Parameter Estimation of BSSRDF for Heterogeneous Materials

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Abstract

Rendering of highly scattering media is computationally expensive in general. While existing BSSRDF models can accurately and efficiently approximate light scattering in homogeneous media, we still have to resort to costly Monte Carlo simulation for heterogeneous media. We propose a simple parameter estimation method which enables homogeneous BSSRDF models to approximate the appearance of heterogeneous media. The main idea is to estimate the input optical parameters of a given homogeneous BSSRDF model such that the output well approximates light transport within heterogeneous media. Our method takes spatially varying optical coefficients into account by taking averages of the coefficients around the incident and exitant points. This approach is motivated by the path integral theory which predicts how wide the beam of light will spread in heterogeneous media. Since our method provides parameters for homogeneous BSSRDF models, it is applicable to many existing BSSRDF models and easy to integrate into existing rendering systems. We show that our modification produces more accurate results than the existing heuristics with the same goal.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Raytracing

1. Introduction

In the past years, several researchers have developed *bidirectional surface scattering reflectance distribution function* (BSSRDF) models to provide an efficient approximation of light transport within highly scattering media. These models typically assume that materials are homogeneous (optical properties are constant) in order to approximate light scattering without relying on costly Monte Carlo simulation. This limitation is problematic in practice since most of highly scattering media we see in our daily life are heterogeneous (optical properties are spatially varying). While there exists heuristics to handle heterogeneous materials using homogeneous BSSRDF models, the accuracy of such heuristics has not been discussed.

We propose another parameter estimation technique for homogeneous BSSRDF models that can more accurately approximate the appearance of heterogeneous materials. Unlike the other heuristics [JMLH01, dI11], our method is built upon existing work on efficient rendering of heterogeneous media. Inspired by aggregation of multiple scattering due to small objects such as hairs and sands, we consider a homogeneous medium which equivalently produces light paths to the given heterogeneous medium. We then estimate optical properties of this homogeneous medium by taking the average of coefficients around the incident and exitant points as predicted by the path integral theory [PAT*04]. Using such average coefficients, we can efficiently approximate the appearance of heterogeneous media with homogeneous BSSRDF models. Figure 1 illustrates the basic concept of our method.

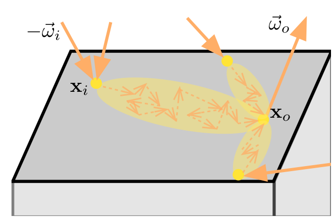


Figure 1: *Conceptual illustration of our method. Our method takes optical properties in wide range (shown in yellow ellipse) into account. The range is decided with path integral approach proposed by Premože et al. [PAT*04], which enables us to predict how wide the beam of light will spread in heterogeneous media. We use the average coefficients within this range as the input to a homogeneous BSSRDF model.*

2. Background and Related Work

2.1. Monte Carlo Simulation

One approach to render highly scattering media is to numerically simulate scattering of lights based on the radiative transfer equation [Cha50]. For example, Novák et al. [NSJ14] proposed an efficient method to estimate light attenuation in heterogeneous media based on this general approach. Although this approach can give accurate results for both homogeneous and heterogeneous materials, it is computationally expensive to obtain plausible results. Highly scattering media can be even more costly to render since there are lots of scattering events to simulate for a single light transport path.

2.2. Diffusion Approximation

Stam [Sta95] introduced the diffusion equation to computer graphics which efficiently approximates multiple scattering in highly scattering homogeneous media. For heterogeneous materials, several approaches based on finite differential [WZT*08, WWH*10] or finite element [AWB10] methods are proposed. Bernabei et al. [BPB*12] proposed a method that combines ray tracing and diffusion equation solver. This approach enables to render heterogeneous objects with the changes of the refractive indices. These approaches do not integrate well with existing rendering systems based on ray tracing and require spatial discretization.

2.3. BSSRDF

Another approach is to use a BSSRDF which is a functional representation of light transport within participating media. The outgoing radiance L_o which leaves from the point $\mathbf{x}_o$ in the direction $\vec{\omega}_o$ is computed by convolving the BSSRDF, S, times the incoming radiance distribution L_i from the direction $\vec{\omega}_i$ at the point $\mathbf{x}_i$:

$$L_o(\mathbf{x}_o, \vec{\omega}_o) = \int_A \int_{2\pi} L_i(\mathbf{x}_i, \vec{\omega}_i)\, S(\mathbf{x}_i, \vec{\omega}_i, \mathbf{x}_o, \vec{\omega}_o | \sigma_s, \sigma_a, g) \cos\theta \, d\omega_i d\mathbf{x}_i, \tag{1}$$

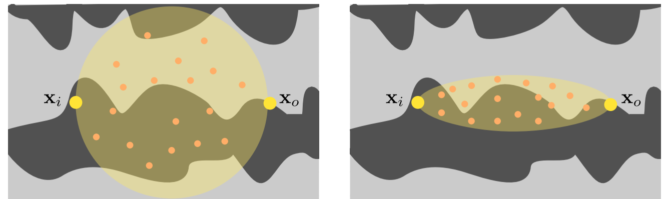
where the parameters σ_s, σ_a, and g are the reduced scattering and absorption coefficients, and the average cosine of scattering which are optical characteristics of materials, and the angle θ is formed by $\vec{\omega}_i$ and the normal vector at $\mathbf{x}_i$. Figure 1 illustrates the configuration.

Jensen et al. [JMLH01] pioneered a practical BSSRDF and several advances have been made in the past years [dI11, HCJ13, FHK14]. Due to the use of the diffusion theory, the existing BSSRDF models assume that σ_s and σ_a are constant over a whole object. This assumption makes the existing BSSRDF models incompatible with heterogeneous media. We propose a technique to estimate alternative parameters $\bar{\sigma}_s$ and $\bar{\sigma}_a$ so that homogeneous BSSRDF models now also approximate light transport within heterogeneous media.

Similar to our work, there are some works which handles heterogeneous media with homogeneous BSSRDFs [JMLH01, dI11], however, there is no discussion regarding how these heuristics work in practice. Donner et al. [DWd*08] proposed to utilize a BSSRDF to render heterogeneous human skin. Their method models the human skin by multiple heterogeneous layers, and approximate light scattering among the layers. In this model, heterogeneity is considered only for the transmittance of lights passing through boundaries between layers, but not for scattering inside one layer. Our method, on the other hand, can handle light scattering within general heterogeneous materials.

The method proposed by Peers et al. [PvBM*06] uses BSSRDFs acquired from the measurement of real materials. In the method, reflectance is represented by a band diagonal matrix which is parameterized over incident and exitant points. The matrix is compactly represented by factorization which makes use of the diagonal symmetry of the matrix. These reflectance profiles need to be precomputed based on the results of time-consuming simulation or measurement. Unlike the method, our method computes BSSRDFs on the fly.

3. Method



Step 1: evaluating $\langle\sigma_s\rangle$ and $\langle\sigma_a\rangle$ Step 2: evaluating $\bar{\sigma}_s$ and $\bar{\sigma}_a$

Figure 2: *Illustration of our method (top view). We evaluate the temporal averaged coefficients $\langle\sigma_s\rangle$ and $\langle\sigma_a\rangle$ in a circle with a number of samples (orange dots). After the evaluation, the new averaged coefficients $\bar{\sigma}_s$ and $\bar{\sigma}_a$ are evaluated inside an ellipse defined with $\langle\sigma_s\rangle$ and $\langle\sigma_a\rangle$.*

Our technique is built upon the concept of virtual homogeneous media which approximate light transport paths in the corresponding heterogeneous media. This concept has been successfully used for approximation of light scattering in discrete media such as hairs [MM06, ZW06]. Our goal is thus to estimate the new coefficients $\bar{\sigma}_s$ and $\bar{\sigma}_a$ so that the resulting evaluation of a given BSSRDF model reproduces light transport in a heterogeneous medium. Our technique consists of the following two steps: (1) determining a region to obtain the averaged coefficients, and (2) evaluating the averaged coefficients and the final BSSRDF. Figure 2 illustrates the overview of our method.

3.1. Region Determination

We first need to determine a region E to obtain the averaged coefficients. We assume that optical properties are varying only along tangent directions. Under this assumption, we consider an ellipse whose antipodal points are $\mathbf{x}_i$ and $\mathbf{x}_o$ on a surface. The length of the major axis is the distance $d = \|\mathbf{x}_i - \mathbf{x}_o\|$, and the length of the minor axis (which is the spatial spreading of the most probable light path) can then be predicted by the path integral [PAT*04]:

$$w = \sqrt{\frac{\langle\theta^2\rangle \ell (d/2)^2}{24\{1 + \langle\theta^2\rangle(\langle\sigma_a\rangle/\langle\sigma_s\rangle)\ell^2/12\}}} \tag{2}$$

where $\ell = \langle\sigma_s\rangle d/2$ is the estimated number of scattering and $\langle\theta^2\rangle$ is the mean square scattering angle. Intuitively, Equation (2) predicts a region E where most of light paths fall within. In order to evaluate $\langle\sigma_s\rangle$ and $\langle\sigma_a\rangle$ in this equation, we use a circle whose center is the midpoint between $\mathbf{x}_i$ and $\mathbf{x}_o$, and radius is $d/2$.

3.2. Evaluation of Averaged Coefficients

After deciding the region E, we estimate the averaged coefficients:

$$\bar{\sigma}_s = \frac{1}{A_E} \int_E \sigma_s(\mathbf{x})\, d\mathbf{x} \tag{3}$$

where $A_E = \int_E d\mathbf{x}$. We can estimate $\bar{\sigma}_a$ in the same manner.

To solve Equation (3), we use Monte Carlo integration as follows. Figure 3 illustrates the procedure of sampling a point $\mathbf{x}$. We first uniformly sample a point $\mathbf{x}'$ in the region E. We then trace two rays from $\mathbf{x}'$: one flights in the positive direction of normal vector, and the other flights in the negative direction. The normal is the same as a modified normal proposed by Frisvad et al [FHK14]. We check if rays hit the object to shade and an intersection is used as a sample $\mathbf{x}$. If both rays hit, the intersection with the distance to $\mathbf{x}'$ is shorter is accepted.

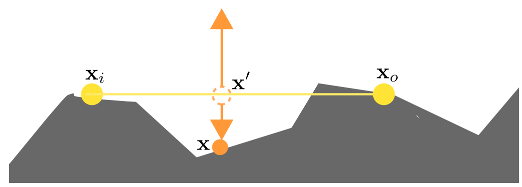


Figure 3: *Illustration of the sampling technique for complex geometries (side view). Firstly we sample a point $\mathbf{x}'$ inside an ellipse lies on the plane that $\mathbf{x}_i$ and $\mathbf{x}_o$ also lie. Next we trace two rays from $\mathbf{x}'$ along the normal vector of the ellipse, and find an intersection $\mathbf{x}$.*

After computing $\bar{\sigma}_s$ and $\bar{\sigma}_a$, a BSSRDF is evaluated. The computation of Equation (1), that is, the procedures of sampling incident lights and evaluation of a BSSRDF model, remains the same as conventional algorithms.

4. Results

We implemented two heuristics to handle heterogeneous media proposed by Jensen et al. [JMLH01] and d'Eon and Irving [dI11], and our technique. The heuristic by Jensen et al. uses coefficients only at incident points, while the heuristic by d'Eon and Irving take averages of coefficients between incident and exitant points. The results shown in this section are rendered on a 6-core Intel Core i7-3930K 3.20GHz CPU.

For quantitative comparison, we plot diffuse reflectance for a beam of light which perpendicularly enters semi-infinite media (Figure 4). For the results with the heuristic by Jensen et al., we use coefficients at exitant points instead of originally suggested incident points since the latter turns these results equivalent to homogeneous media. This scenario is the ideal case for the existing BSSRDF models, except that we now consider heterogeneous media. Figure 4 shows the results generated by path tracing, BSSRDF with the two heuristics, and BSSRDF with our method. For the top *stripe* scene the heuristics and our method utilize the original dipole model [JMLH01], while the photon beam diffusion [HCJ13] is used for the bottom *check* scene.

The heuristic by Jensen et al. fails to match with path tracing, since the heuristic does not consider the mixture of optical properties. The heuristic by d'Eon and Irving and our method capture the characteristic attenuation and blur similar to path tracing. However, especially in the stripe scene, the heuristic by d'Eon and Irving causes notable artifacts in diagonal lines, while our method does not. While there are still some differences from path traced results, our method matches better than any other heuristics.

Figure 5 compares path tracing, an original with the heuristic by Jensen et al., and our method for a practical scene. This heuristic by Jensen at el. does not reproduce blurring within heterogeneous media. While our method is more costly than the heuristic by Jensen et al., it is still roughly eight times faster than path tracing while producing a visually similar result.

5. Discussions

Application to Real-time Rendering Our method is potentially applicable to real-time rendering. The most general approach to render translucent materials in real time is filtering irradiance texture via a sum-of-Gaussians [dLE07]. We however cannot directly utilize such approaches for our method since they assume that diffuse reflectance is radially symmetrical with respect to the incident point.

Region for Averaging Our method uses an ellipse for the region E. We can trivially replace E to any other shape (e.g., rectangle). In many cases, the difference of the shape does not affect the results. We also set the length of the major axis to be the distance between incident and exitant points. This could mean that we ignore some lights that go in the opposite to the exitant point and get around the ellipse. While we can trivially include such lights by expanding the region E, it usually causes the loss of illumination features as far as we have experimented.

Limitations Our method assumes that optical properties are varying along only tangent directions. Taking variation along normal directions into account is left as a future work. Another limitation is that our method can be slow compared to evaluating only BSSRDF due to the averaging computation. Accelerating this computation is similar to the problem of fast blurring of an image with a spatially varying kernel, which might be possible via precomputation. While our method is built upon the well-established path integral theory, additional theoretical analysis might help us identify failure cases and lead to further improvement.

6. Conclusion

We introduced a new method which captures characteristic appearance of heterogeneous translucent materials. Our method takes the average optical properties around incident and exitant points of light as predicted by the path integral theory. We quantitatively demonstrate that, compared to previous work, our method provides accurate results without relying on costly full Monte Carlo simulation. We believe that our work is the first step to make advances in practical techniques to accurately render heterogeneous materials based on BSSRDFs. The use of the path integral theory within BSSRDF models should be a promising direction of research to develop a generic BSSRDF model.

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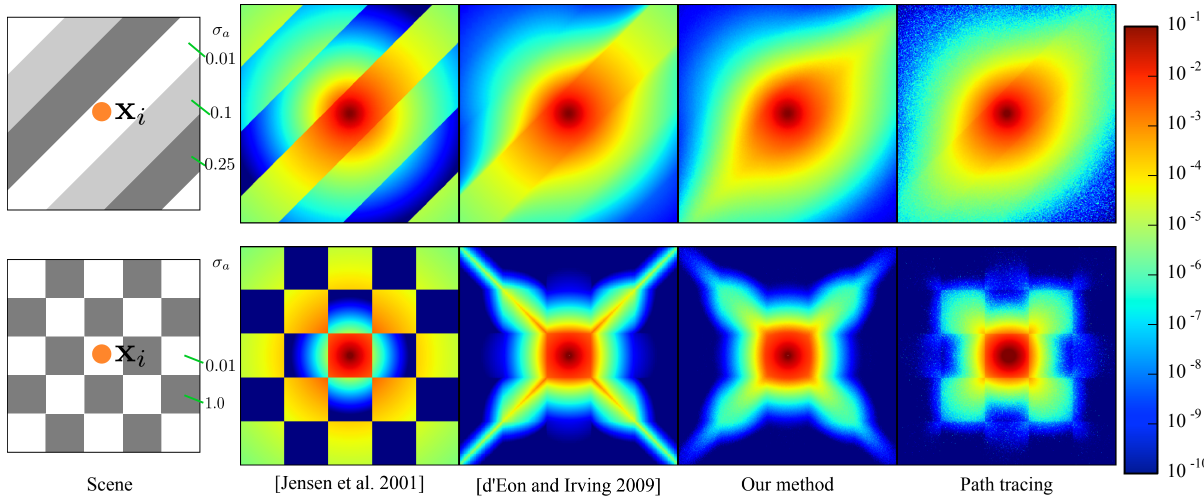

Figure 4: *Comparisons with path tracing, a heuristic proposed by Jensen et al. [JMLH01], a heuristic proposed by d'Eon and Irving [dI11] and our technique. A beam of light enters at $\mathbf{x}_i$ on a nonrefractive semi-infinite medium with the absorption coefficients σ_a given by the texture and the constant scattering coefficient $\sigma_s = 1.0$. The plot regions are $[-15, 15] \times [-15, 15]$. Single scattering is excluded for all the results.*

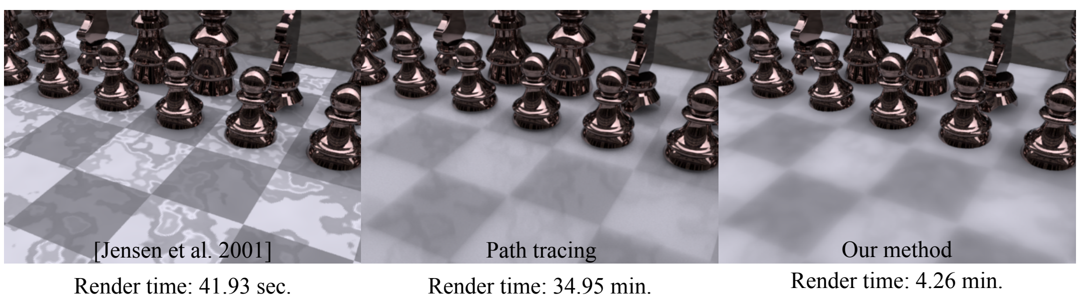

Figure 5: *Comparison of rendered images. Our method reproduces the blurred pattern of the chessboard where the existing method fails.*

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