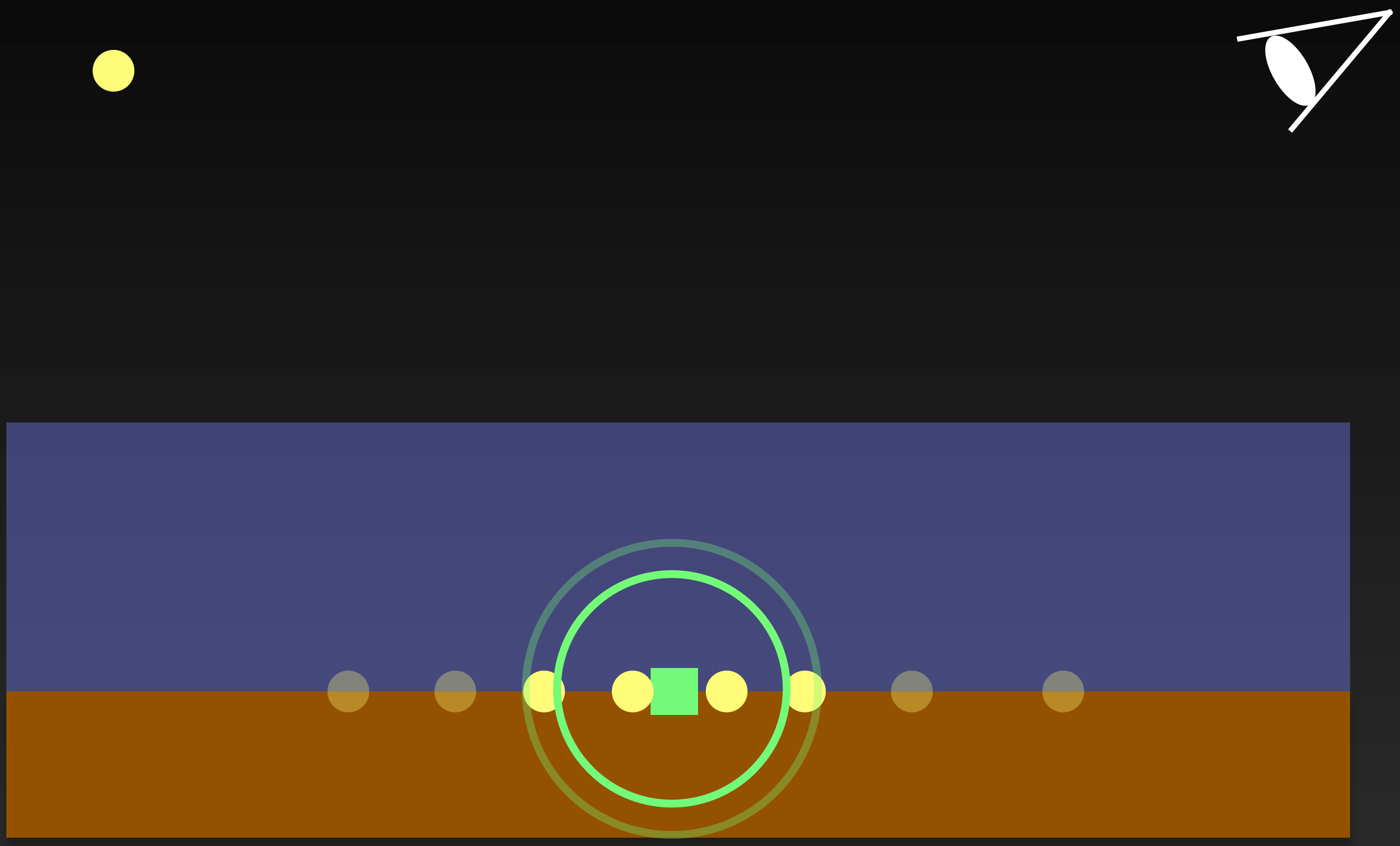
Radius reduction

- Reduce radius based on sample statistics
  - Number of photons found so far N_n
  - Number of newly found photons M_n
  - Reduction rate (user-defined) $\alpha \in]0,1[$

$$r_{n+1} = \frac{N_n + \alpha M_n}{N_n + M_n} r_n$$

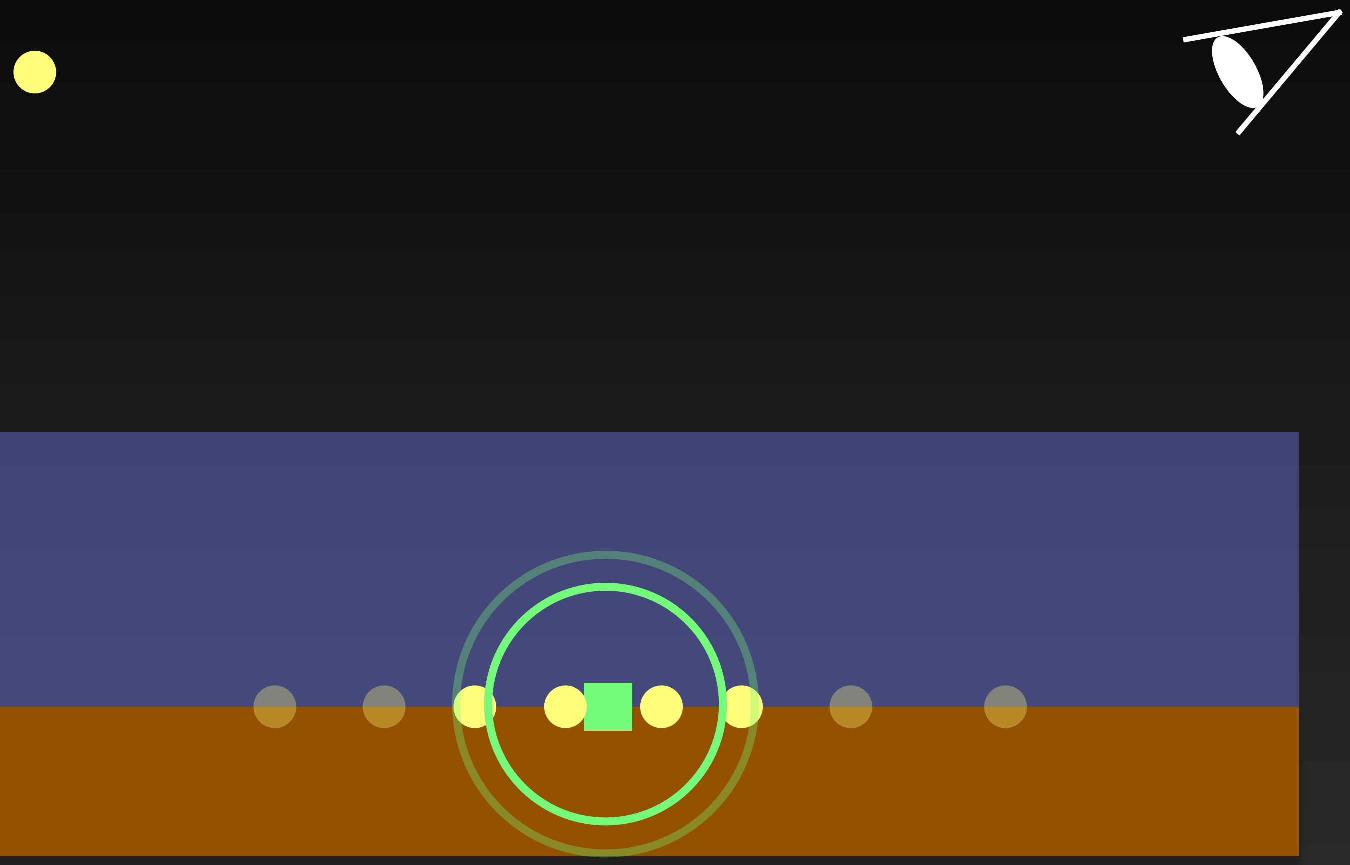
$$N_{n+1} = N_n + \alpha M_n$$

1st pass



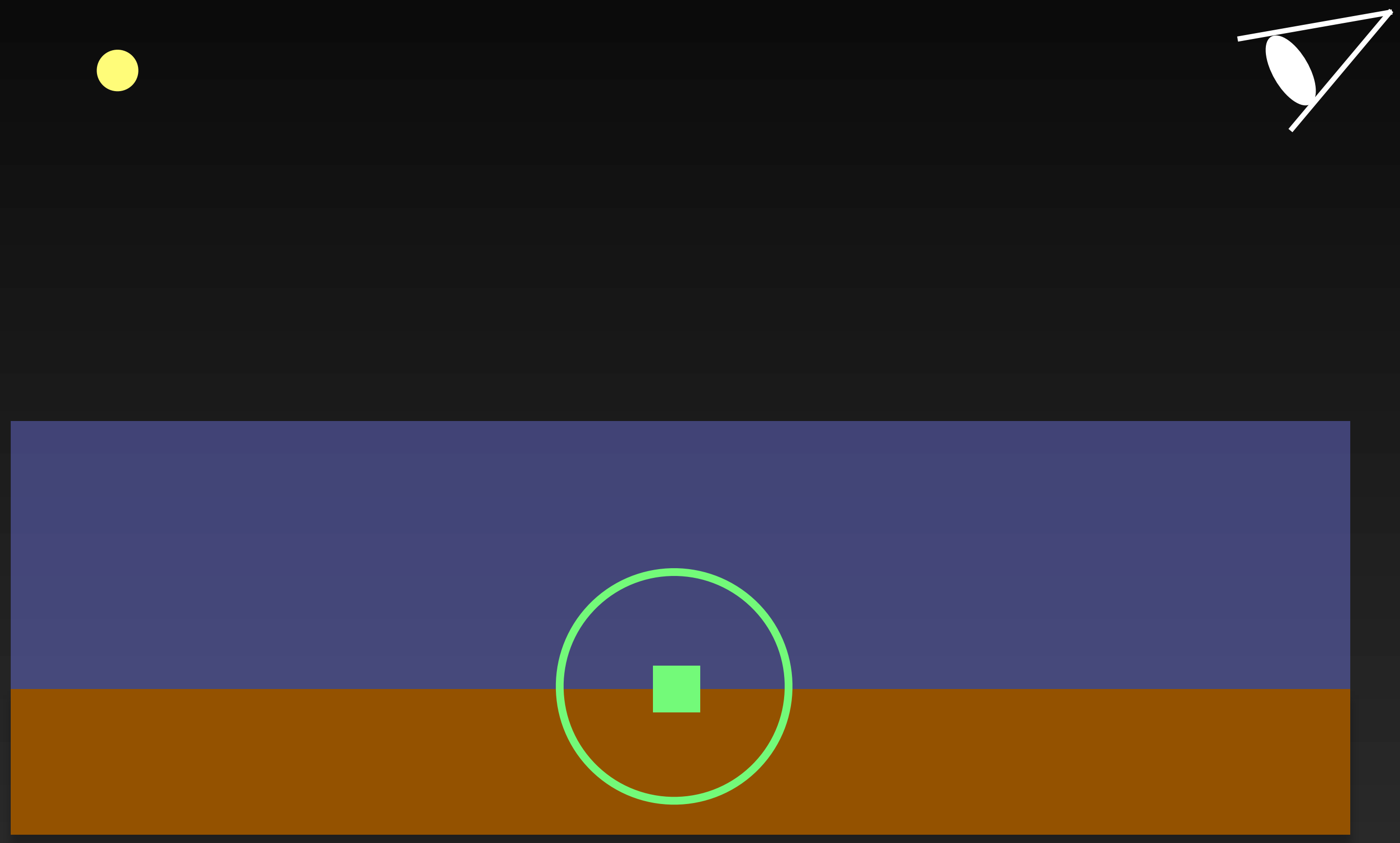
$$r_1 = \frac{N_0 + \alpha M_0}{N_0 + M_0} r_0 = \frac{0 + \alpha 4}{0 + 4} r_0$$

1st pass

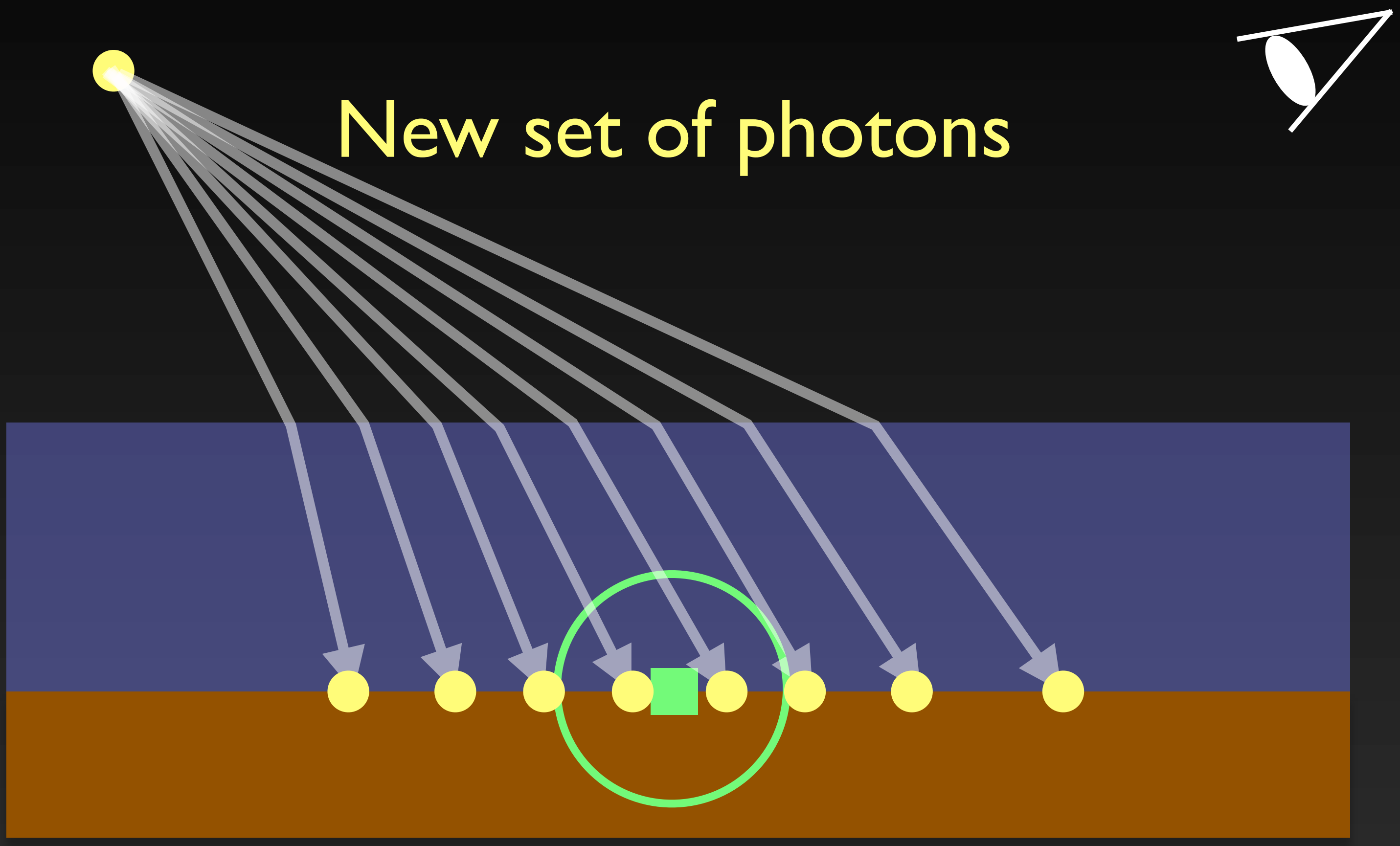


$$N_1 = N_0 + \alpha M_0 = 0 + \alpha 4$$

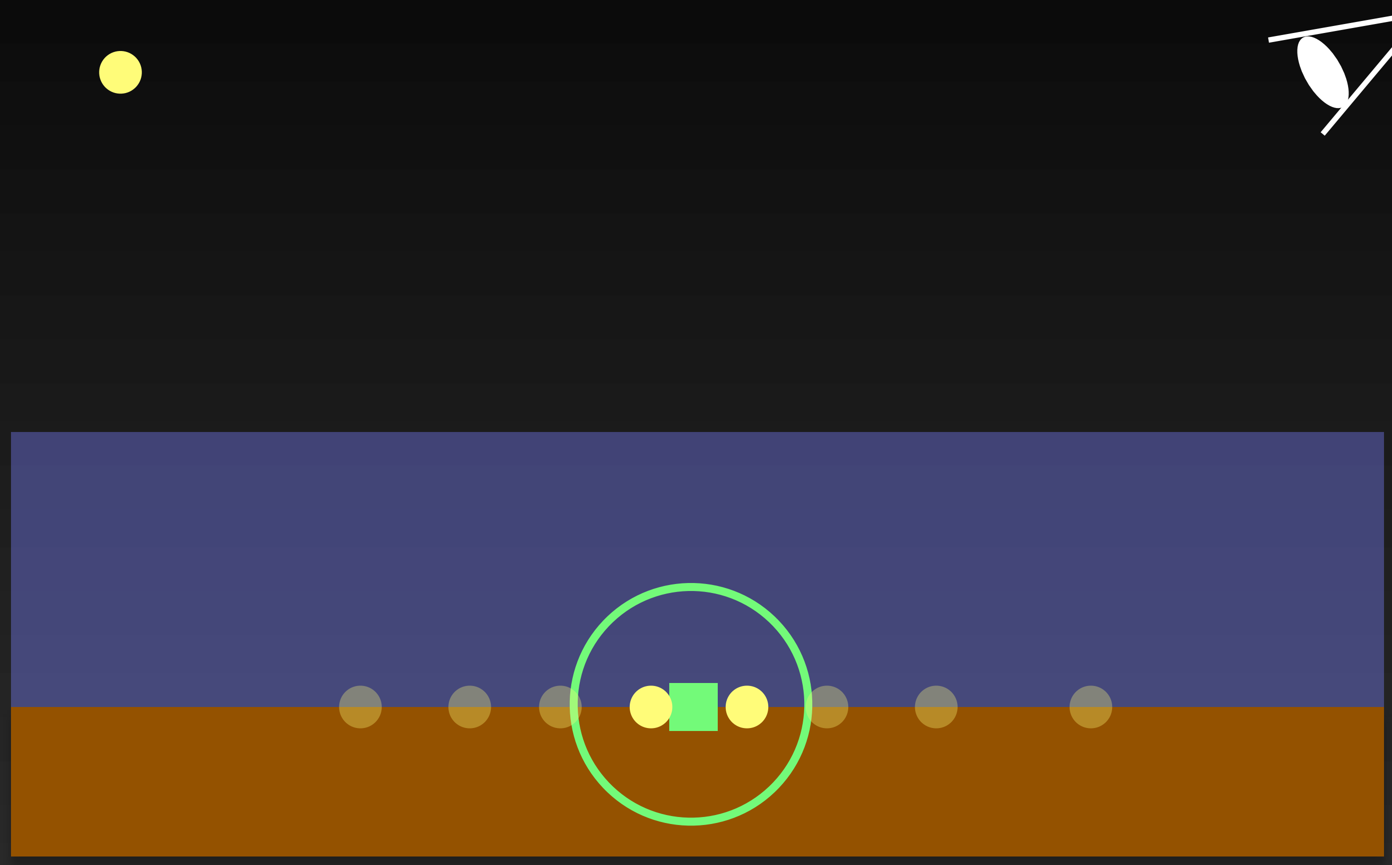
2nd pass



2nd pass

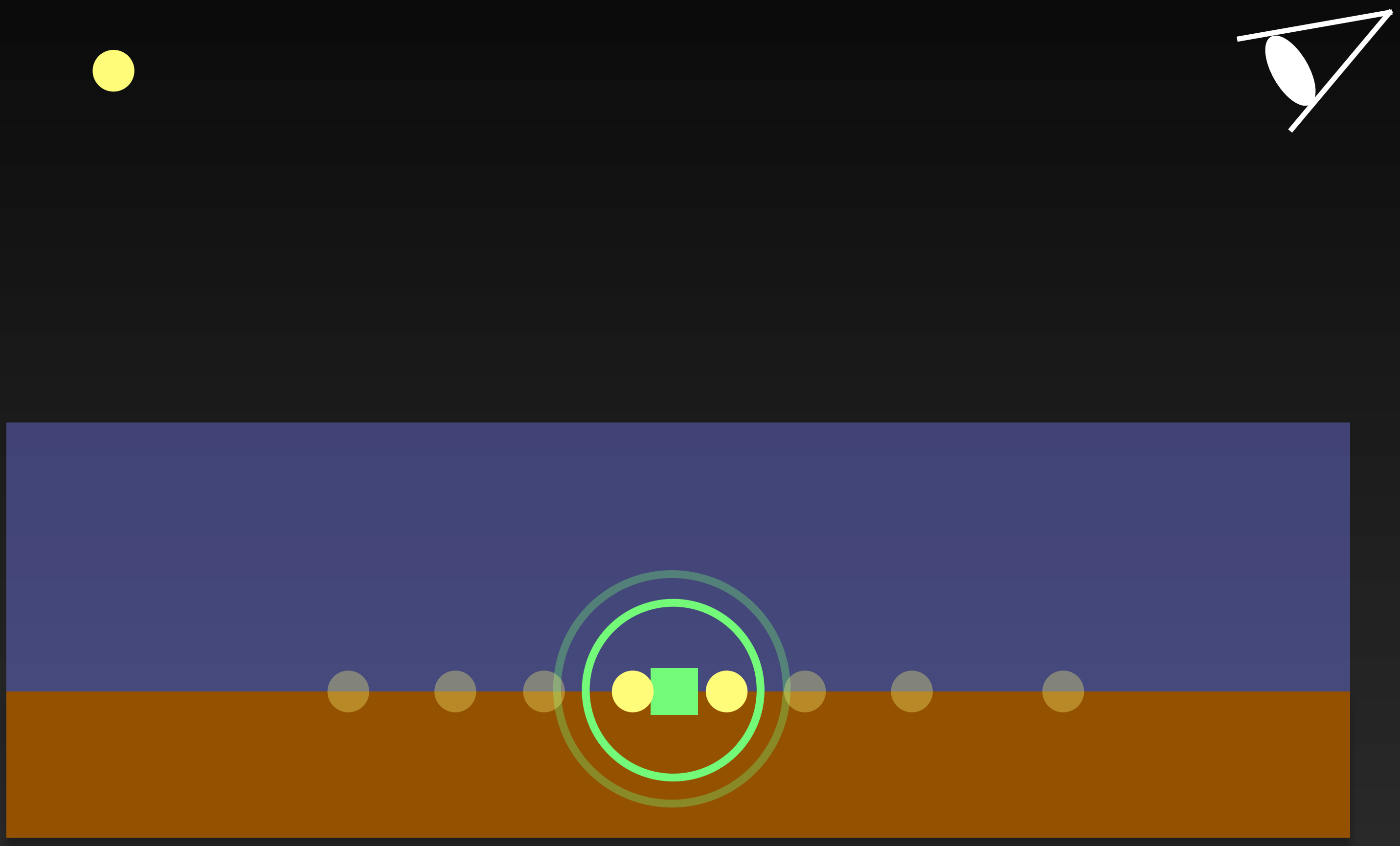


2nd pass



Range query with r_1

2nd pass



$$r_2 = \frac{N_1 + \alpha M_1}{N_1 + M_1} r_1 \qquad N_2 = N_1 + \alpha M_1$$

3rd and n-th passes

$$r_{n+1} = \frac{N_n + \alpha M_n}{N_n + M_n} r_n \qquad N_{n+1} = N_n + \alpha M_n$$

MC integration

Progressive density estimation

Equal time

Convergence

$$L = \lim_{n \to \infty} \frac{N_n}{\pi r_n^2}$$

- Converges to the correct solution
  - Infinite number of neighboring samples
  - Infinitely small radius

Convergence

$$L = \lim_{n \to \infty} \frac{N_n}{\pi r_n^2}$$

- Converges to the correct solution
  - $\lim_{i \to \infty} N_i(R_i) = \infty$
  - $\lim_{i \to \infty} R_i(N_i) = 0$

Convergence

$$L = \lim_{n \to \infty} \frac{N_n}{\pi r_n^2}$$

- Converges to the correct solution
  - $\lim_{i \to \infty} N_i(R_i) = \infty$
  - $\lim_{i \to \infty} R_i(N_i) = 0$
  - Convergence rate
    - Variance $O(n^{-\alpha})$
    - Bias $O(n^{\alpha-1})$

Balanced with $\alpha = \frac{2}{3}$

Relationship to recursive density estimation

- Recursive density estimation
  - Predefined sequence of radii
  - Assume stationary density distribution

[Wolverton and Wagner 1969, Yamato 1971, Knaus and Zwicker 2011]

- Progressive density estimation
  - Adaptive sequence from sample statistics
  - No assumption on stationarity

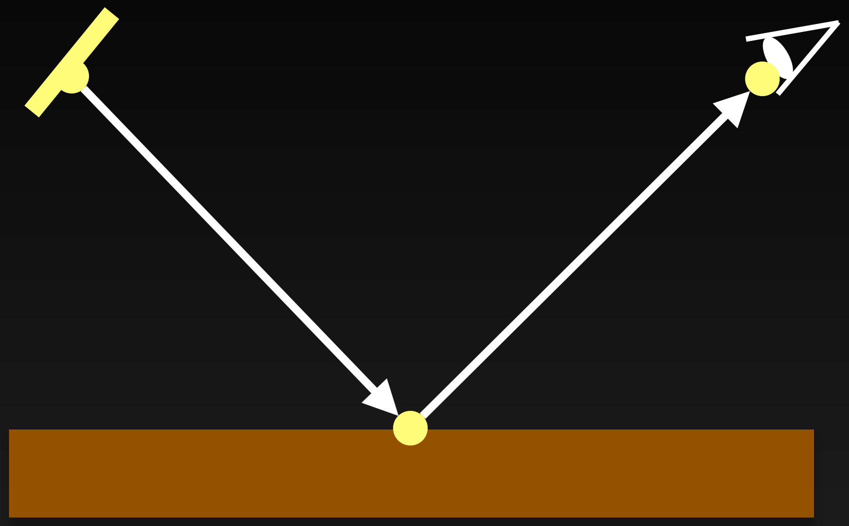
[Hachisuka et al. 2008, Hachisuka and Jensen 2009]

Progressive vs Ordinary

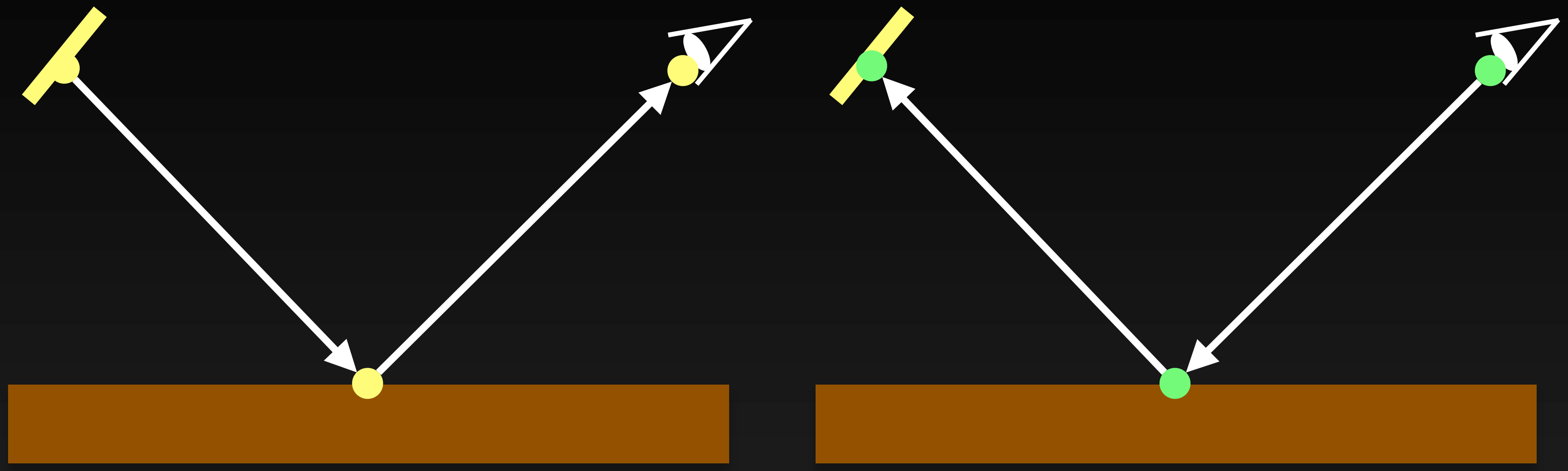
	Progressive	Ordinary
Computation	Unbounded	Unbounded
Storage	Bounded	Unbounded
Convergence	Single run	Multiple runs

Primary Space Serial Tempering

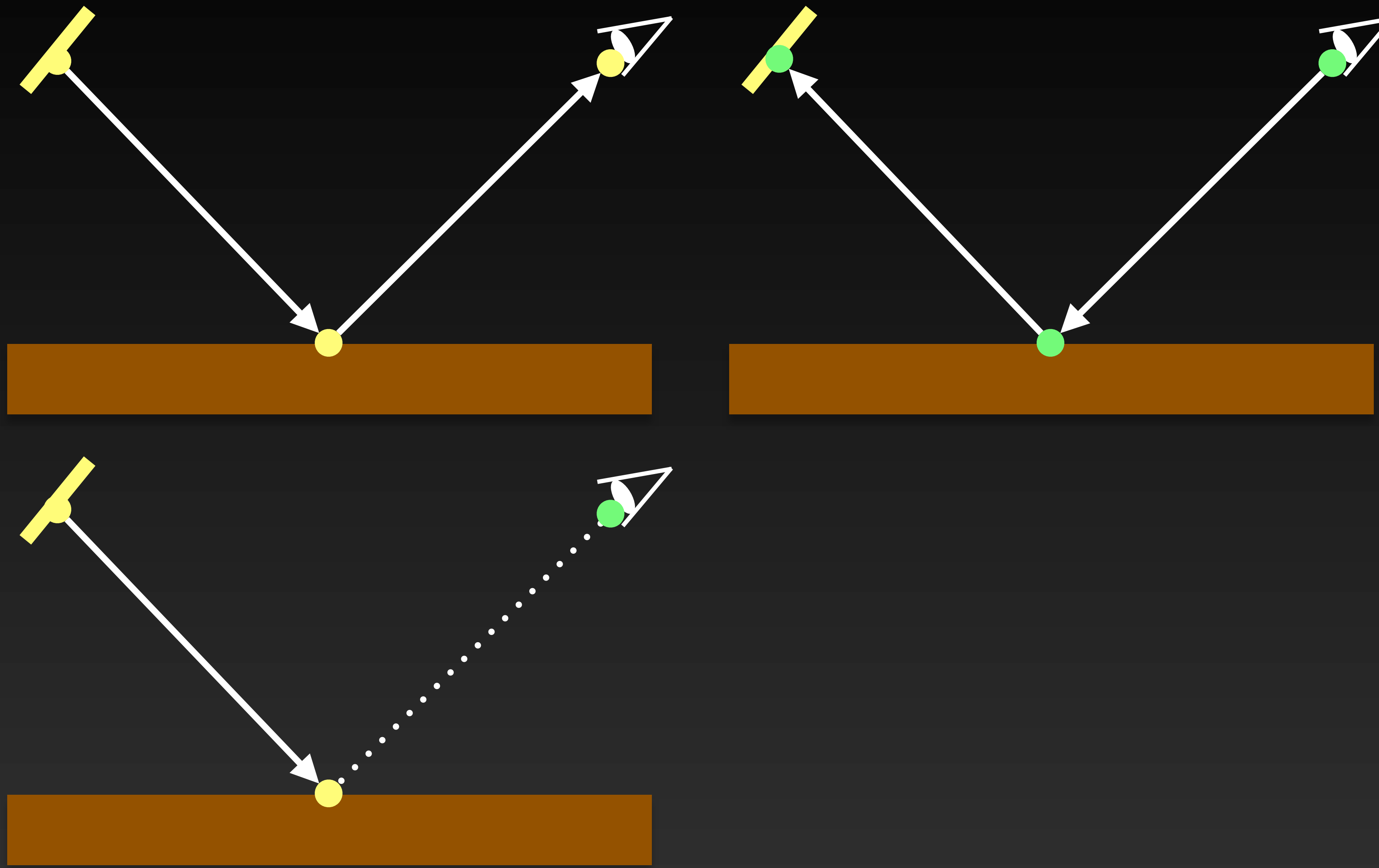
Multiple sampling methods



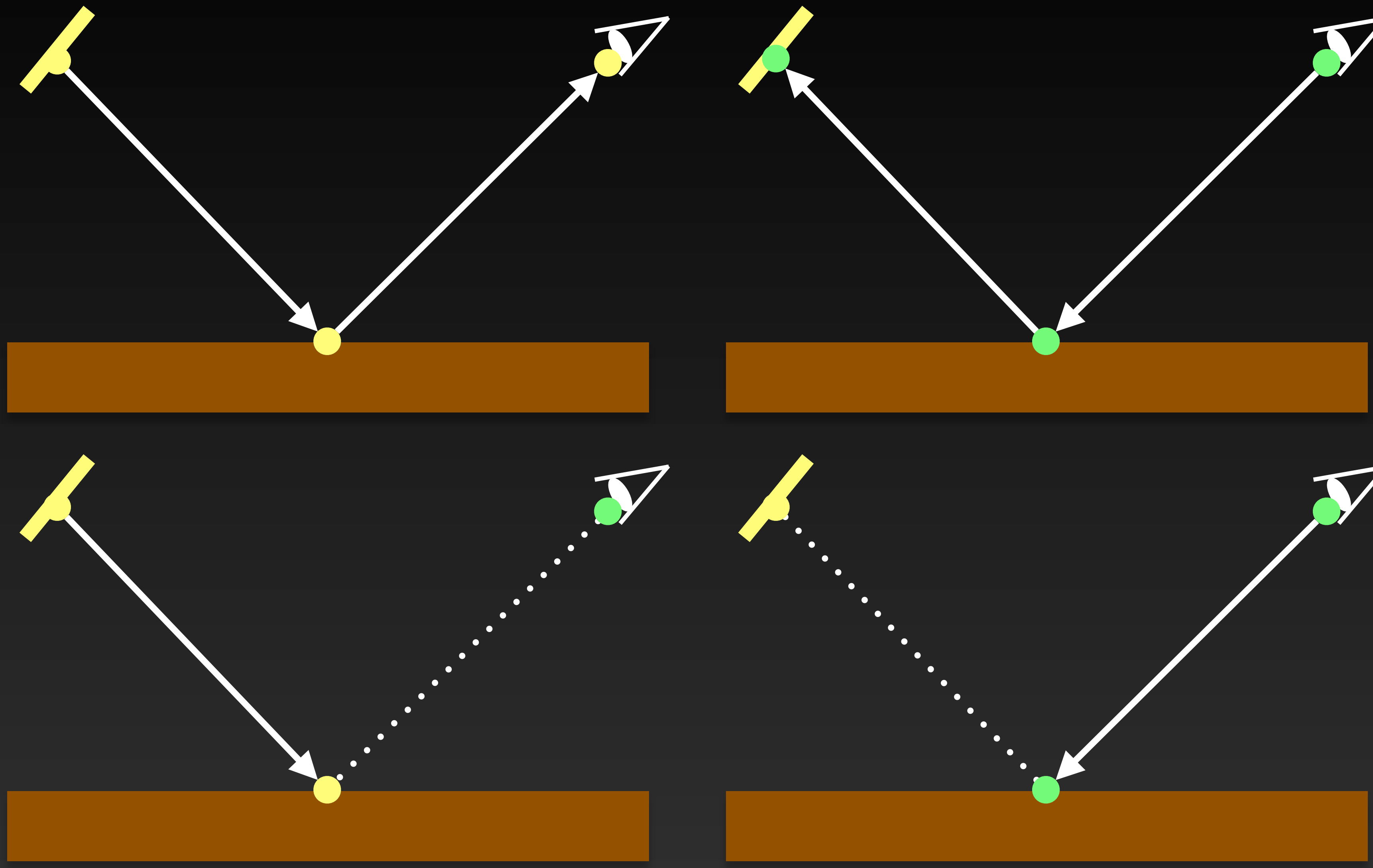
Multiple sampling methods



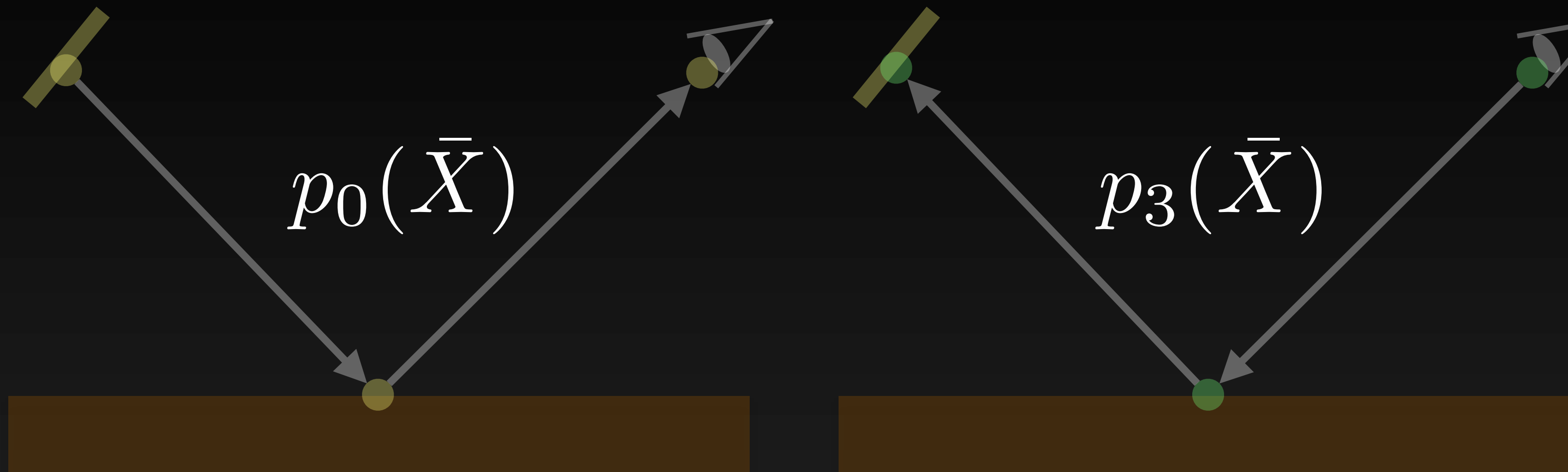
Multiple sampling methods



Multiple sampling methods



Multiple sampling methods

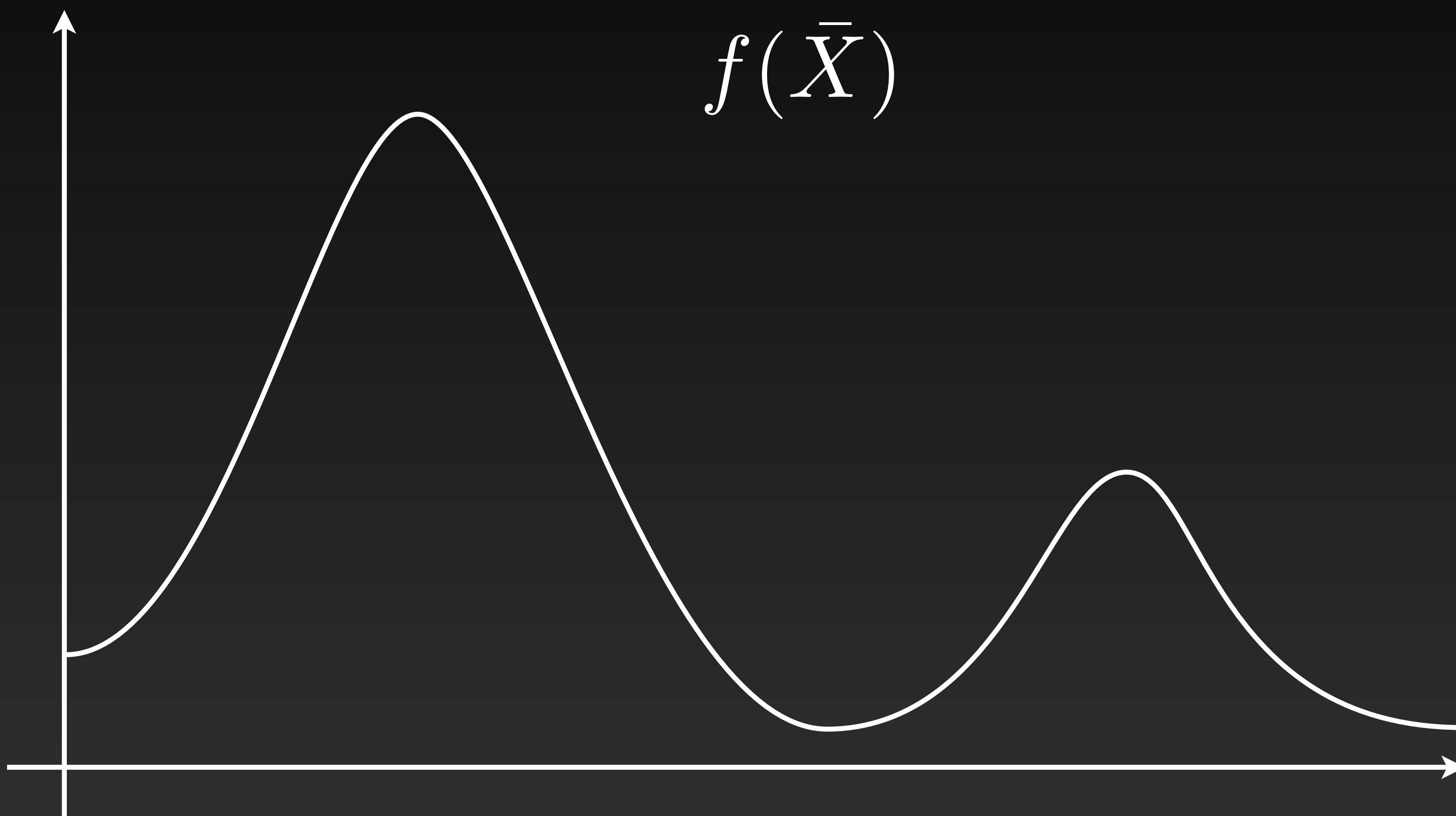


Given the same integrand, there are many known PDFs

Multiple importance sampling

[Veach and Guibas 1995]

- Utilizes samples from multiple PDFs by weighting



Multiple importance sampling

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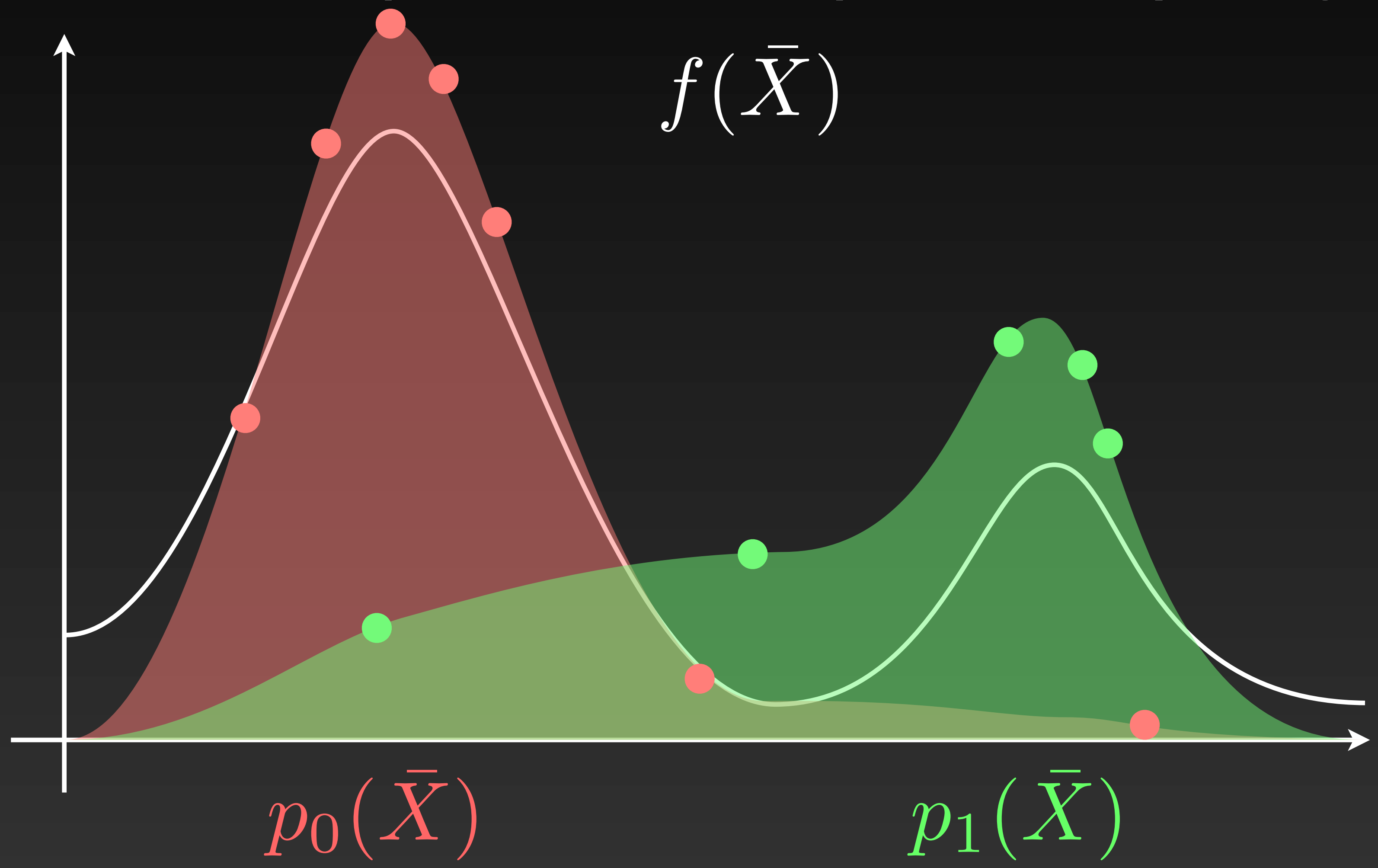


Multiple importance sampling

[Veach and Guibas 1995]

- Importance sampling

$$\int f(\bar{X})d\mu(\bar{X}) \approx \frac{1}{N}\sum_i \frac{f(\bar{x}_i)}{p_0(\bar{x}_i)} \quad \text{or} \quad \frac{1}{N}\sum_i \frac{f(\bar{x}_i)}{p_1(\bar{x}_i)}$$

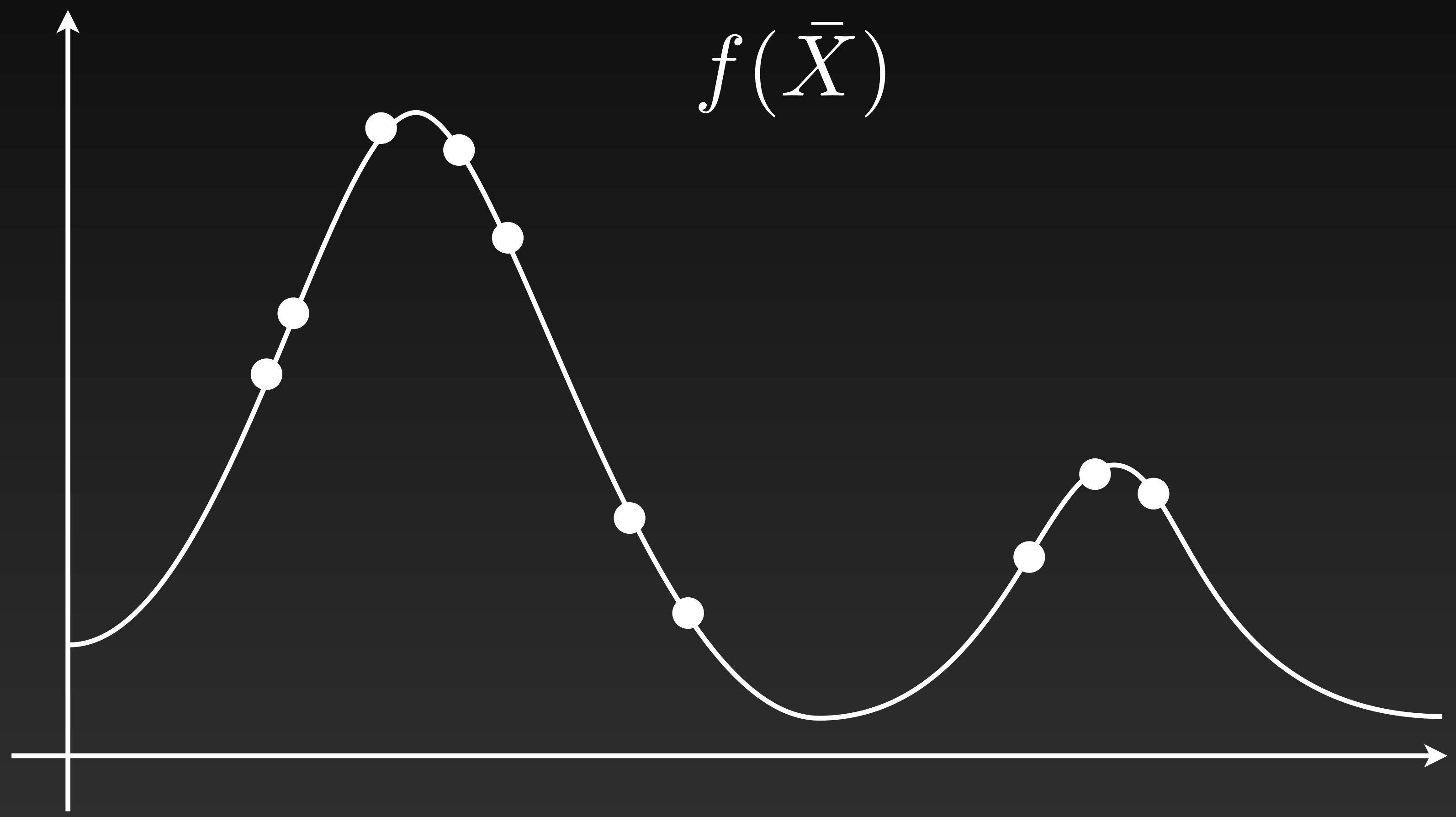
- Multiple importance sampling

$$\int f(\bar{X})d\mu(\bar{X}) \approx \frac{1}{N}\left(w_0(\bar{x}_i)\frac{f(\bar{x}_i)}{p_0(\bar{x}_i)} + w_1(\bar{x}_i)\frac{f(\bar{x}_i)}{p_1(\bar{x}_i)}\right)$$

Uses both

Markov chain Monte Carlo sampling

- Requires no knowledge of PDFs



How can we combine MIS and MCMC?

Combination

- Multiple importance sampling
  - Ordinary Monte Carlo method
  - Utilizes the knowledge of all the PDFs
- Markov chain Monte Carlo sampling
  - Markov chain Monte Carlo method
  - No usage (or requirement) of PDFs

Primary Space Serial Tempering

Primary sample space

[Kelemen et al. 2002]

- Hypercube of random numbers $\bar{U} \in \,]0,1[^N$
  - Mapping from a point to a sample
  - Inverse CDF = mapping function

$$\int f(\bar{X})d\mu(\bar{X}) \approx \frac{1}{N}\sum_i \frac{f(\bar{x}_i)}{p(\bar{x}_i)} = \frac{1}{N}\sum_i C(\bar{u}_i)$$

$$C(\bar{U}) = \frac{f(P^{-1}(\bar{U}))}{p(P^{-1}(\bar{U}))}$$

Primary sample space

[Kelemen et al. 2002]

- Hypercube of random numbers $\bar{U} \in \,]0,1[^N$
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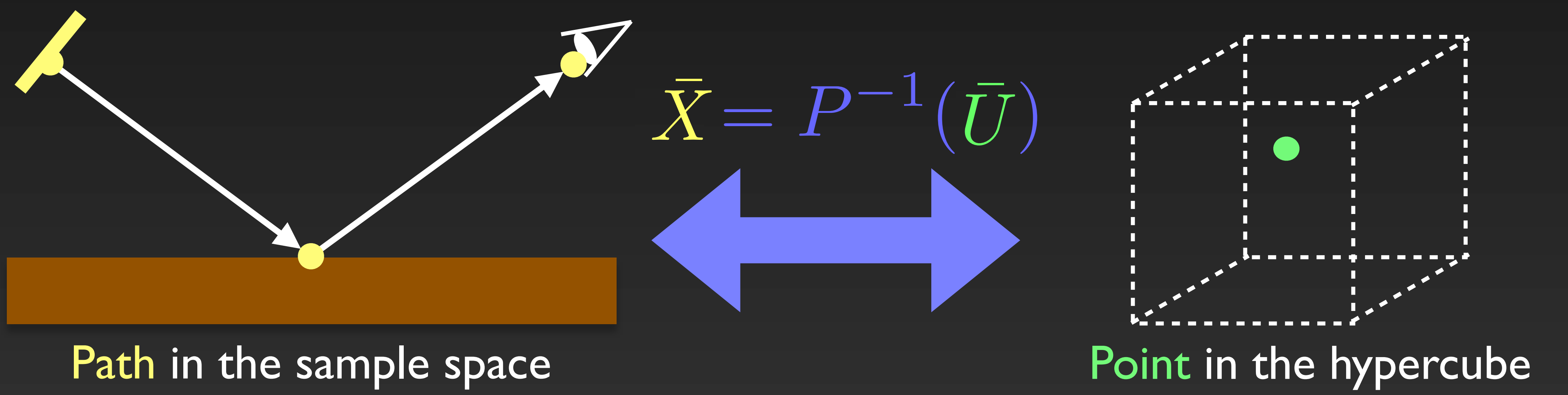


Serial tempering

[Marinari and Parisi 1992, Geyer and Thompson 1995]

- MCMC sampling of a sum of multiple distributions

$$\bar{x} \sim \sum_j f_j(\bar{x})$$

- Extended states of the Markov chain $(\bar{x}, j)$
  - Index j is updated via MCMC
  - Extended state is originally temperature
  - Still does not use any PDFs

Primary space serial tempering

[Hachisuka et al. 2014] (TBA)

- Use a weighted sum of primary space distributions
  - Extended states is now $(\bar{u}, j)$
  - Index j is the index to the j-th PDF/CDF

$$\bar{u} \sim \sum_j w_j(\bar{u}) C_j(\bar{u})$$

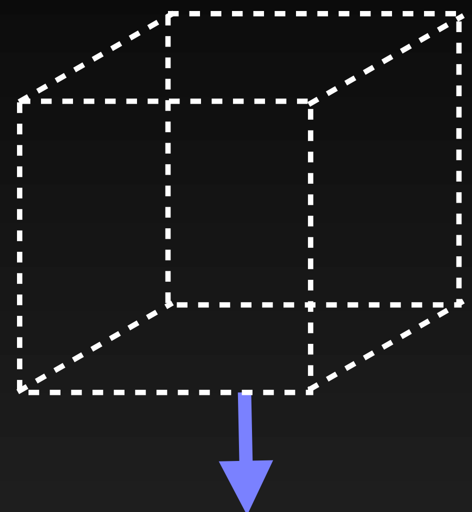
$$C_j(\bar{U}) = \frac{f(P_j^{-1}(\bar{U}))}{p_j(P_j^{-1}(\bar{U}))} \qquad w_j(\bar{U}) = w(P_j^{-1}(\bar{U}))$$

Primary space distribution

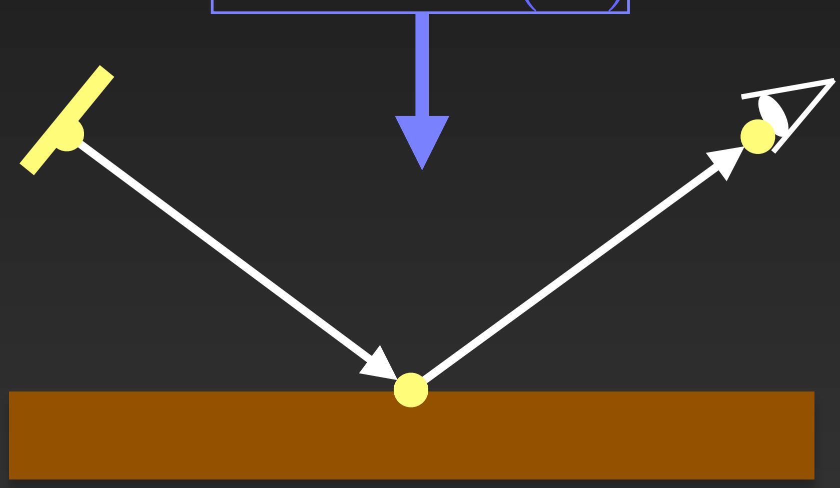
MIS weight

Original primary space

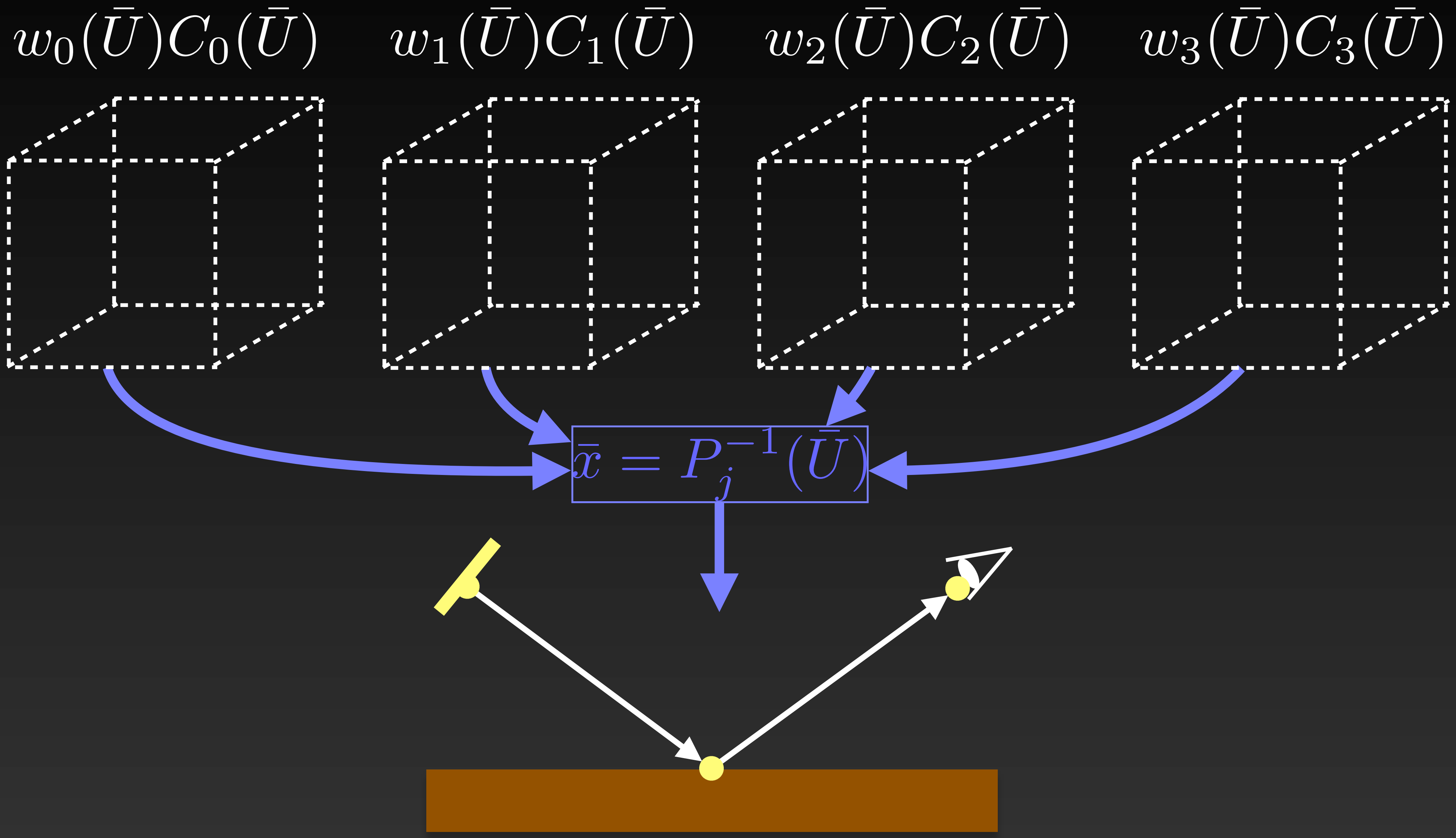
$C(\bar{U})$



$\bar{x} = P^{-1}(\bar{U})$

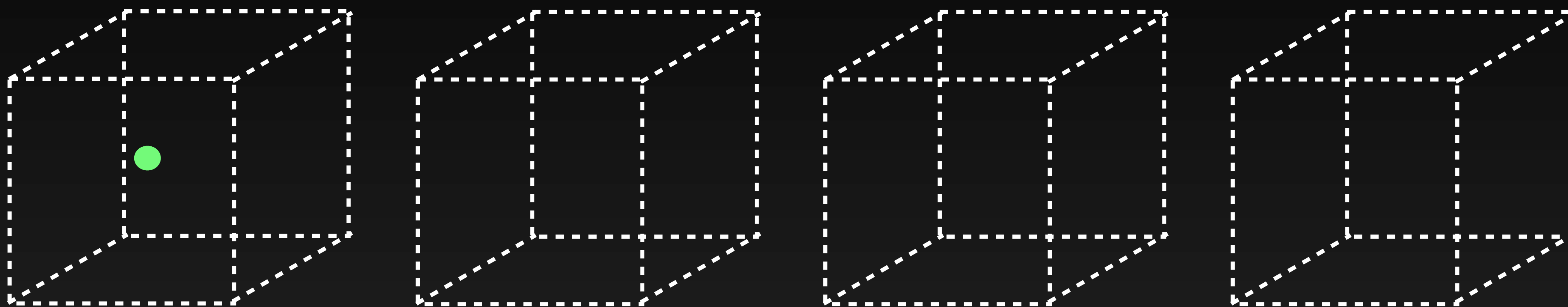


Multiplexed primary space

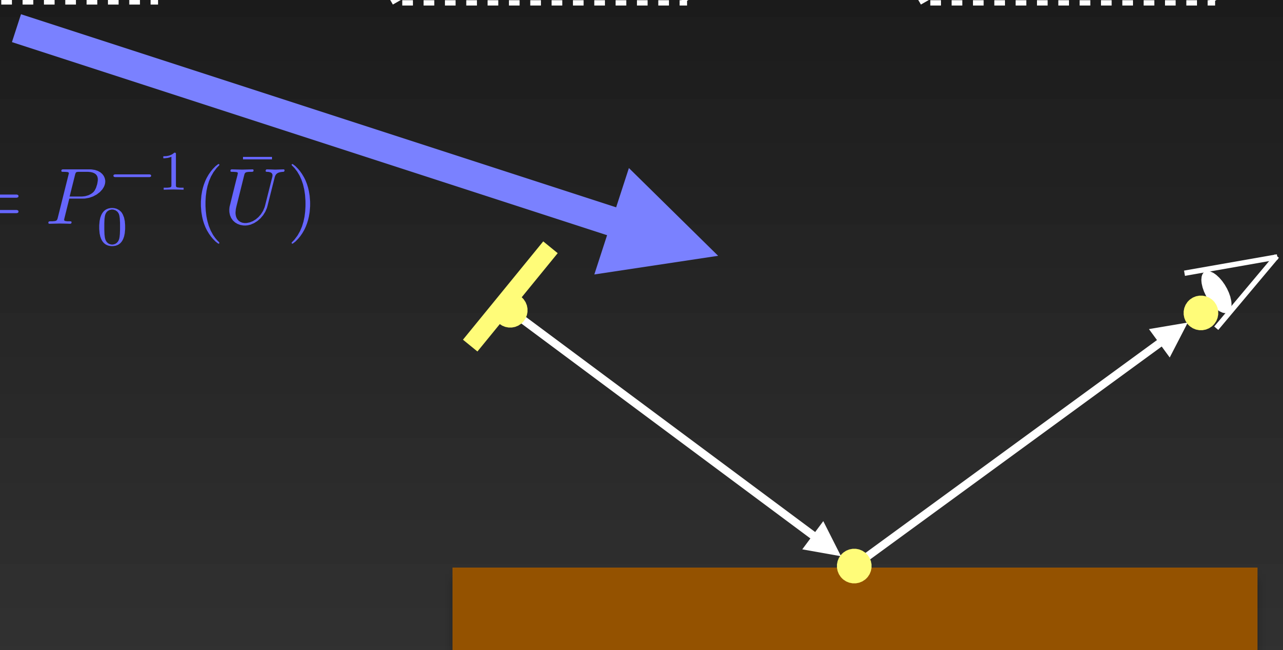


Primary space serial tempering

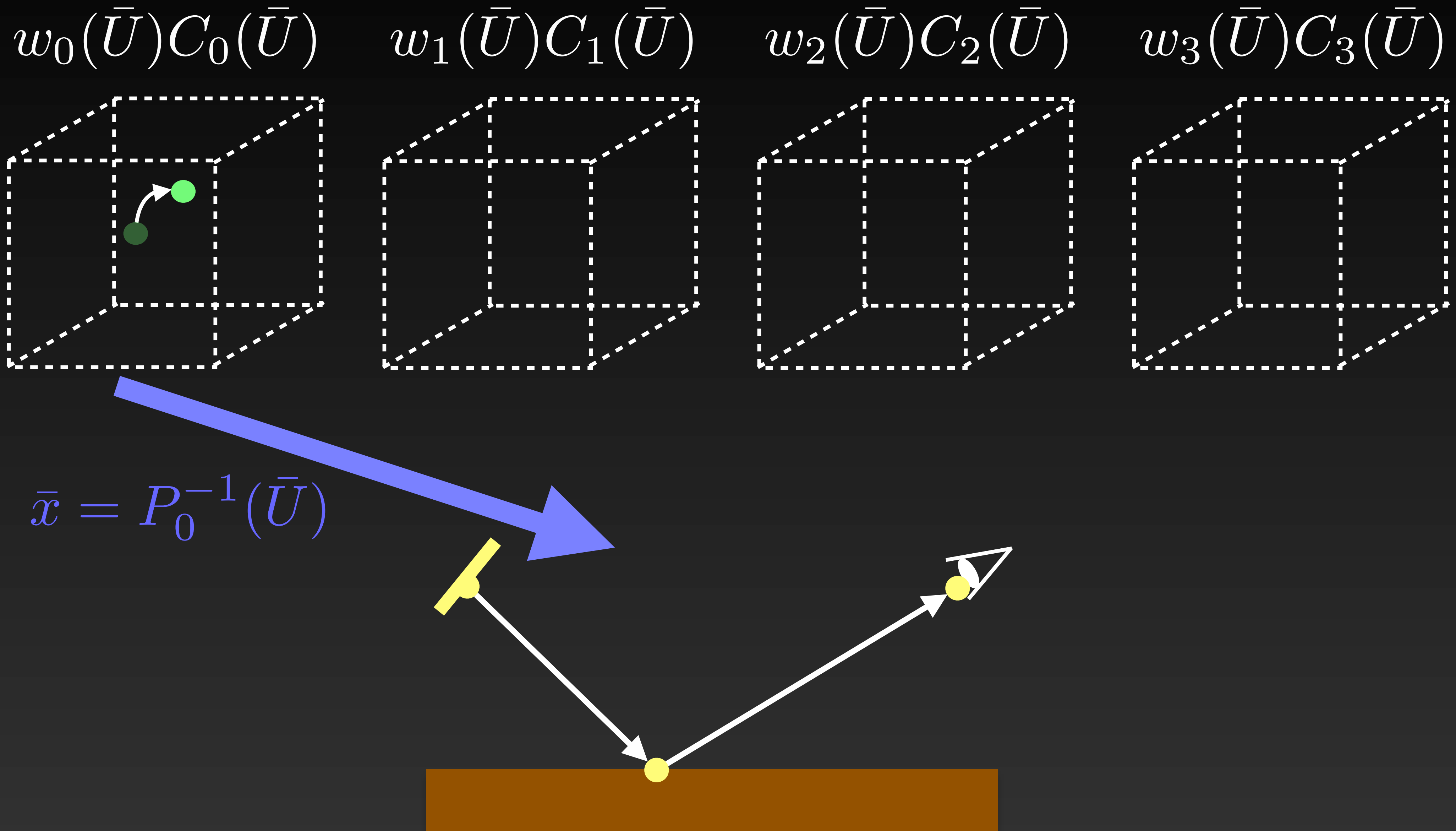
$w_0(\bar{U})C_0(\bar{U})$ $w_1(\bar{U})C_1(\bar{U})$ $w_2(\bar{U})C_2(\bar{U})$ $w_3(\bar{U})C_3(\bar{U})$



$\bar{x} = P_0^{-1}(\bar{U})$

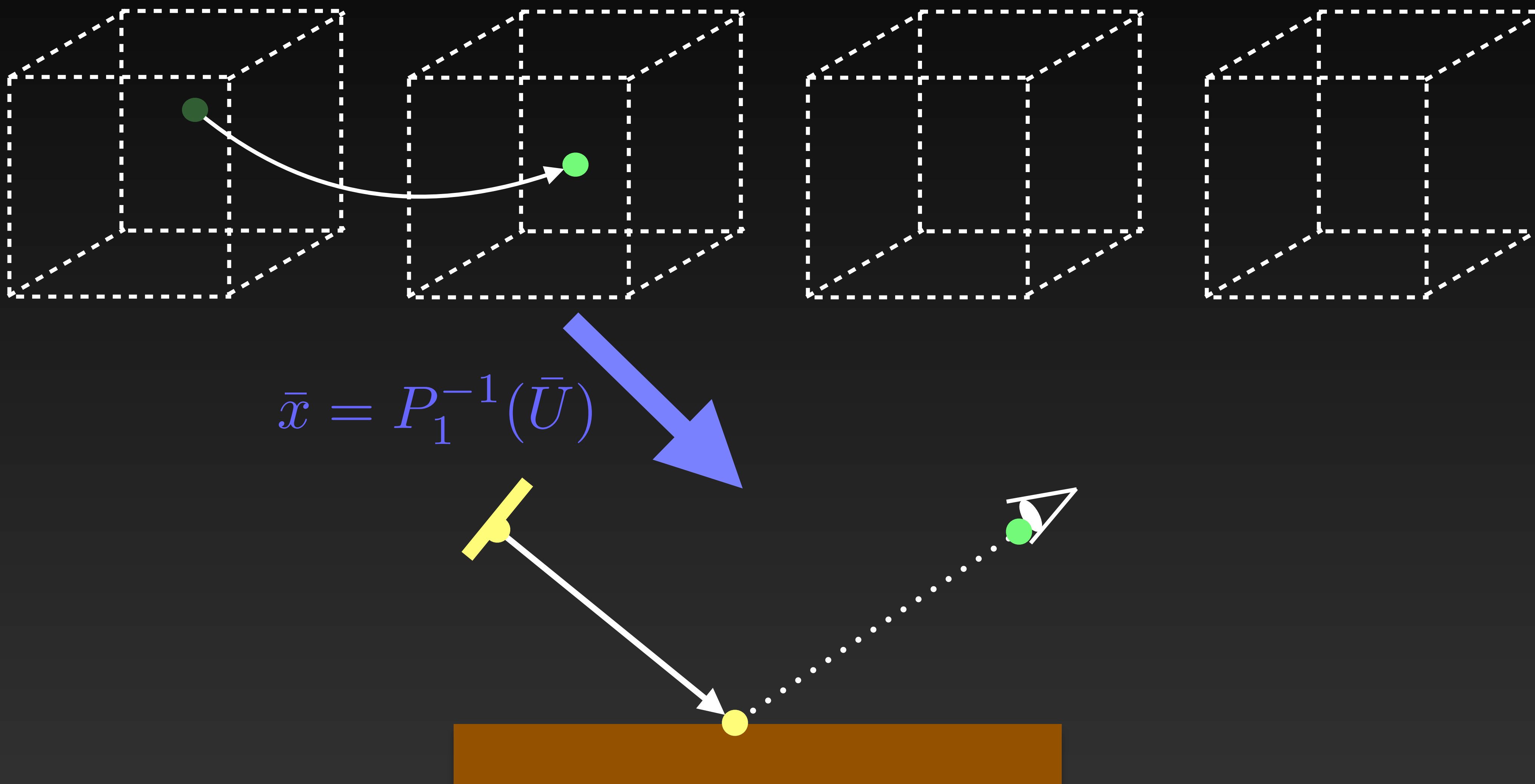


Primary space serial tempering

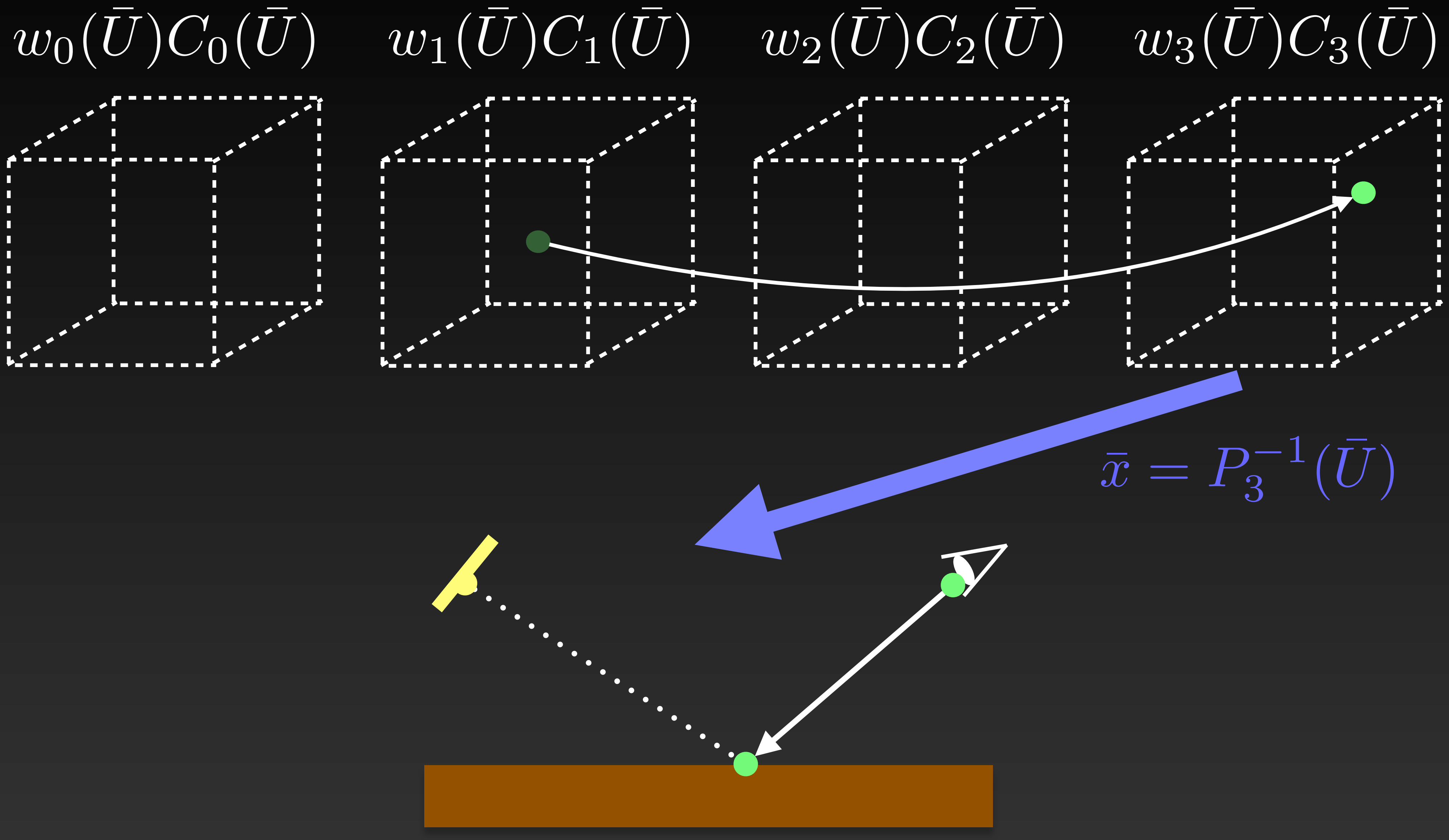


Primary space serial tempering

$w_0(\bar{U})C_0(\bar{U})$ $w_1(\bar{U})C_1(\bar{U})$ $w_2(\bar{U})C_2(\bar{U})$ $w_3(\bar{U})C_3(\bar{U})$

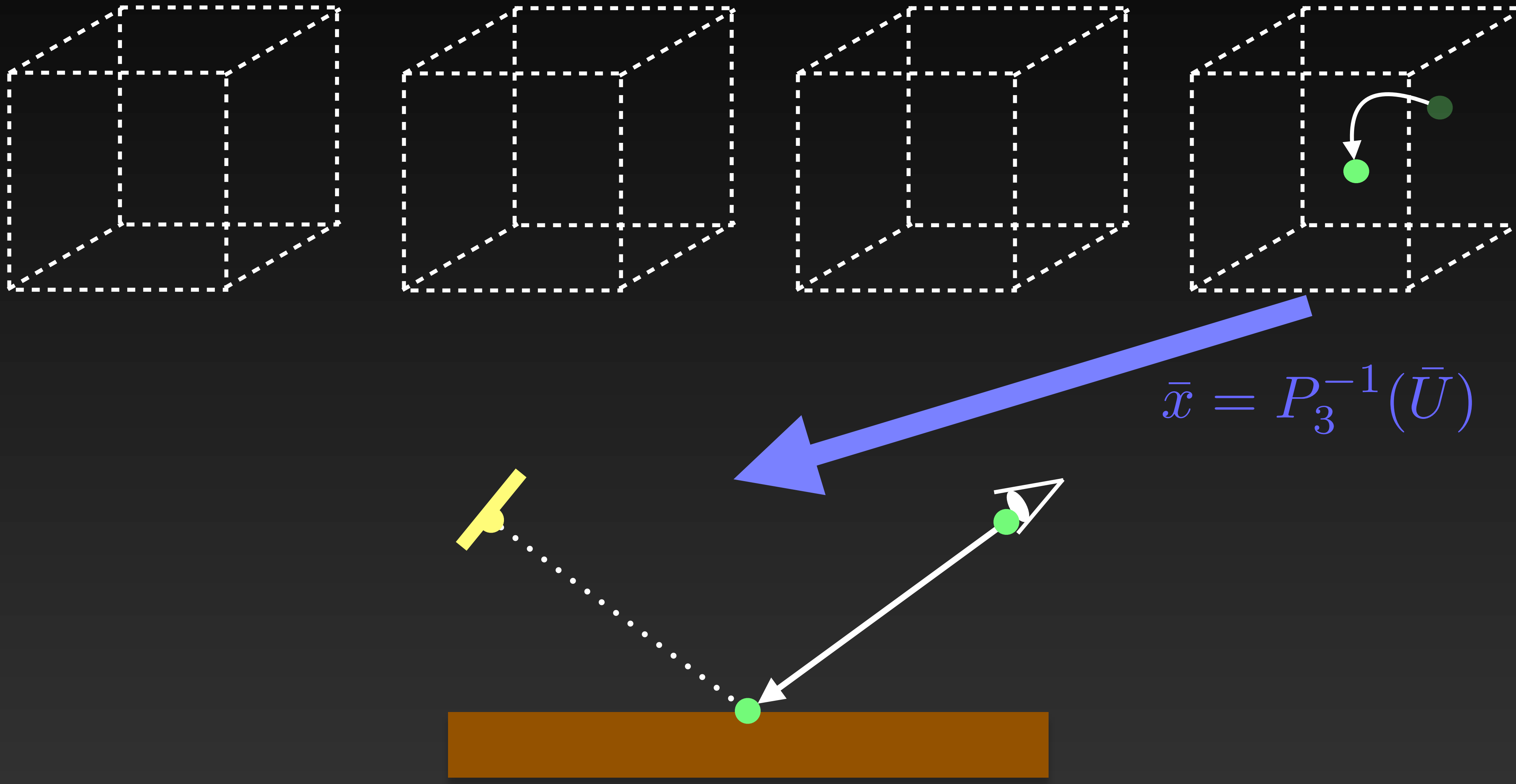


Primary space serial tempering



Primary space serial tempering

$w_0(\bar{U})C_0(\bar{U})$ $w_1(\bar{U})C_1(\bar{U})$ $w_2(\bar{U})C_2(\bar{U})$ $w_3(\bar{U})C_3(\bar{U})$



$\bar{x} = P_3^{-1}(\bar{U})$

Metropolis-Hastings

[Veach and Guibas 1997]

Equal time

Primary space only

[Kelemen et al. 2002]

Equal time

Primary space serial tempering

Equal time

Metropolis-Hastings

[Veach and Guibas 1997]

Equal time

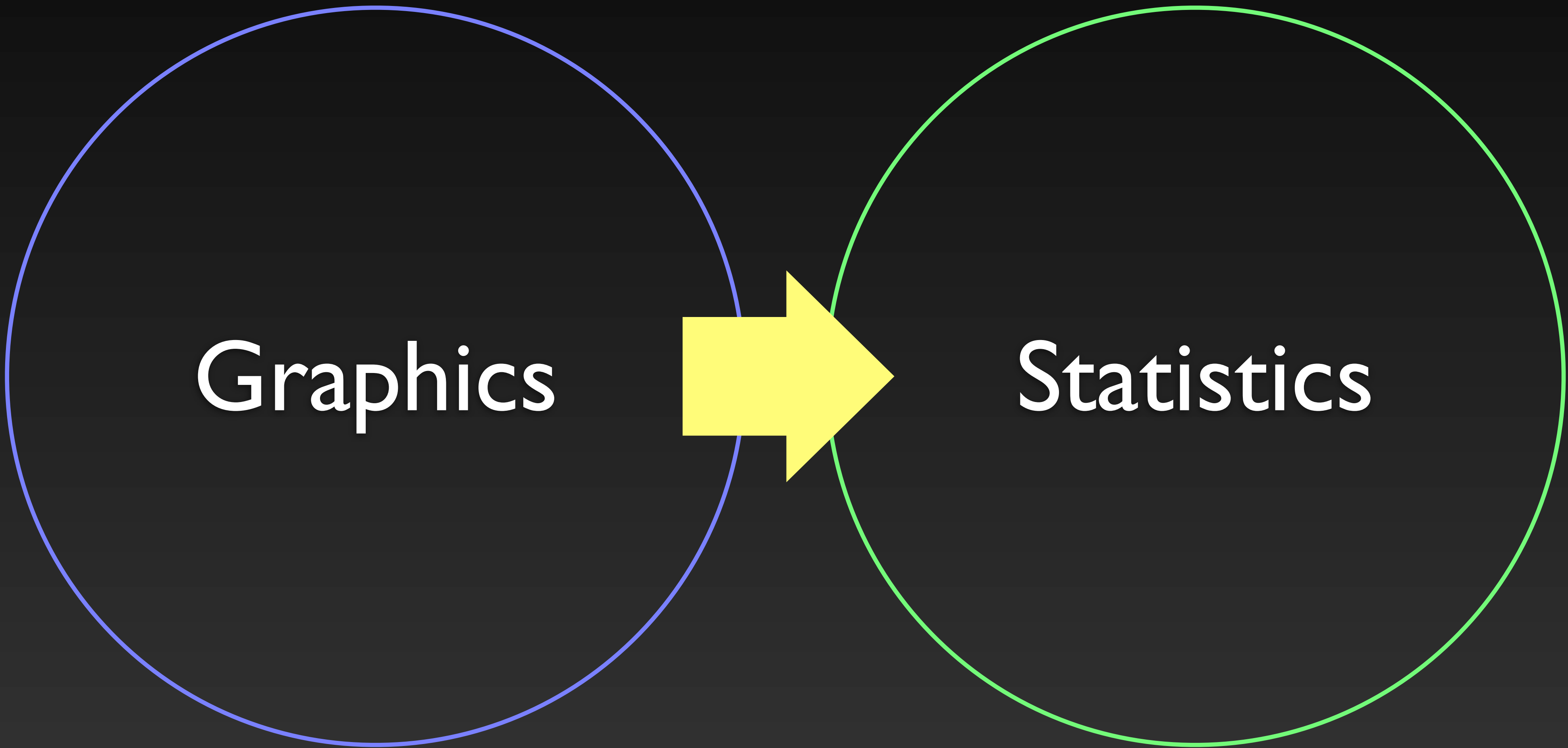
Summary

Statistics / Graphics

- Progressive density estimation
  - Statistics: new density estimator
  - Graphics: simulation of complex light paths
- Primary space serial tempering
  - Statistics: MCMC driven MIS
  - Graphics: robust and efficient path sampling

What I hope to see

- Statistics communities take something from graphics



What I **really** hope to see

- We collaborate to develop something interesting!