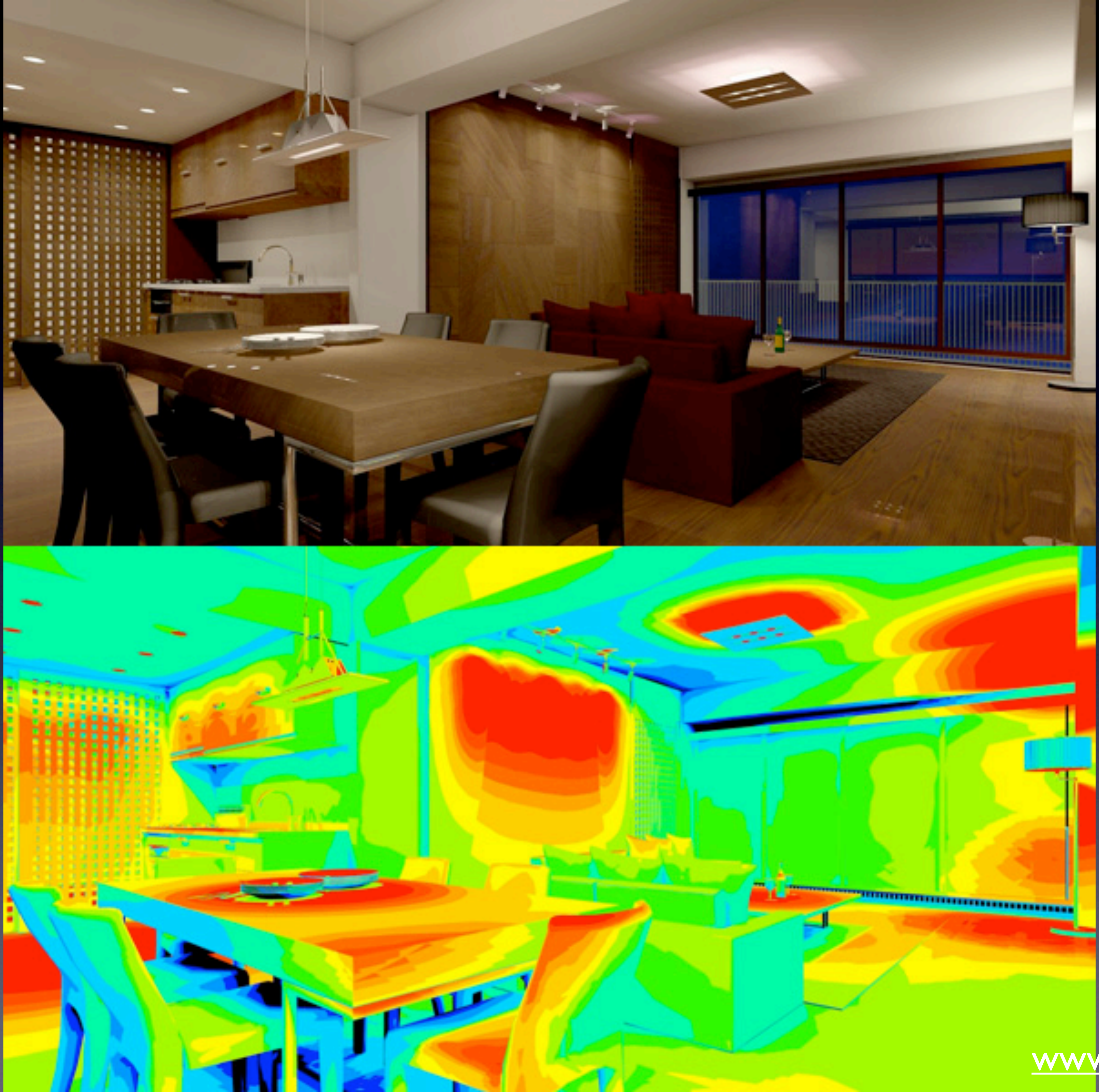
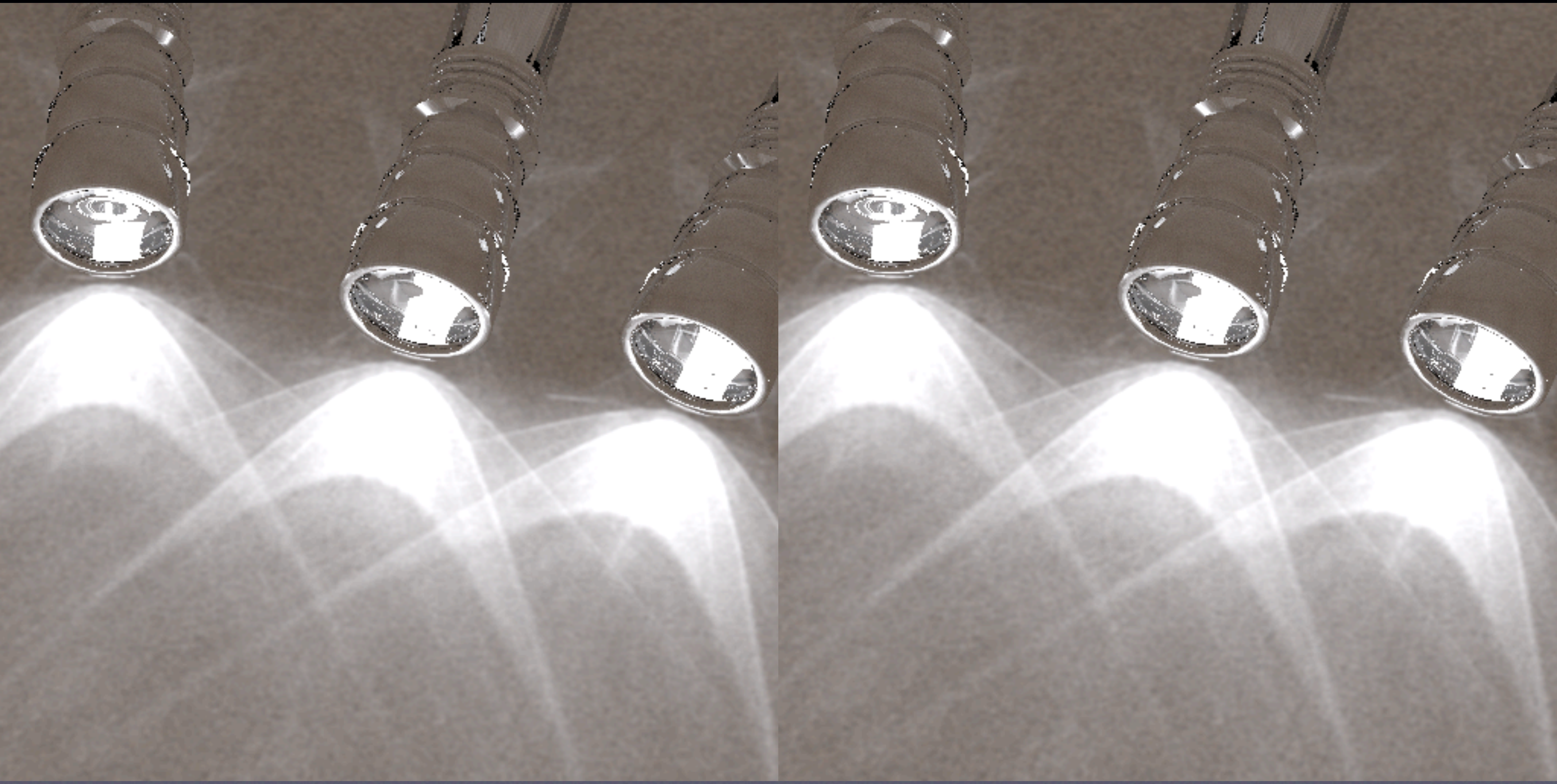
A Progressive Error Estimation Framework for Photon Density Estimation

Toshiya Hachisuka[*] Wojciech Jarosz[†*] Henrik Wann Jensen[*]

[*]University of California, San Diego
[†]Disney Research Zürich



Quiz



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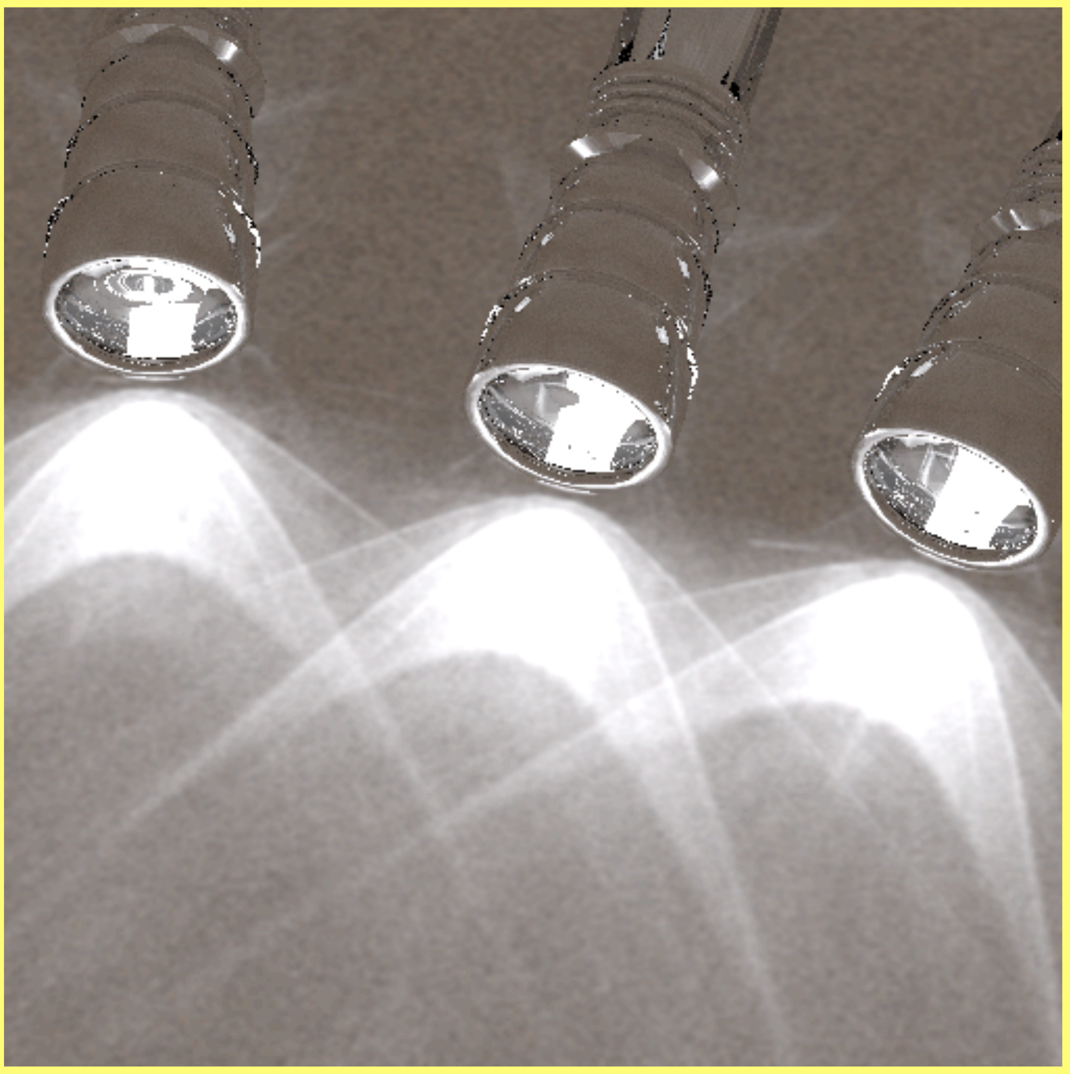

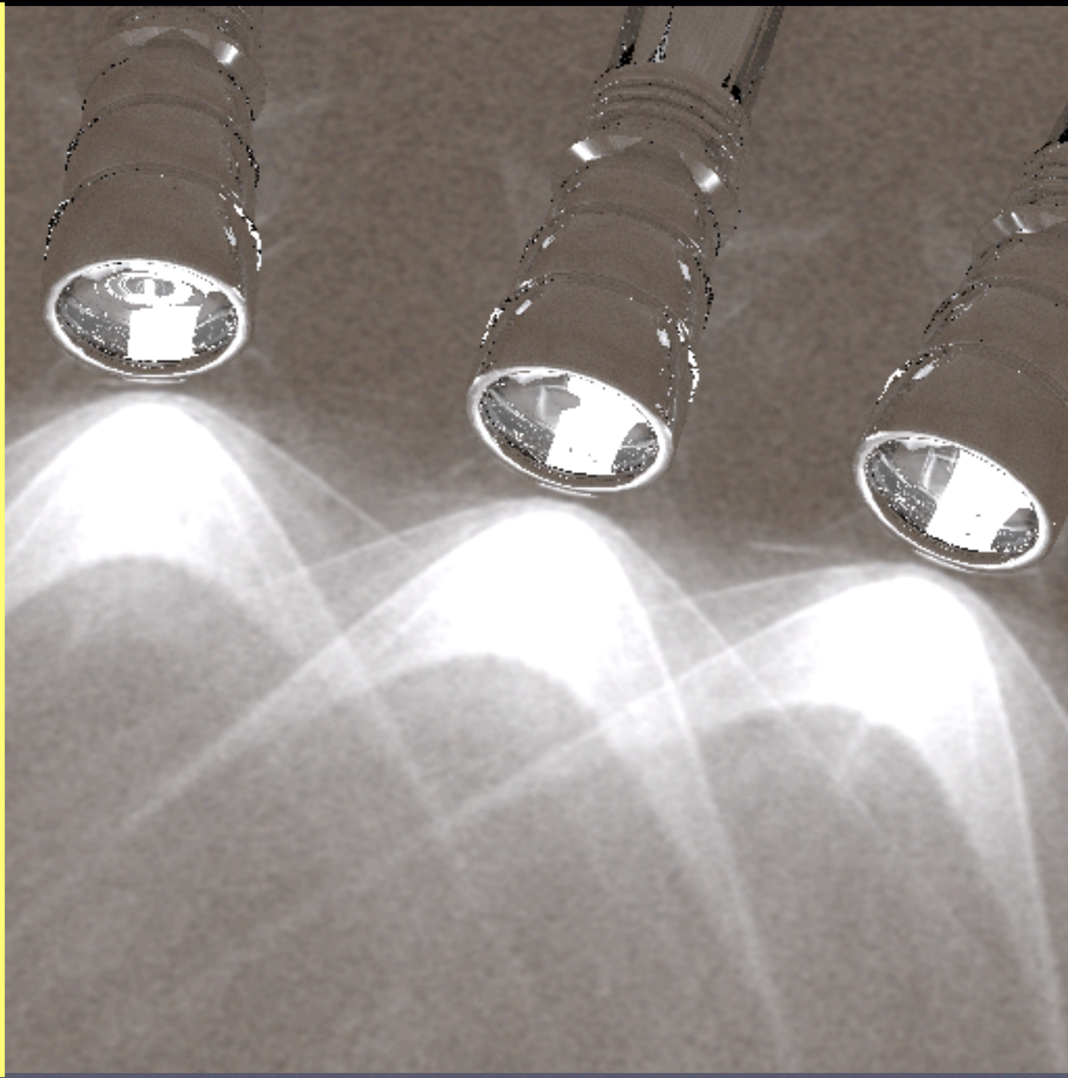

Importance of Error Estimation

- ‘Guesswork’ does not work!

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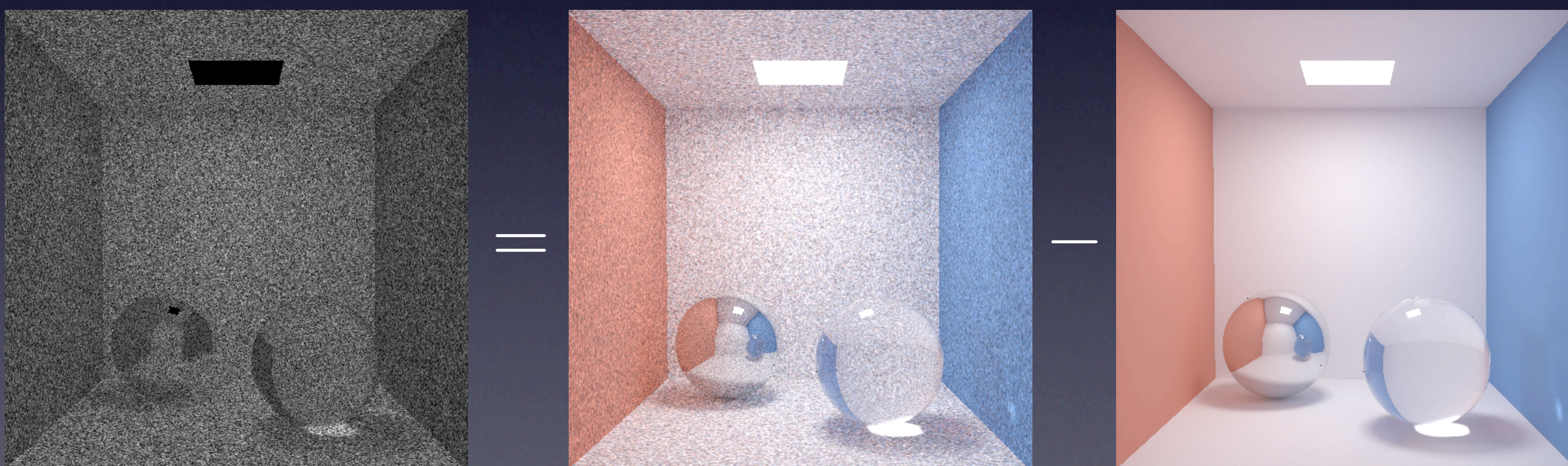
- ‘Guesswork’ does not work!
- Key element for many applications
  - Predictive rendering (e.g., lighting engineering)
  - Error-driven computation
  - Theoretical error analysis

Definition of Error

- Difference between computed and exact

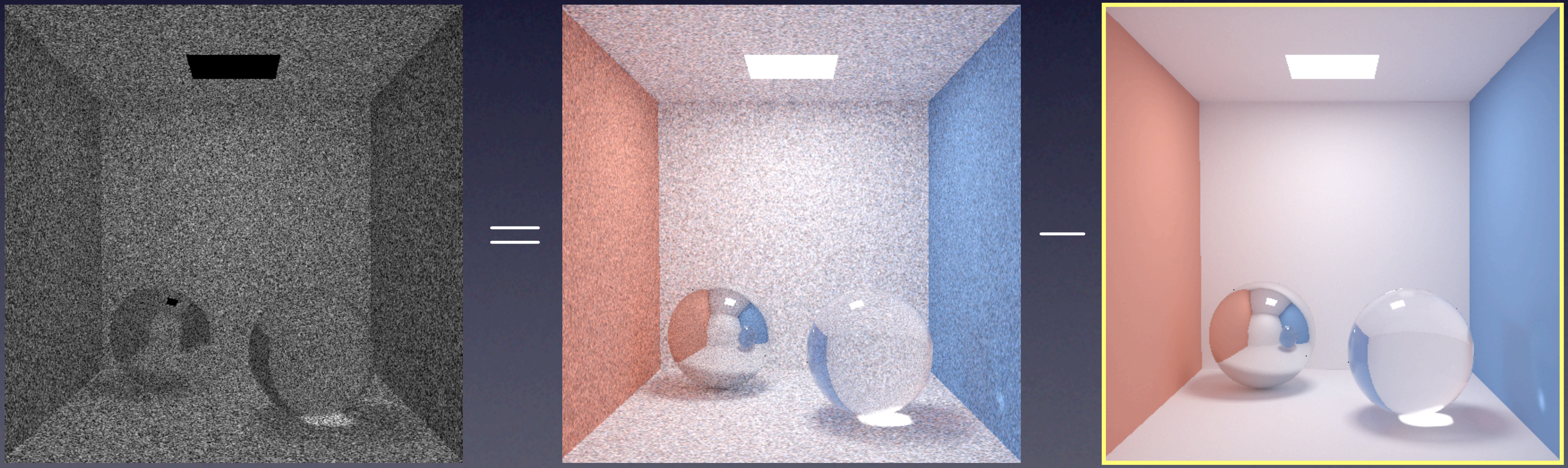
$$E_i = L_i - L$$



Definition of Error

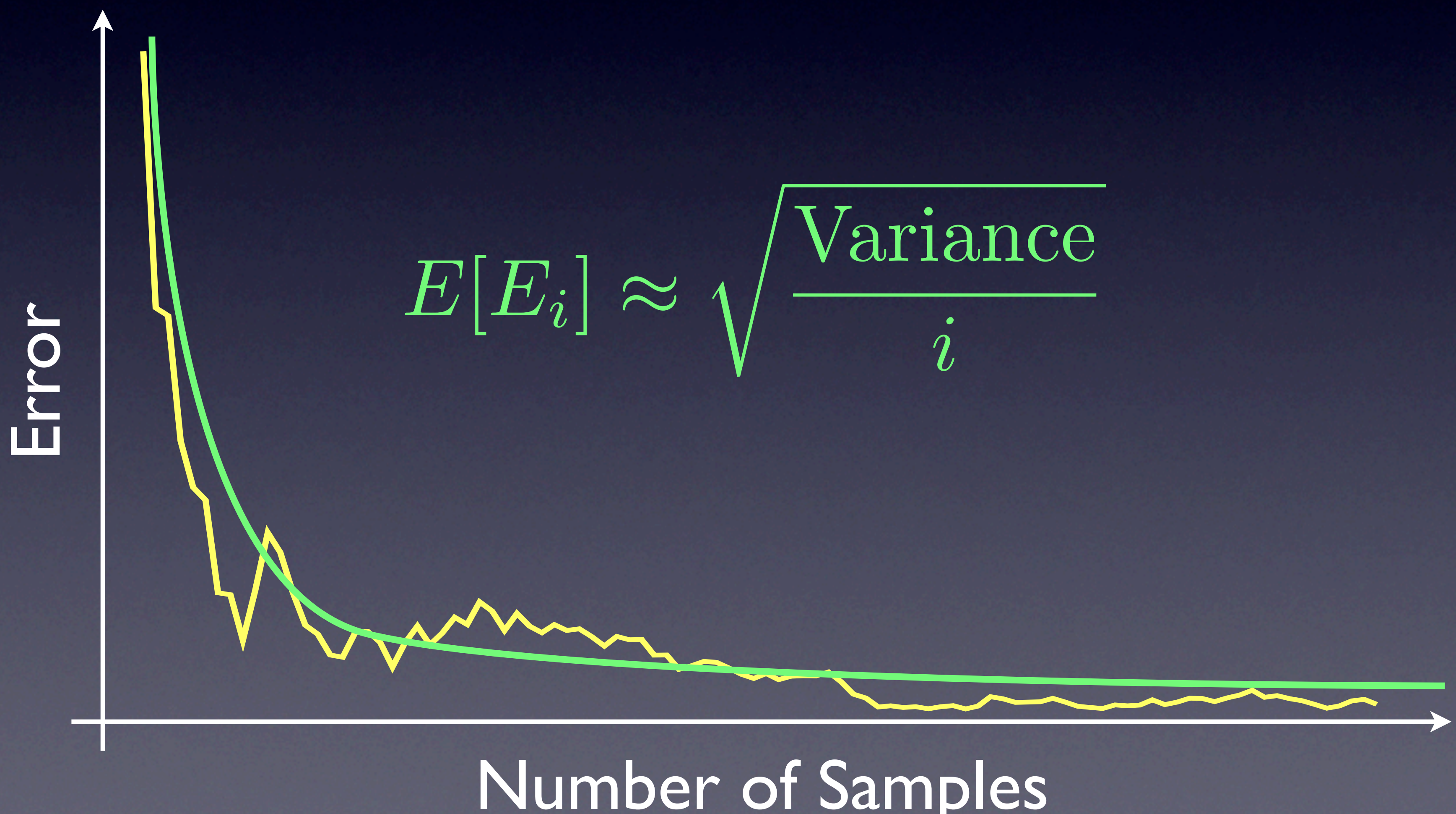
- Difference between computed and exact

$$E_i = L_i - \boxed{L}$$ Unknown



Error in Unbiased Methods

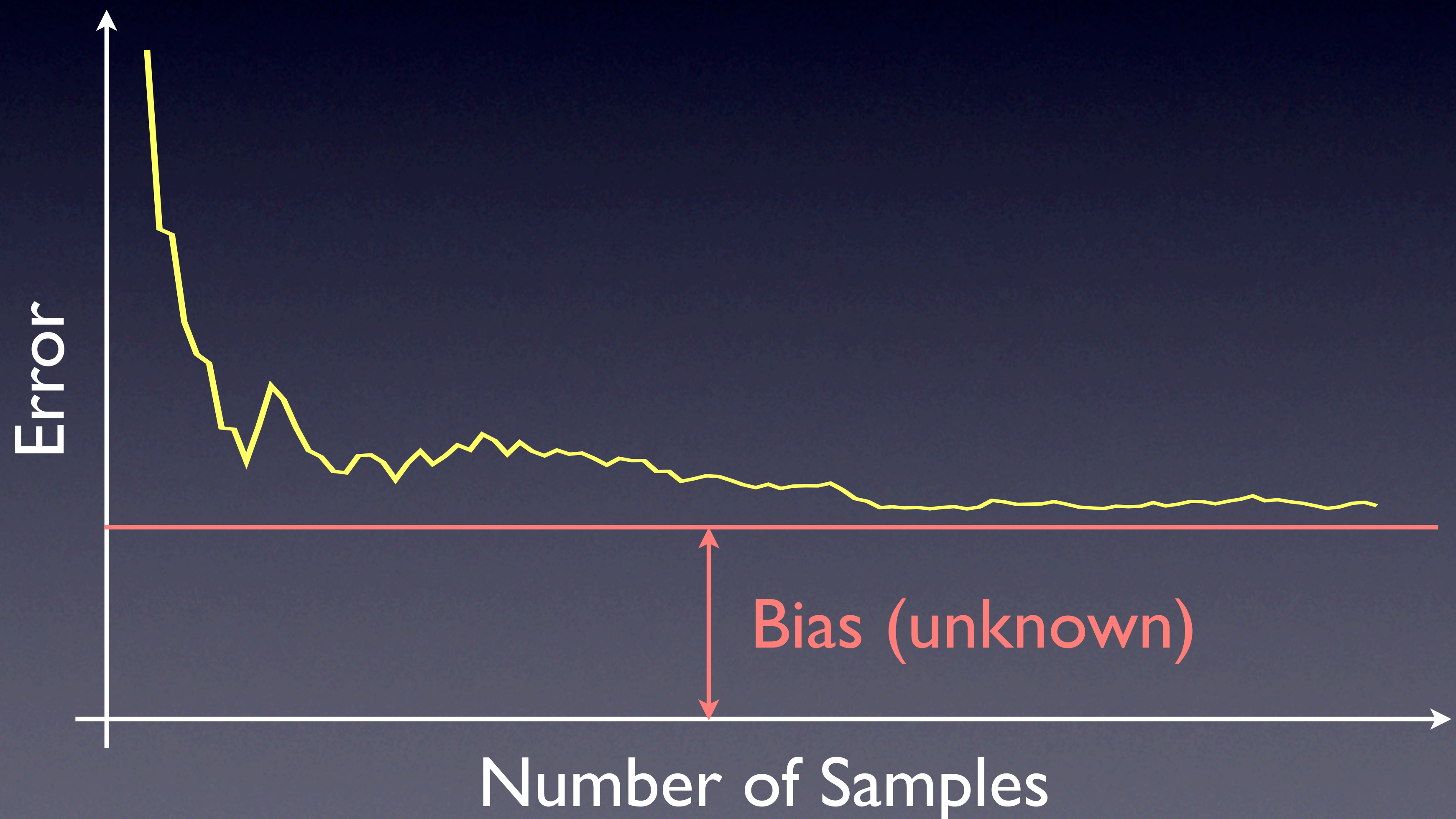
- Path Tracing [Kajiya 86]



$$E[E_i] \approx \sqrt{\frac{\text{Variance}}{i}}$$

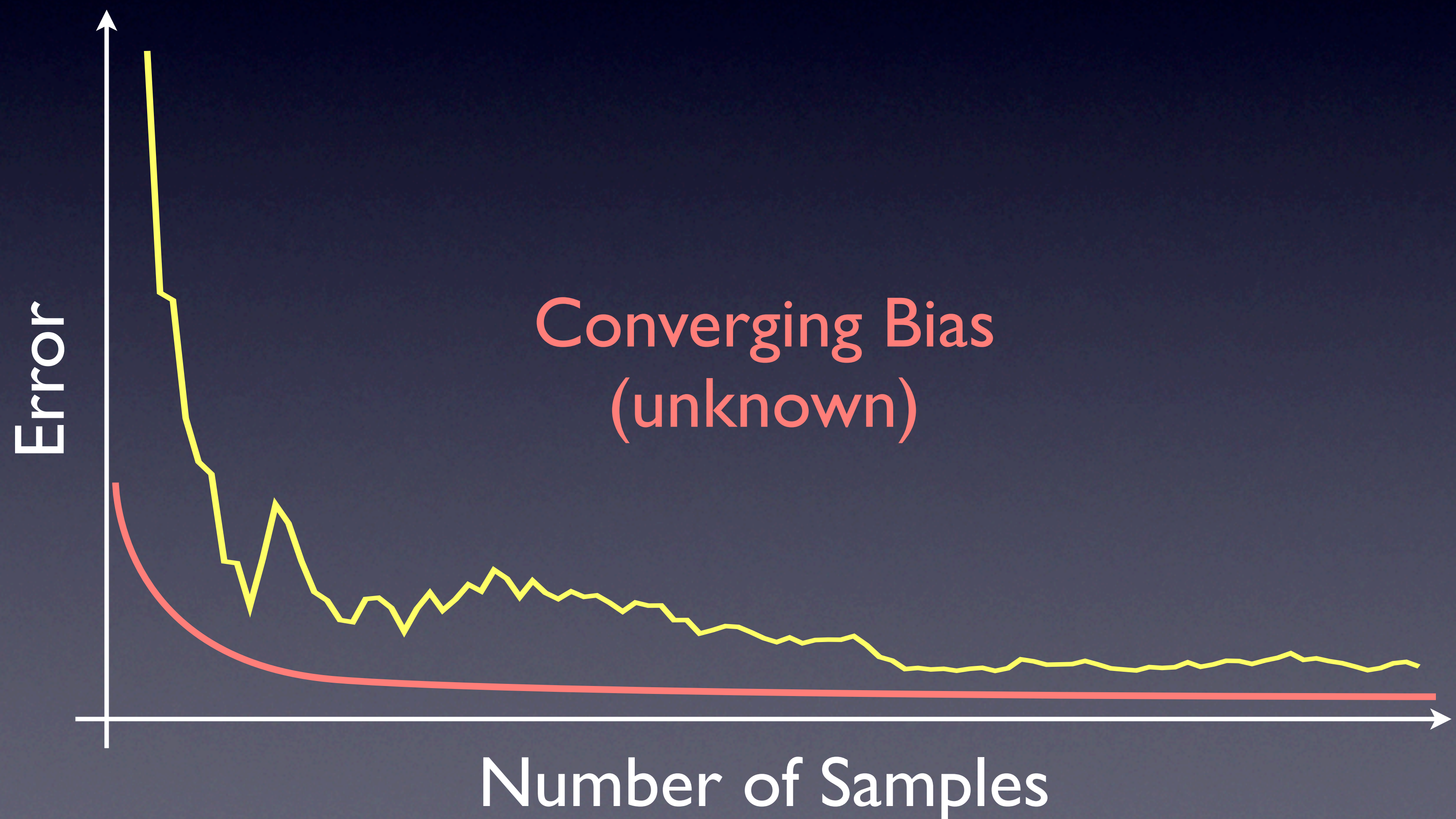
Error in Biased Methods

- Photon Density Estimation [Jensen 96][Walter 98]



Error in Biased Methods

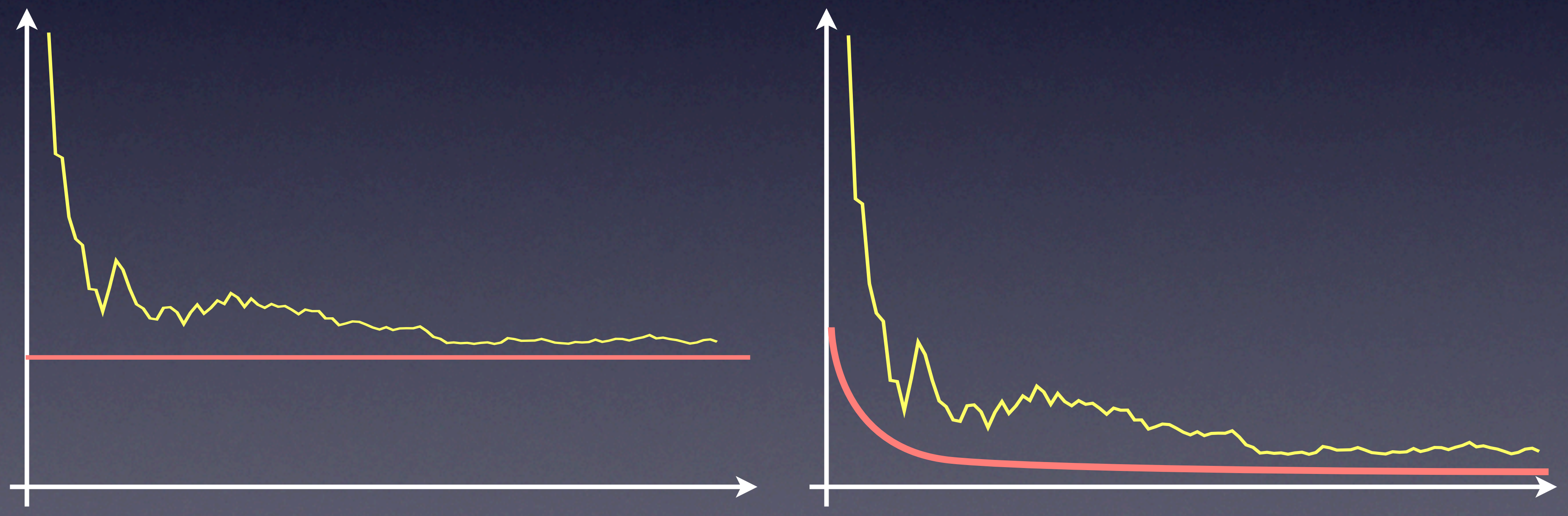
- Progressive Photon Mapping [Hachisuka et al. 08]



Error in Biased Methods

- Error estimation in biased methods is challenging

$$E_i = ?$$



Related Work

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- Variance-based estimation for unbiased methods [Lee et al. 85][Purgathofer 87][Tamstorf et al. 97]

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- Bias reduction [Myszkowski 97][Schregle 03]
- Heuristic error estimation [Walter 98]

Contribution

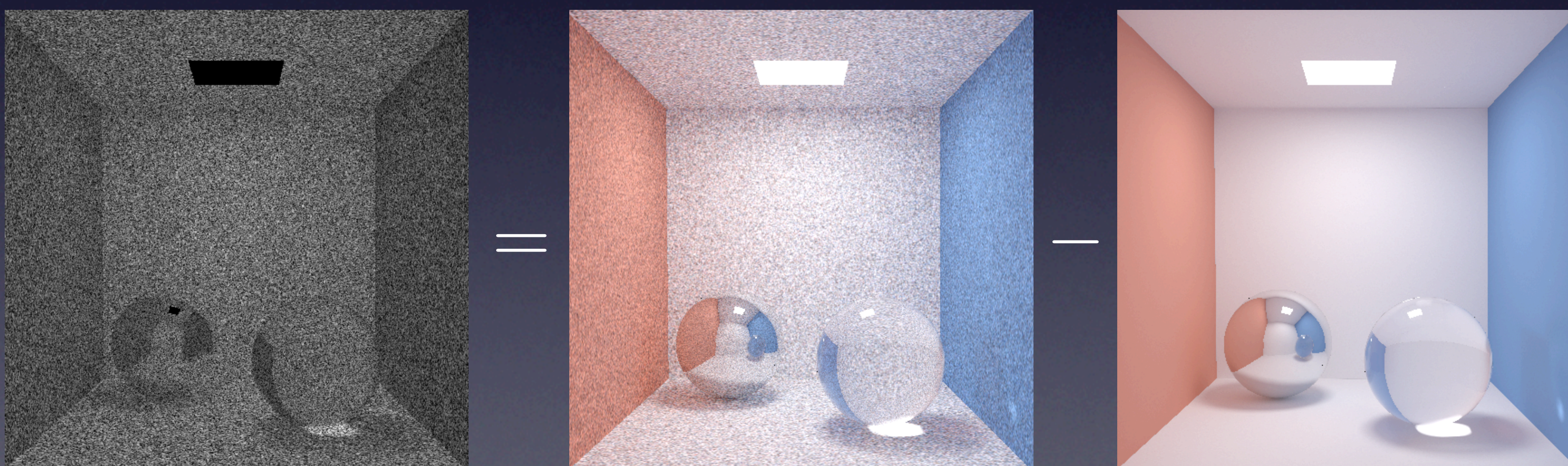
Error estimator for photon density estimation

Method

Error in Biased Methods

- Difference between computed and exact

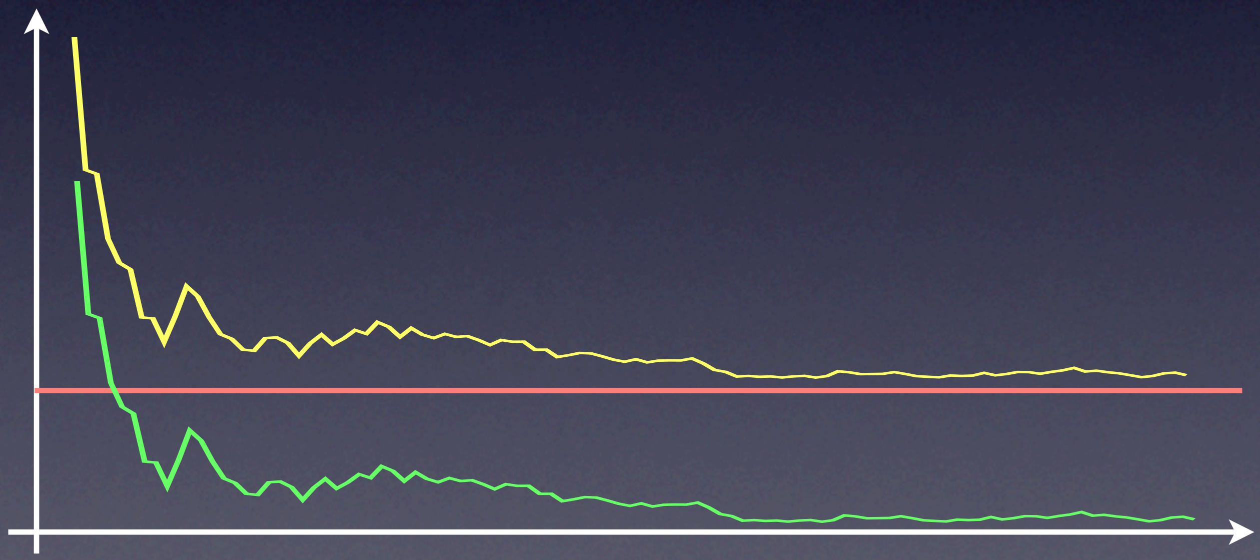
$$E_i = L_i - L$$



Error in Biased Methods

- Bias-Noise decomposition

$$E_i = L_i - L = B_i + N_i$$



Photon Mapping

Error in Biased Methods

- Bias-Noise decomposition

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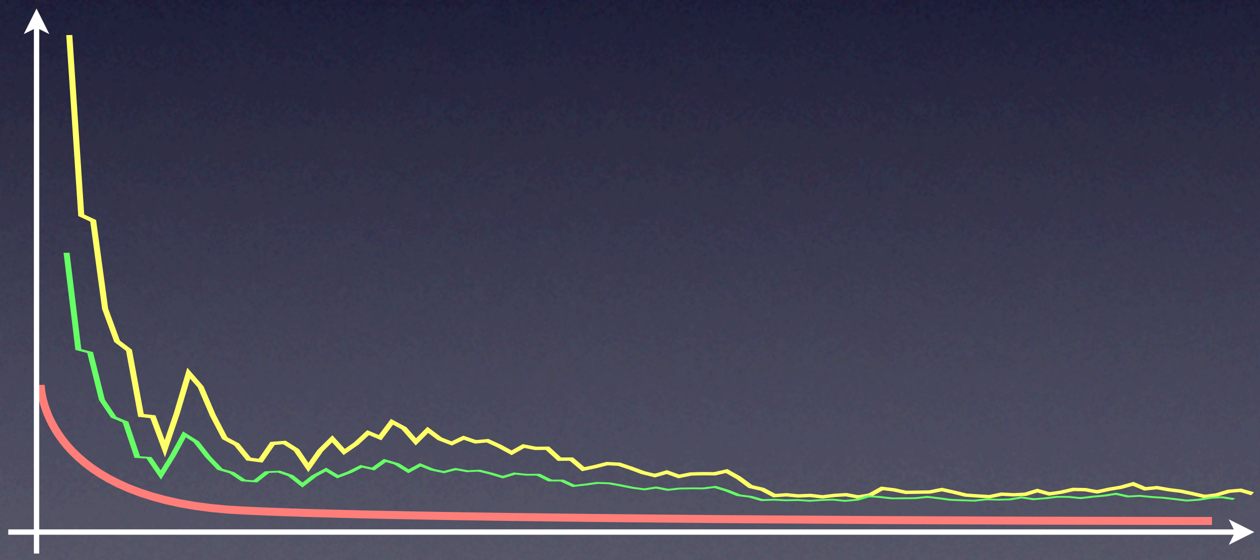


Progressive Photon Mapping

Error Estimation in Monte Carlo Methods

- Can we estimate E_i?

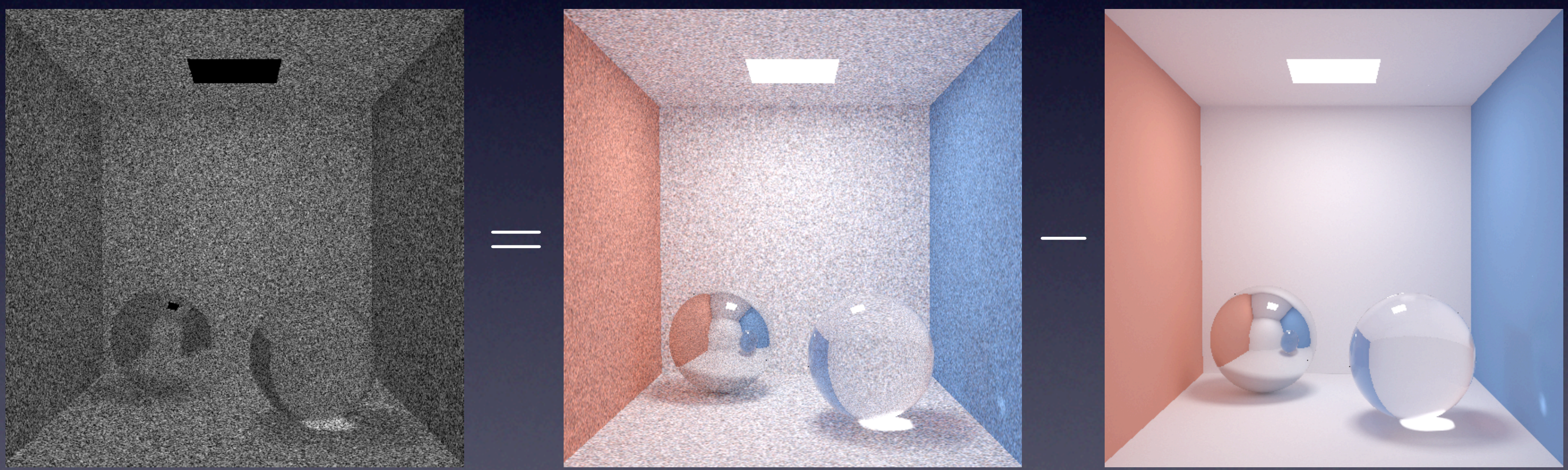
Error Estimation in Monte Carlo Methods

- Can we estimate E_i?

Probably Not

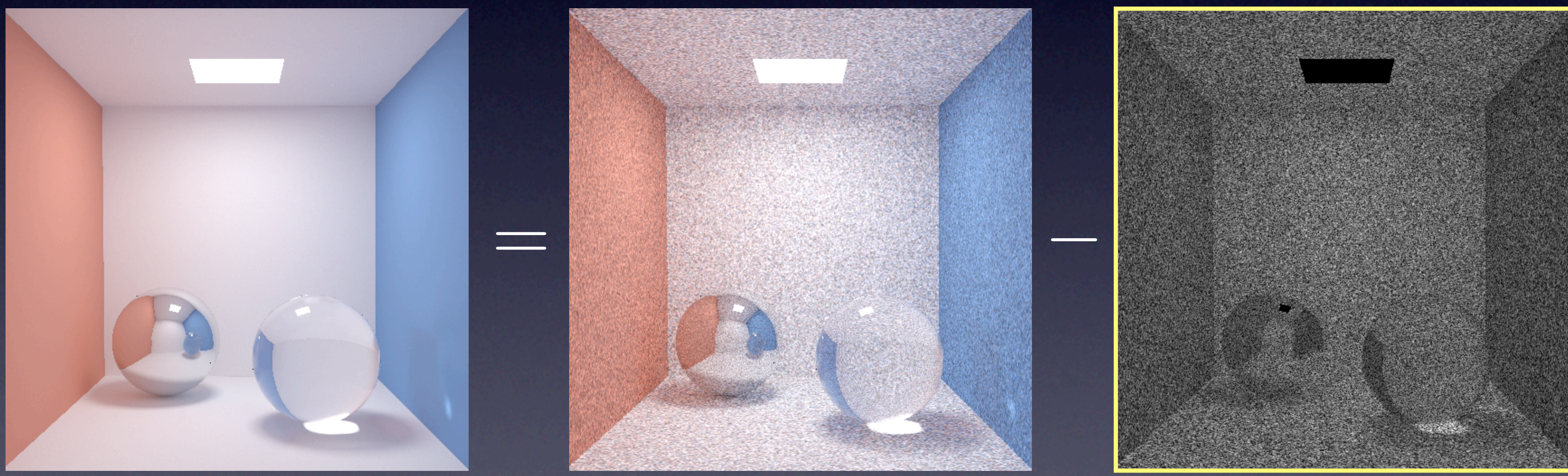
Error Estimation in Monte Carlo Methods

$$E_i = L_i - L$$



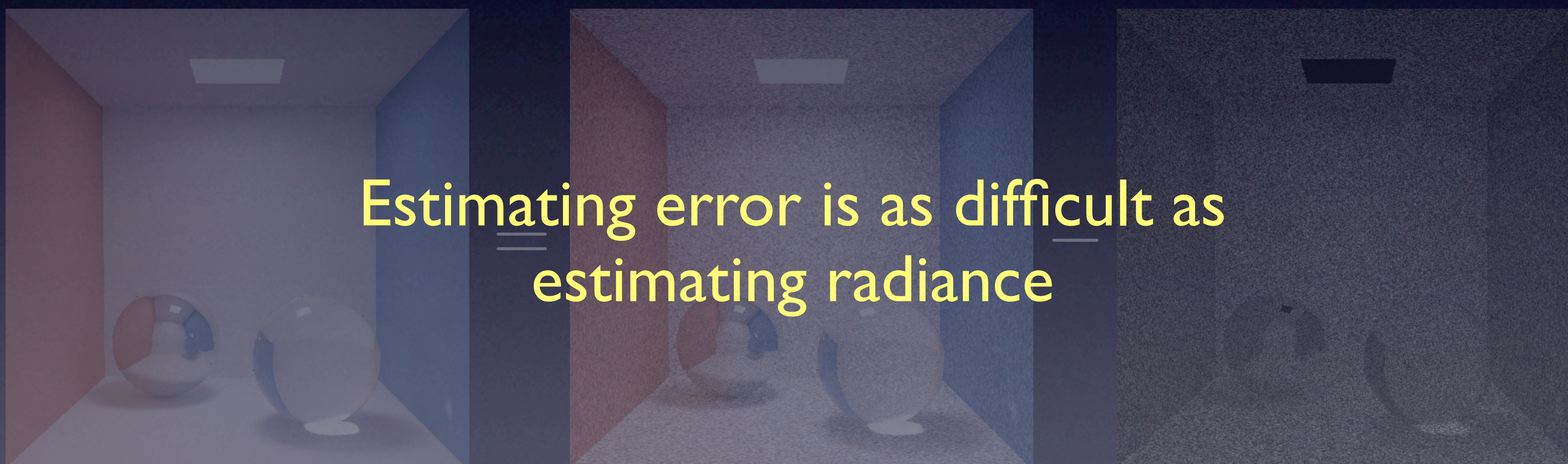
Error Estimation in Monte Carlo Methods

$$L = L_i - \boxed{E_i}$$



Error Estimation in Monte Carlo Methods

$$L = L_i - E_i$$



Estimating error is as difficult as estimating radiance

Error Estimation in Monte Carlo Methods

- Can we estimate bounds of E_i ?

$$E_{\min,i} \leq E_i \leq E_{\max,i}$$

Error Estimation in Monte Carlo Methods

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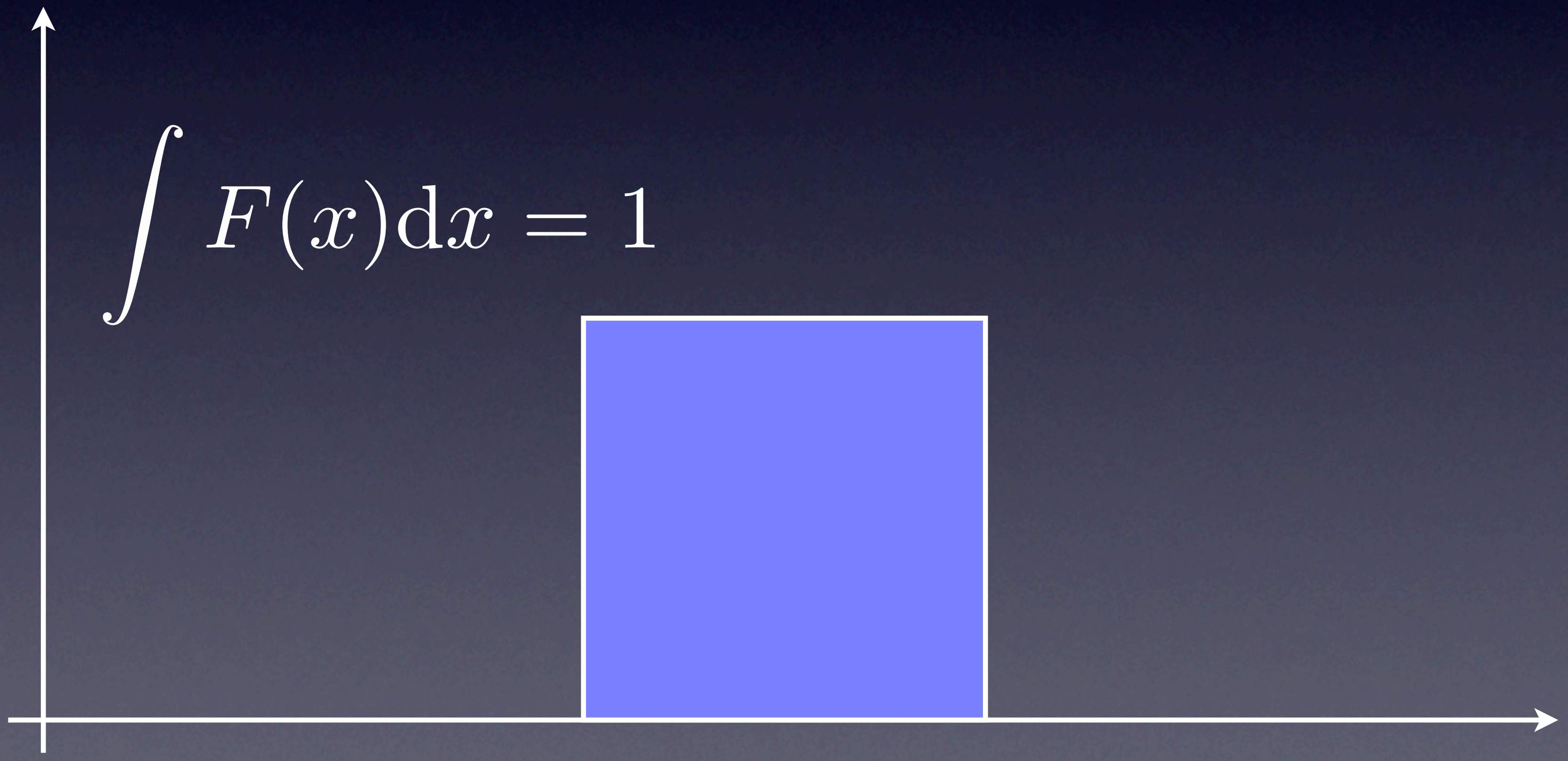
Probably Not

Error Estimation in Monte Carlo Methods

- Integration of a rectangular function

$$\int F(x)\mathrm{d}x = 1$$



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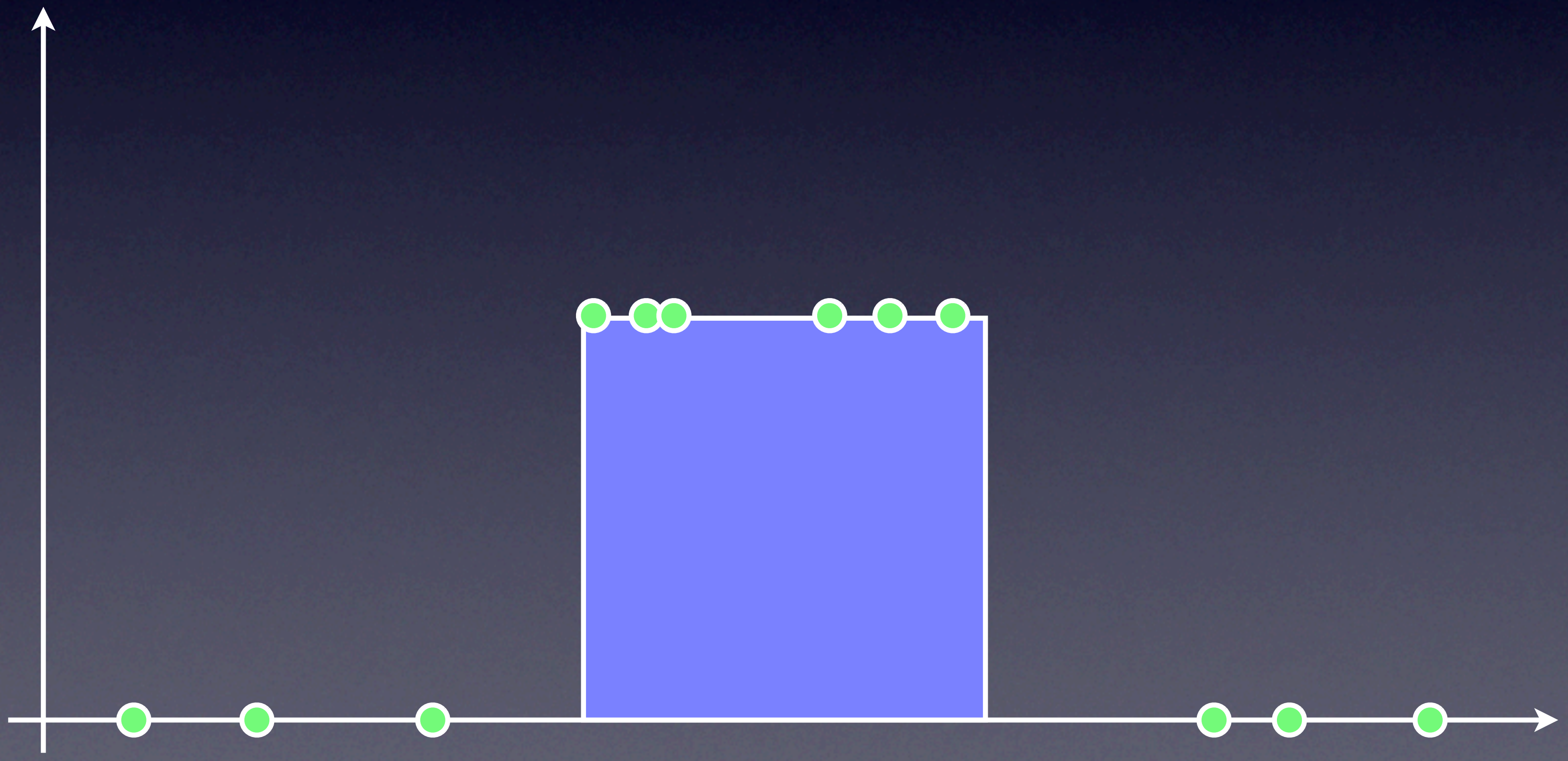

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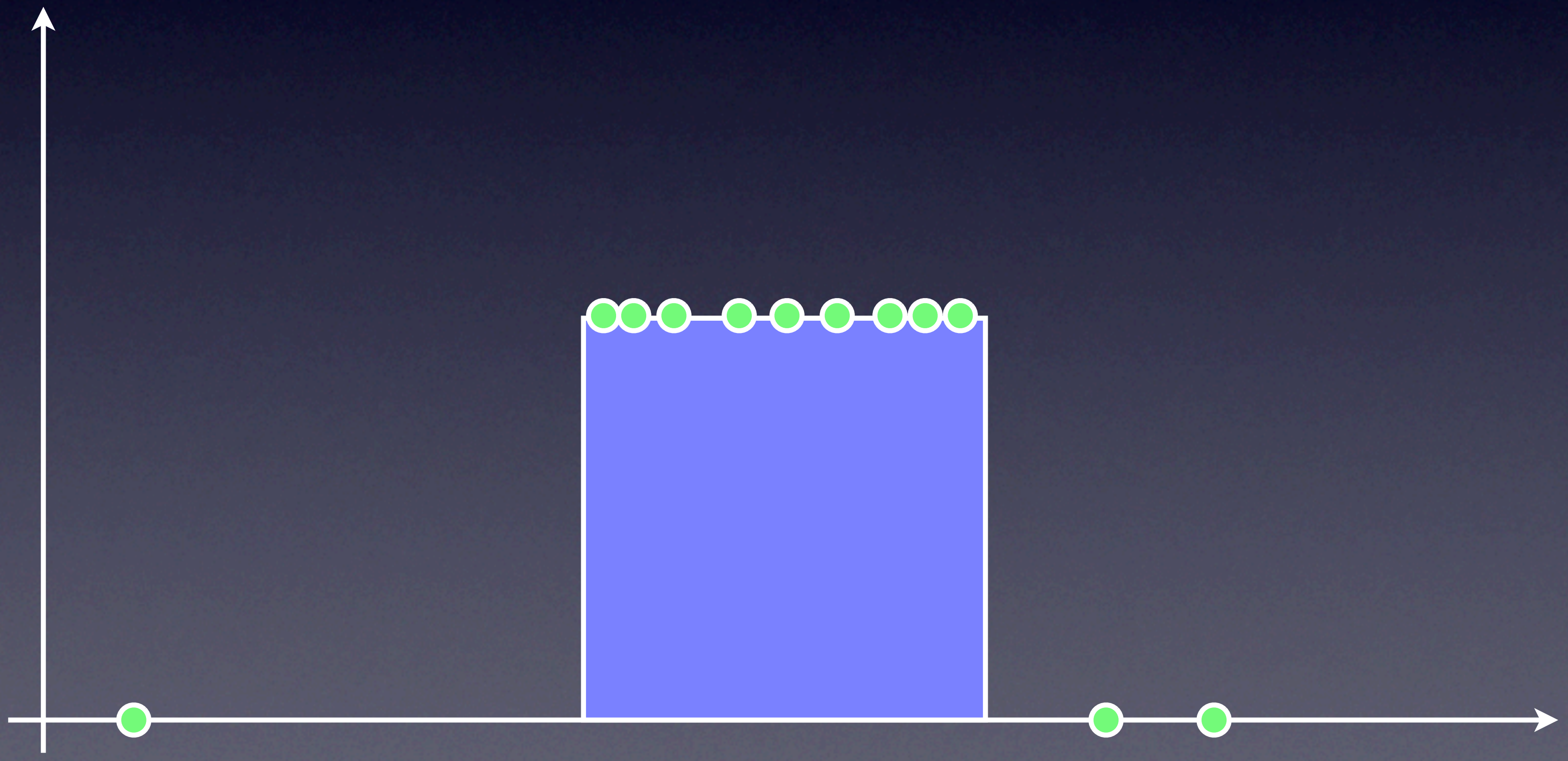

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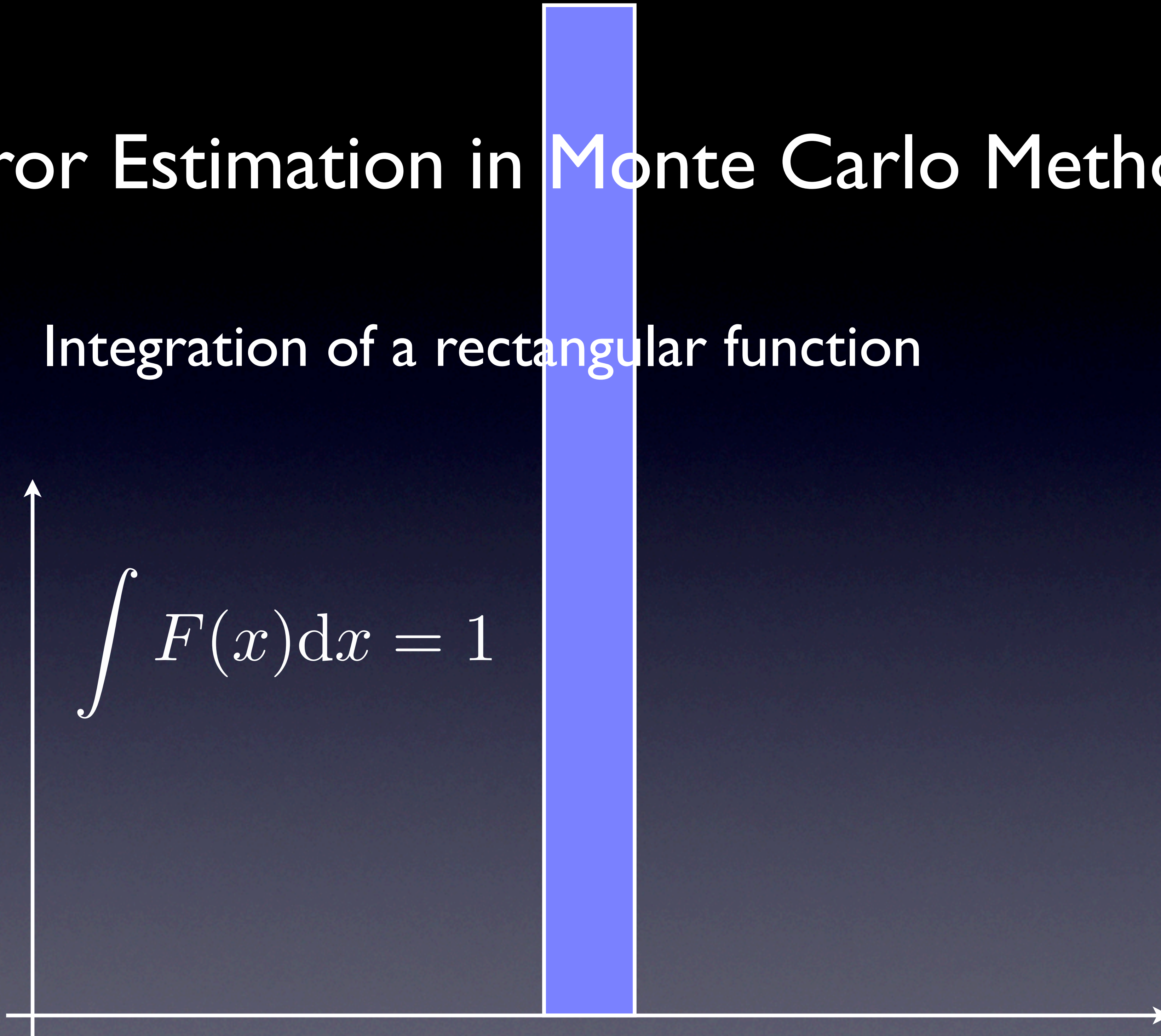

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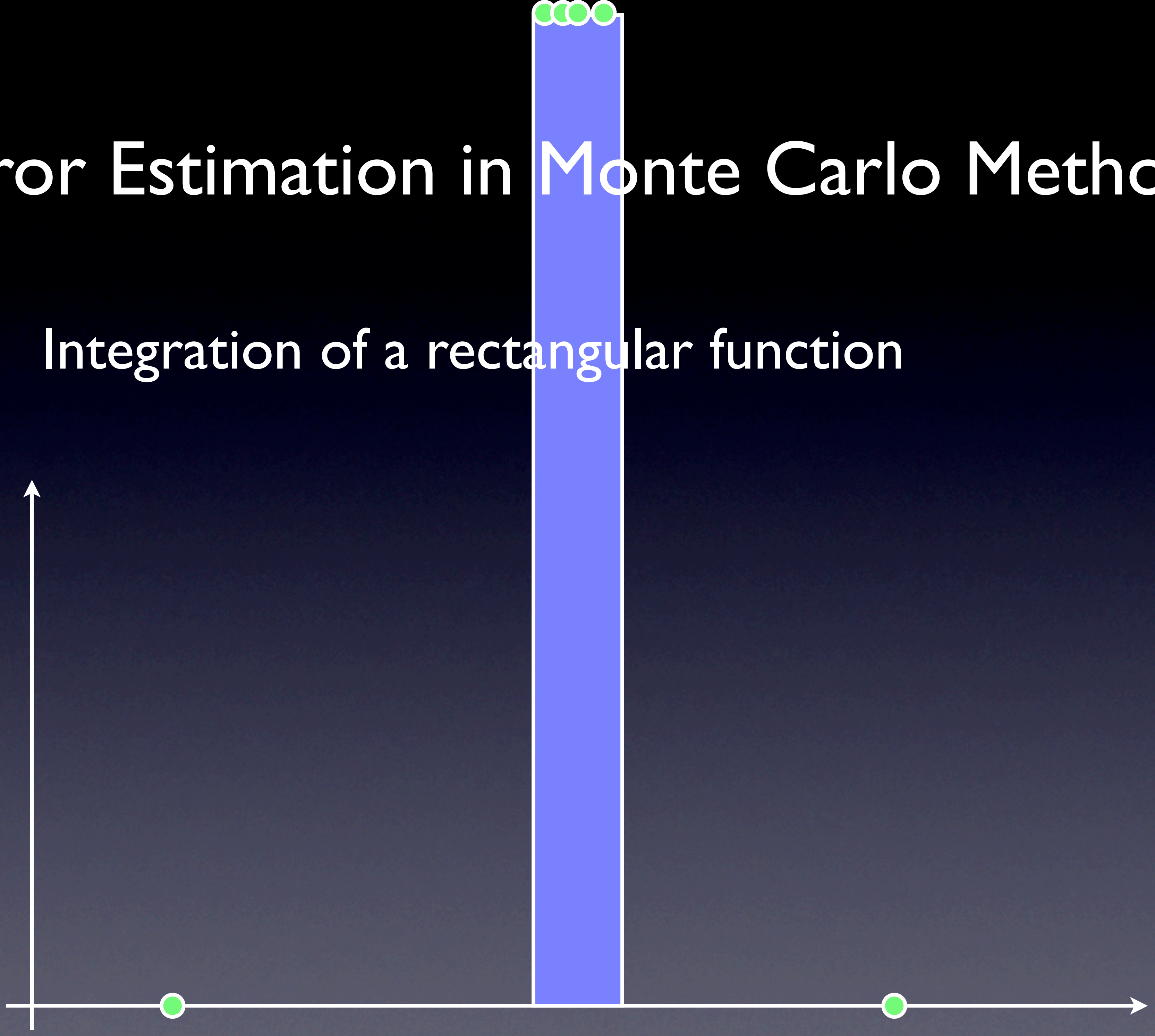

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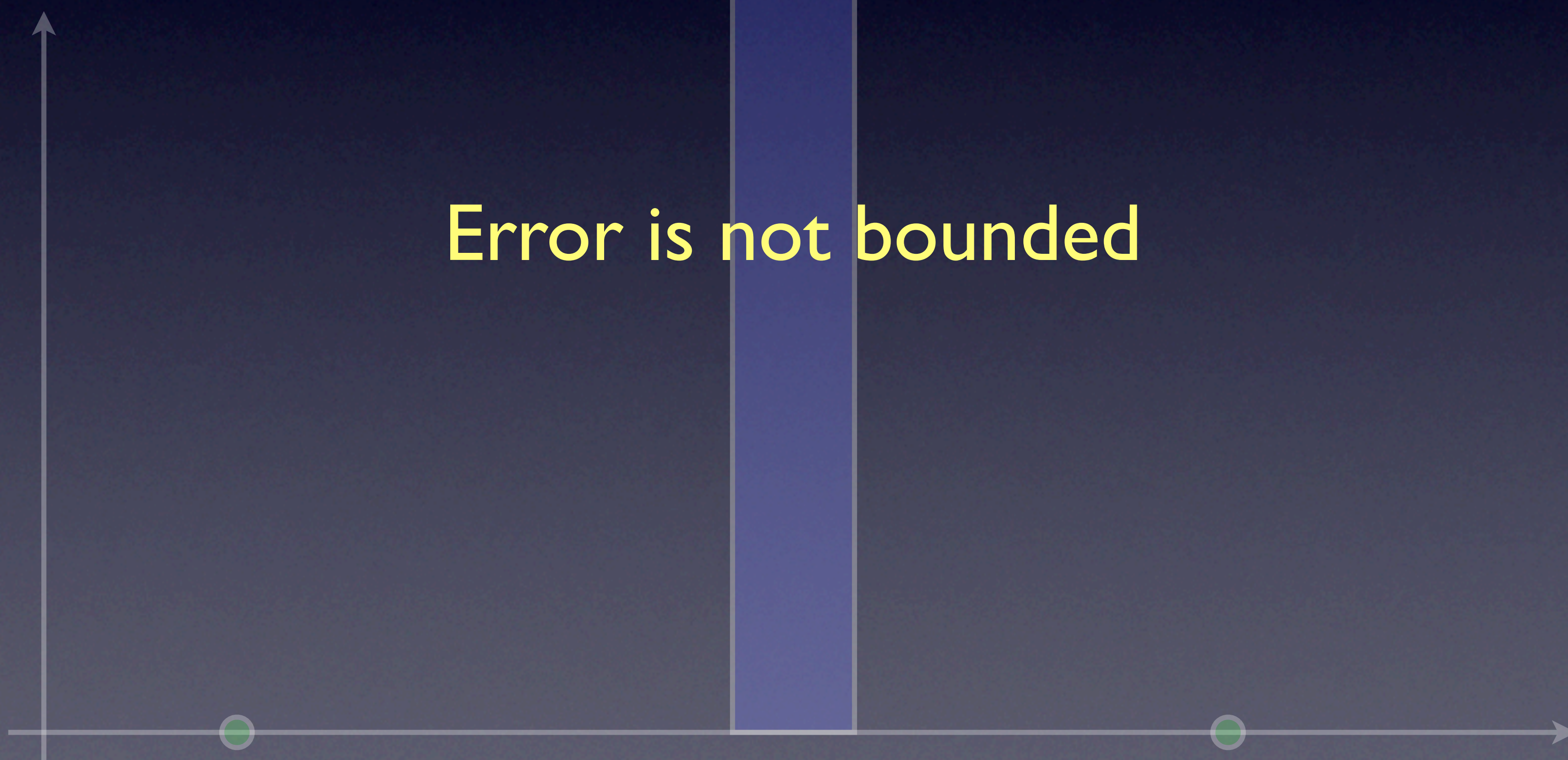

Error is not bounded

Error Estimation in Monte Carlo Methods

$$\text{Probability}(E_{\min,i} \le E_i \le E_{\max,i}) = 90\%$$

Drop a few cases where

$$E_{\min,i} \le E_i \le E_{\max,i}$$

is false

Stochastic Error Bounds

- Bounds that are true with some probability
- Well-known concept in computational statistics

$$P(\boxed{E_{\min,i} \leq E_i \leq E_{\max,i}}) = \boxed{1 - \beta}$$

Stochastic error bound

User-defined Confidence

Stochastic Error Bound Derivation

$$L_i - L = E_i = B_i + N_i$$

Stochastic Error Bound Derivation

- Subtract bias

$$L_i - L - B_i = E_i - B_i = N_i$$

Stochastic Error Bound Derivation

- Noise follows the t-distribution

$$L_i - L - B_i = E_i - B_i = N_i$$

$$P(-N_b \leq N_i \leq N_b) = 1 - \beta$$

$$N_b = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}}$$

Stochastic Error Bound Derivation

- Add back bias

$$L_i - L = E_i = B_i + N_i$$

$$P(-N_b + B_i \le E_i \le N_b + B_i) = 1 - \beta$$

$$N_b = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}}$$

Stochastic Error Bound Derivation

- Take the absolute value

$$L_i - L = E_i = B_i + N_i$$

$$P(|E_i| \leq |N_b| + |B_i|) \leq 1 - \beta$$

$$N_b = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}}$$

Stochastic Error Bound Derivation

$$L_i - L = E_i = B_i + N_i$$

Stochastic error bound

User-defined Probability

$$\boxed{P(|E_i| \leq E_{b,i})} \leq \boxed{1 - \beta}$$

$$E_{b,i} = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}} + |B_i|$$

Stochastic Error Bound Derivation

$$L_i - L = E_i = B_i + N_i$$

$$P(|E_i| \leq E_{b,i}) \leq 1 - \beta$$

$$E_{b,i} = \boxed{C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}}} + |B_i|$$

Error due to Noise

Stochastic Error Bound Derivation

$$L_i - L = E_i = B_i + N_i$$

$$P(|E_i| \leq E_{b,i}) \leq 1 - \beta$$

$$E_{b,i} = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}} + \boxed{|B_i|}$$

Error due to Bias

Challenges

$$E_{b,i} = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}} + |B_i|$$

- B_i (bias) is unknown

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$$E_{b,i} = C_{i,1-\frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}} + |B_i|$$

- B_i (bias) is unknown
- Variance estimation assumes i.i.d.
    - independent and identically distributed
    - not true in progressive photon mapping

Bias Estimation

- B_i (bias) is unknown
- Well-known approximation [Silverman 86]

$$B_i \approx k_2 R_i^2 \Delta L$$

k_2 constant

R_i search radius

ΔL Laplacian of radiance

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- B_i (bias) is unknown
- Well-known approximation [Silverman 86]

$$B_i \approx k_2 R_i^2 \Delta L$$

k_2 constant

R_i search radius

ΔL Laplacian of radiance

Unknown

Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

Kernel

$$L_i(x) = \frac{\sum \boxed{K(x_p - x)} f_r(x, \omega, \omega_p) \Phi(x_p, x)}{\pi R_i^2}$$

Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

Laplacian of the kernel

$$\Delta L_i(x) = \frac{\sum \boxed{\Delta K(x_p - x)} f_r(x, \omega, \omega_p)\Phi(x_p, x)}{\pi R_i^2}$$

Radiance Derivative Estimation

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Laplacian of the kernel

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Extended to progressive photon mapping

Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

Laplacian of the kernel

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Extended to progressive photon mapping

- Applicable to any order

Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

Laplacian of the kernel

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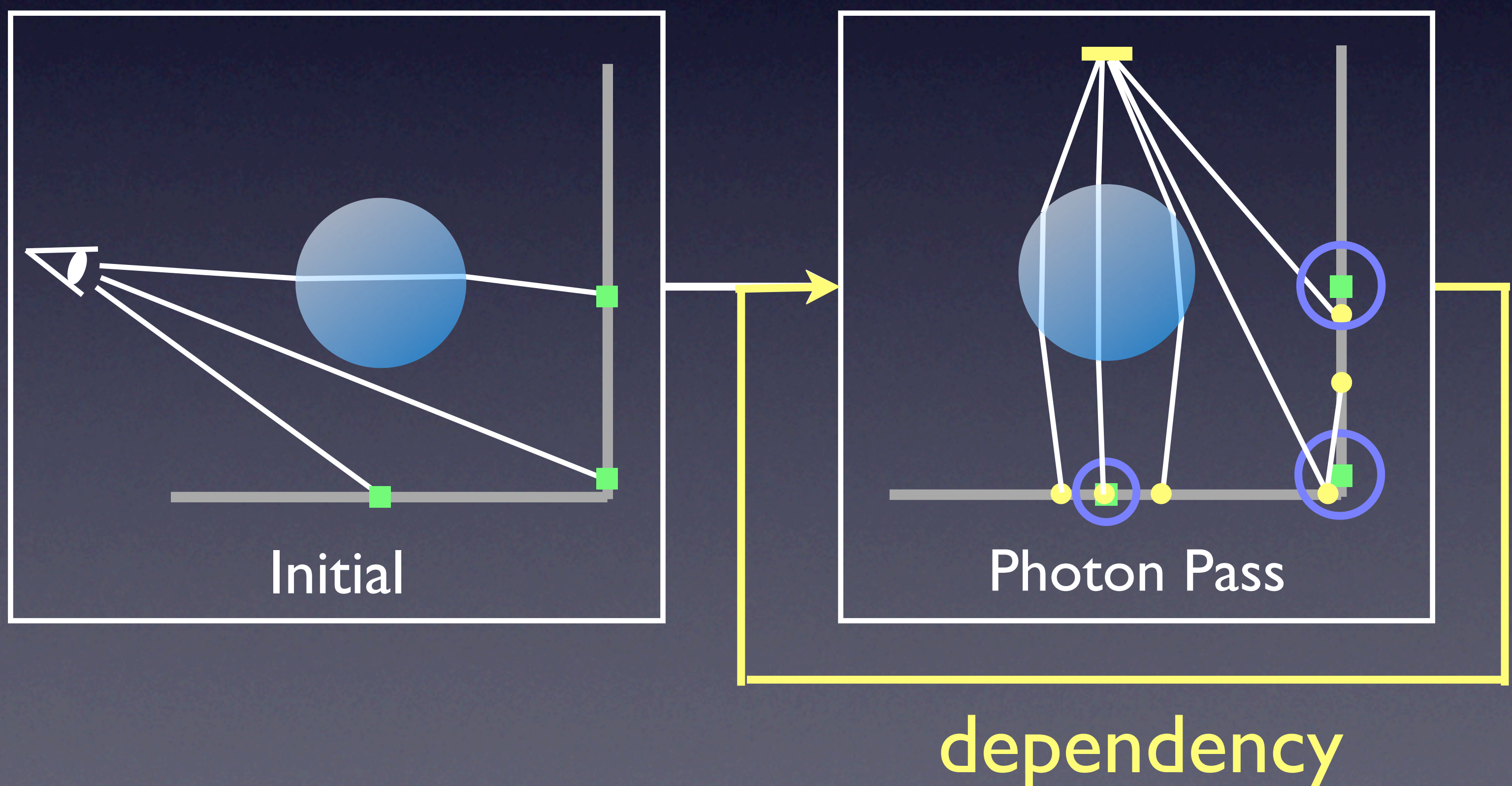
Extended to progressive photon mapping

- Applicable to any order
- Convergent

Variance Estimation

- Variance estimation assumes i.i.d.
  - not true in progressive photon mapping



Variance Estimation

- Two key observations
  - Photon tracing itself is independent
  - Dependency is only in radius reduction

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Bias-corrected radiance

$$L_i' = L_i - B_i$$

$$\text{Variance} \approx \frac{\sum {L_i'}^2 - \frac{\left(\sum L_i'\right)^2}{i}}{i-1}$$

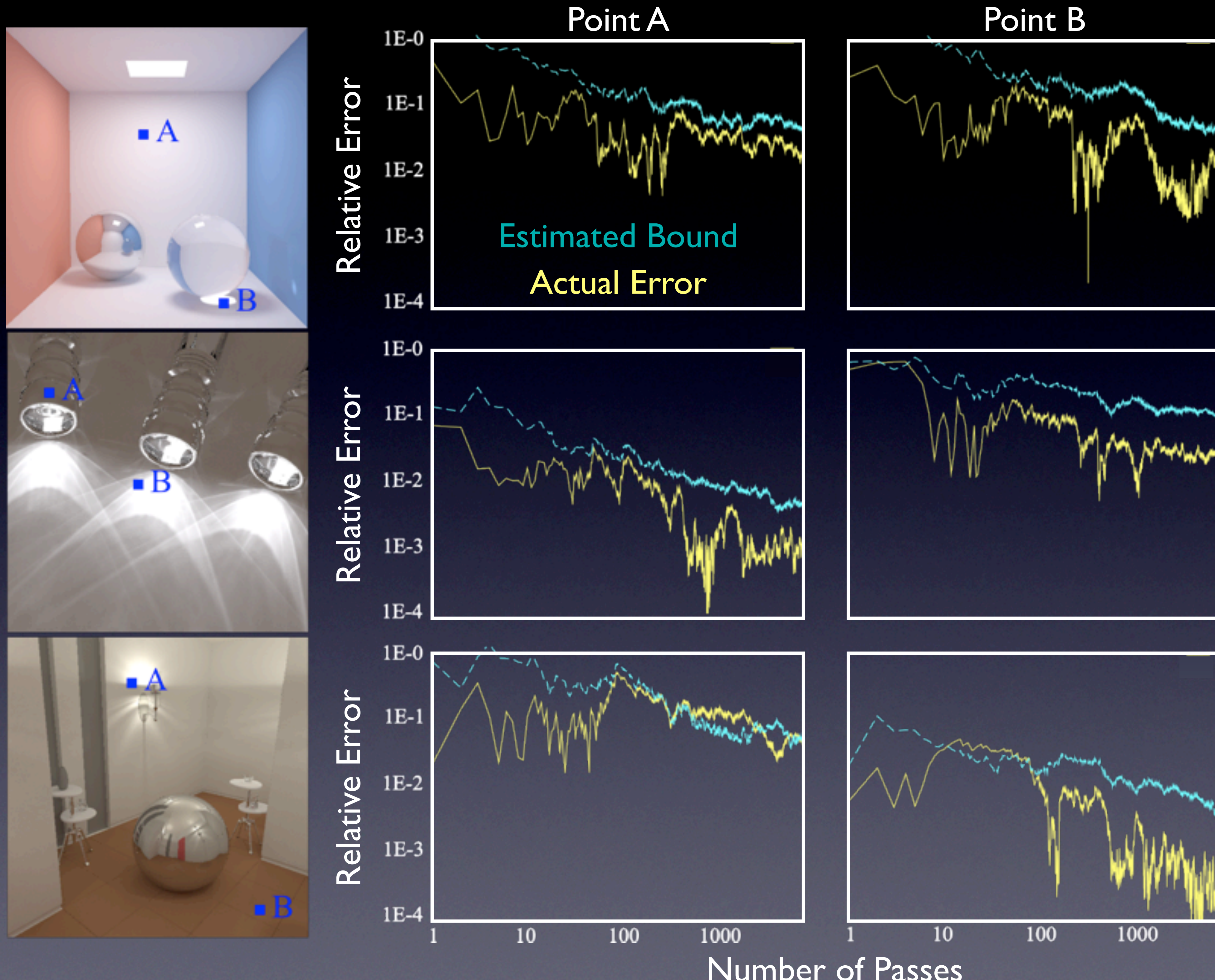
Key Points

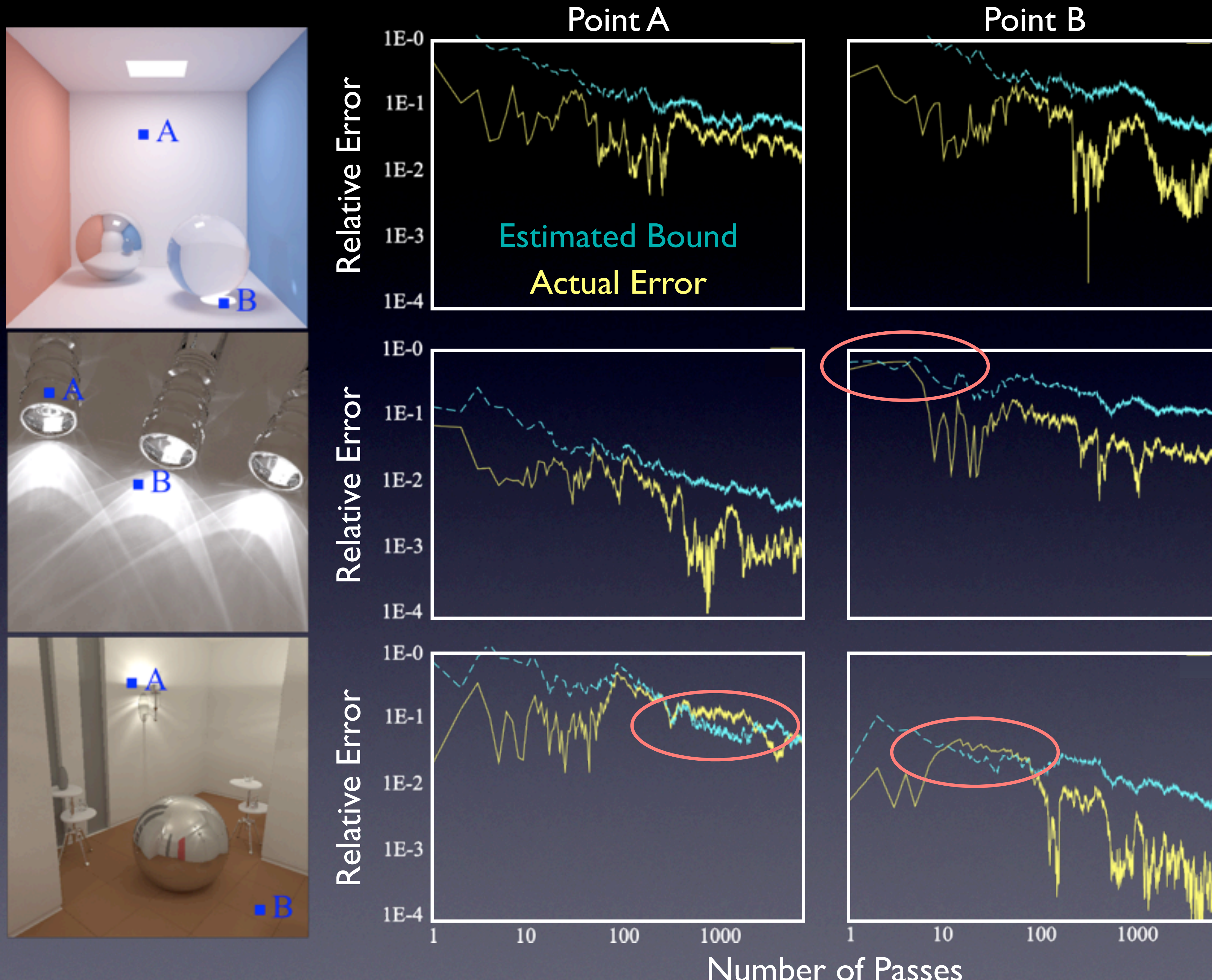
- Approximate stochastic error bounds
- Convergent derivative estimator
- Bias/Noise estimators valid for PPM

Results

Experiments Setup

- Progressive Photon Mapping [Hachisuka et al. 08]
- 15k photons per pass
- Three test scenes with full global illumination


Point A
Point B
A
B
A
B
A
B
Relative Error
Relative Error
Relative Error
1E-0
1E-1
1E-2
1E-3
1E-4
Estimated Bound
Actual Error
1
10
100
1000
Number of Passes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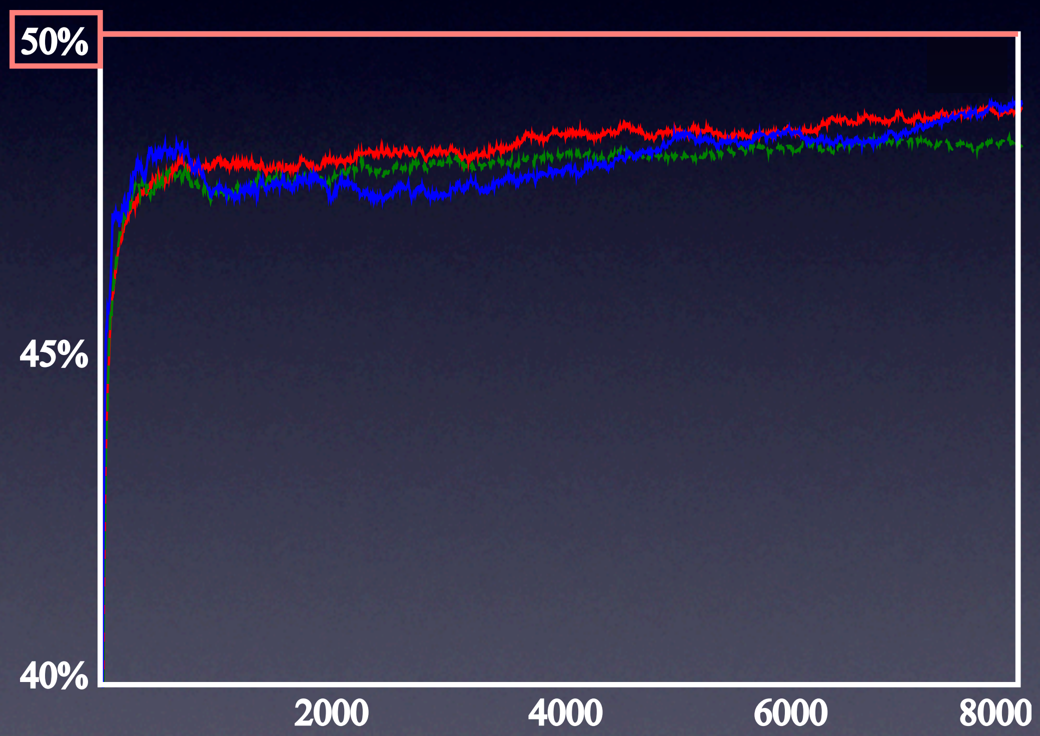
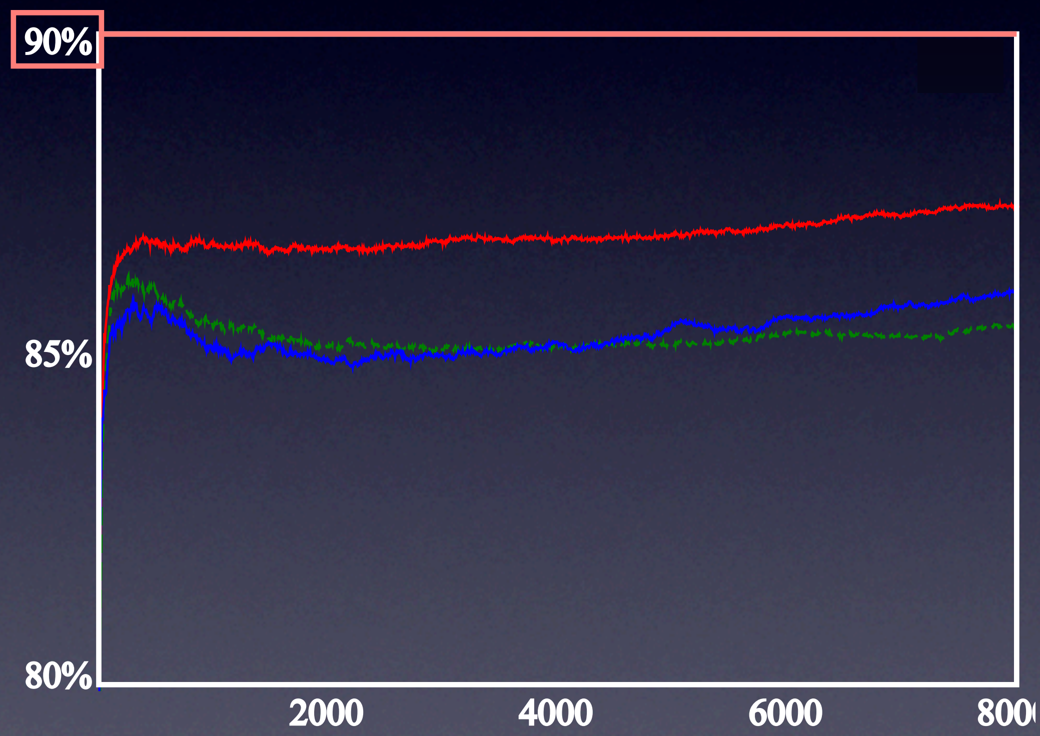
Calculated Probability of Bounds

$$P(|E_i| \leq E_{b,i}) \leq 1 - \beta$$



$$1 - \beta = 50\%$$

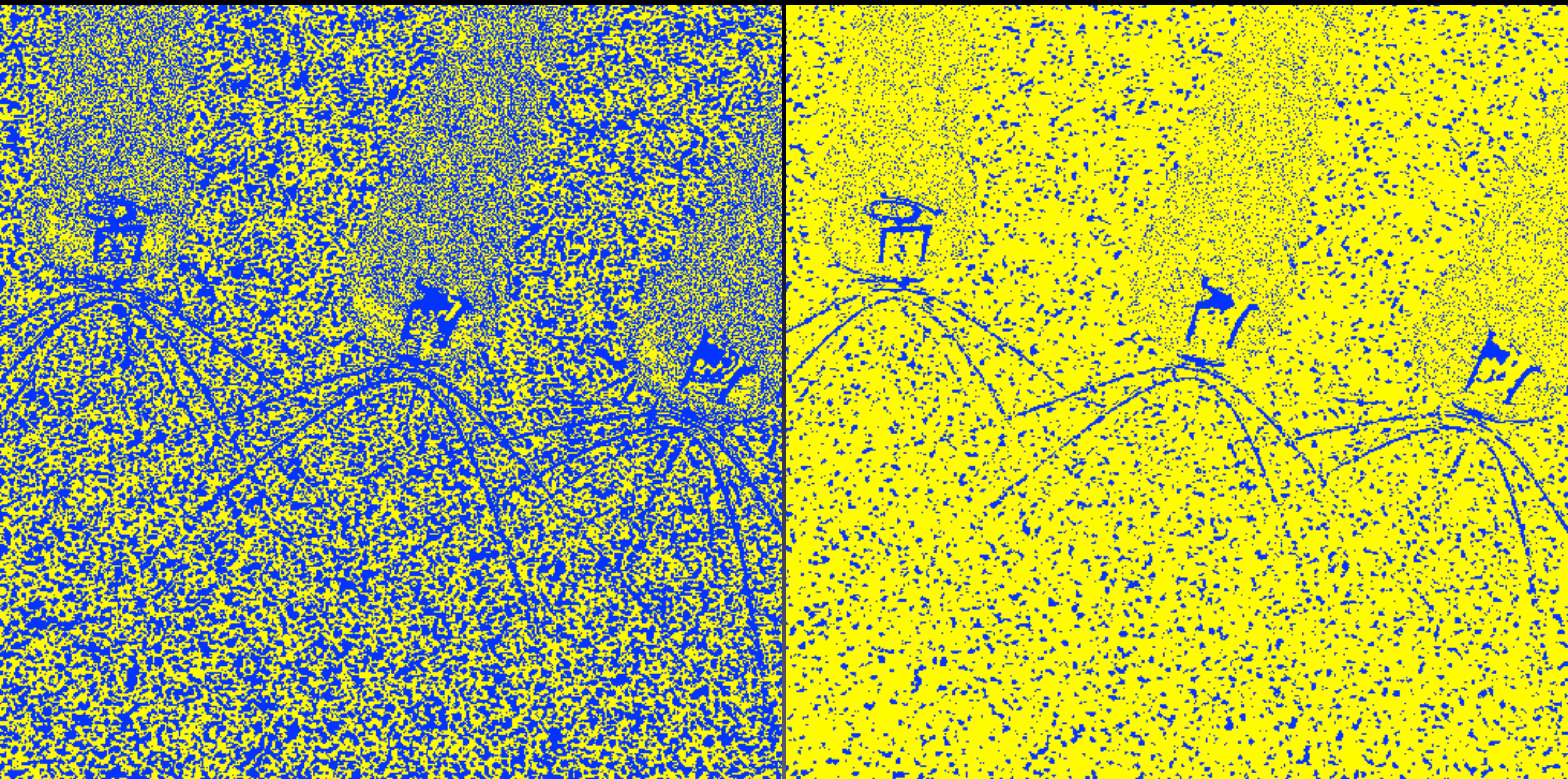


$$1 - \beta = 90\%$$

within 5% deviation

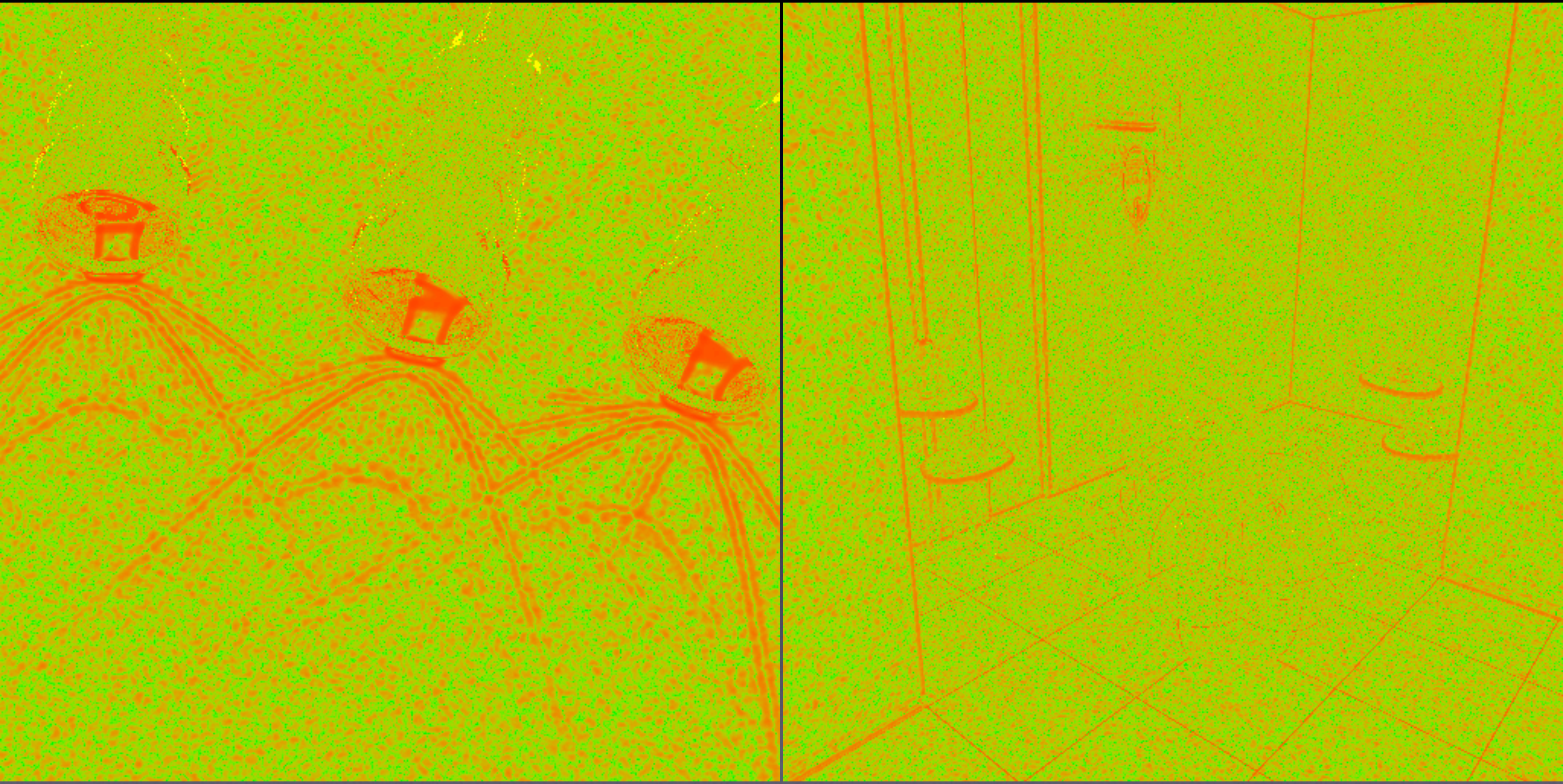
Bounded Pixel Visualization

Bounded/Not bounded



$1 - \beta = 50\%$ $1 - \beta = 90\%$

Noise-Bias Ratio



Automatic Rendering Termination

Stochastic Bound Per Pixel (50%)

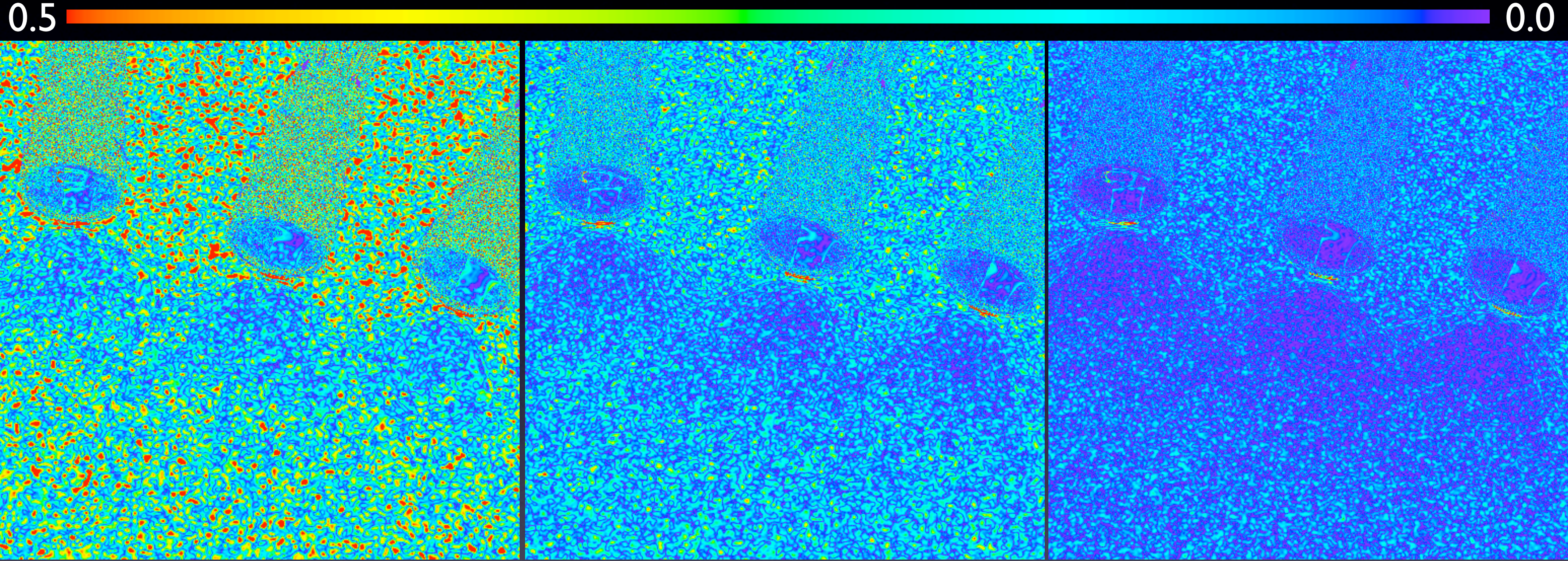
$$P(|E_i| \leq E_{b,i}) \leq 50\%$$

Stop rendering if

$$\text{Average}[E_{b,i}] < \boxed{E_{\text{thr}}}$$

User-specified allowable error

Automatic Rendering Termination

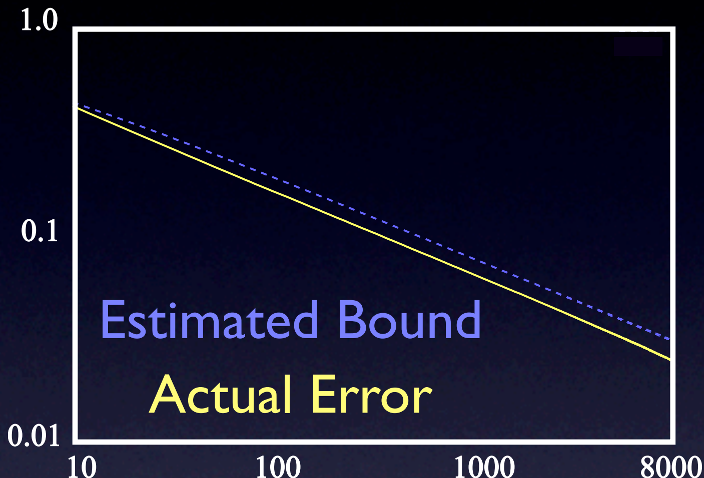


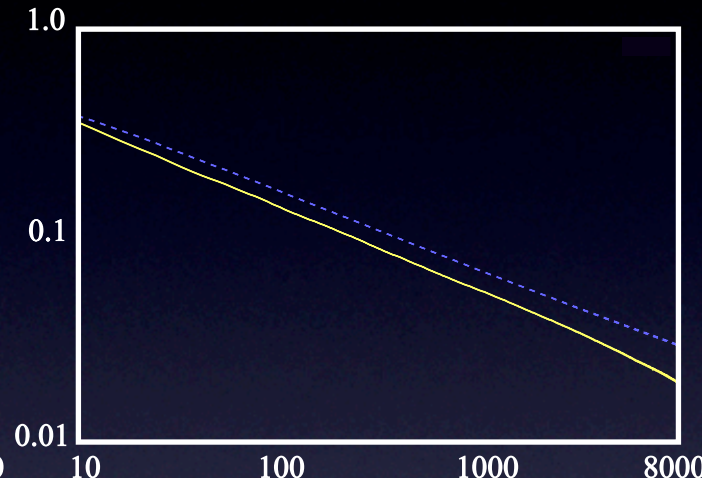
specified: 0.25 | specified: 0.125 | specified: 0.0625

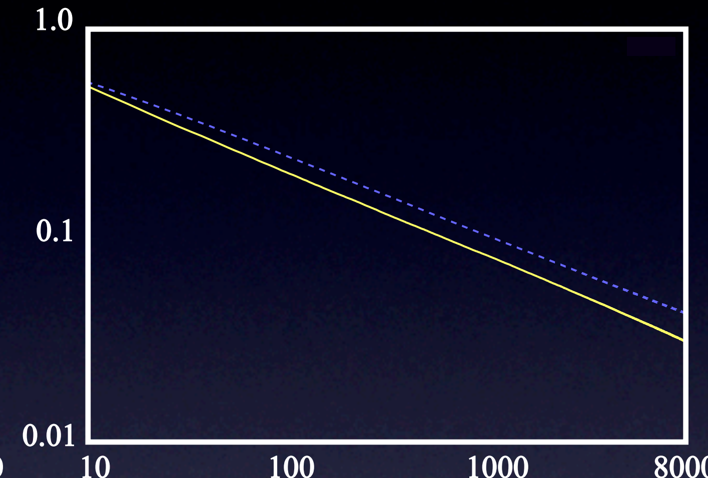
actual: 0.1916 | actual: 0.09294 | actual: 0.04482

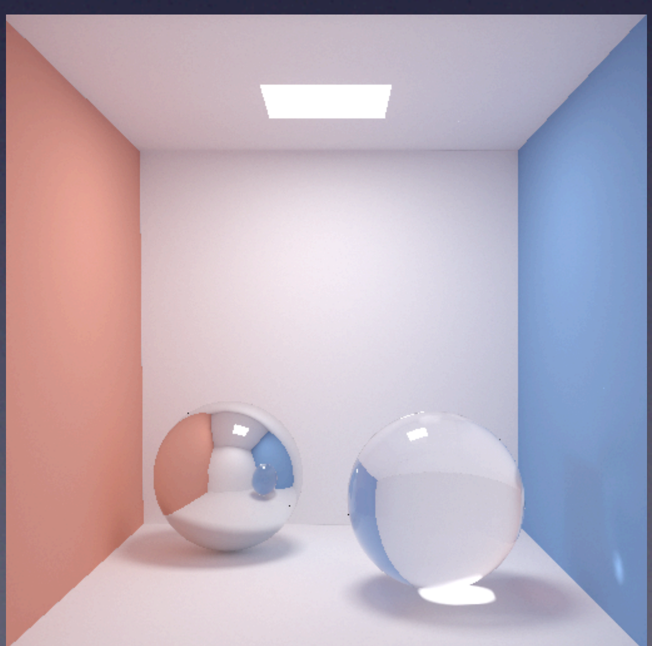
1.3 times overestimation on average

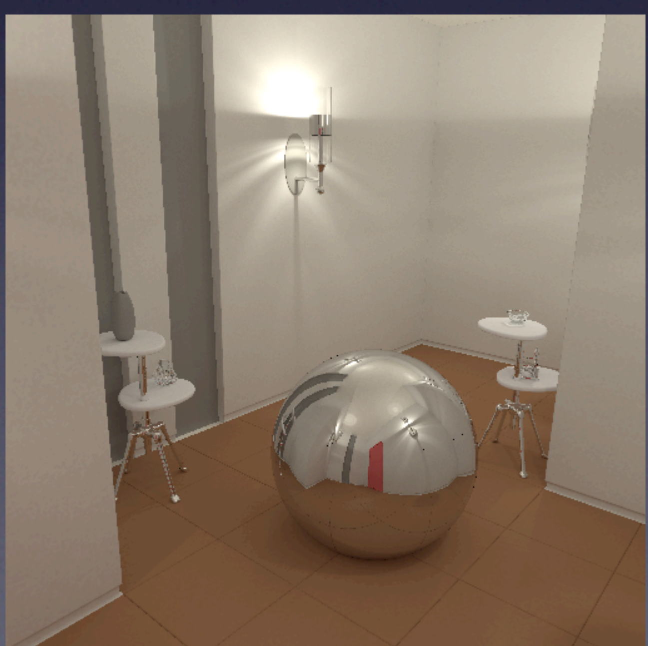
Automatic Rendering Termination











Future Work

- Various applications of error estimates

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  - Better stopping criterion

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- Extension to stochastic PPM [Hachisuka et al. 09]

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- Various applications of error estimates
  - Better stopping criterion
  - Error-driven adaptive sampling
  - Optimal search radius based on error
  - ... and many more
- Extension to stochastic PPM [Hachisuka et al. 09]
- More accurate bias estimation

Conclusion

- Error estimation for photon density estimation
  - General and non-heuristic (gives an error-bar)
  - Estimator applies to progressive photon mapping

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- Take-home message:

First step toward answering:
"How many photons are enough?"

Acknowledgements

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- Youichi Kimura (Studio Azurite): modeling
- Matus Telgarsky, Daniel Hsu (UCSD): discussion

Thank You