# State Complexity of Suffix Distance

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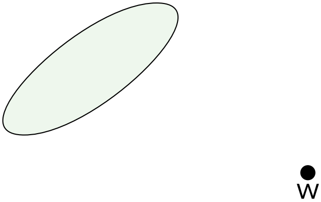
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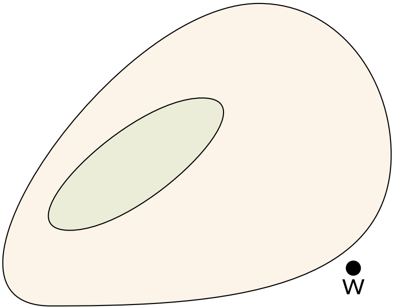
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- regular languages
- finite languages
- suffix-closed regular languages

A distance is a function $d : \Sigma^* \times \Sigma^* \to [0, \infty)$ such that

1. $d(x, y) = 0$ if and only if $x = y$
2. $d(x, y) = d(y, x)$
3. $d(x, y) \leq d(x, w) + d(w, y)$

The neighbourhood of a language $L \subseteq \Sigma^*$ of radius $k \geq 0$ with respect to a distance measure $d$ is the set of all words $u$ with $d(w, u) \leq k$ for some $w \in L$,

$$E(L, d, k) = \{u \in \Sigma^* : (\exists w \in L)\, d(w, u) \leq k\}.$$

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- Additive distances are regularity preserving (Calude, Salomaa, Yu 2002)

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- The state complexity of these neighbourhoods is $(k+2)^n$
  - Upper bound (Salomaa, Schofield 2007)
  - Lower bound (Ng, Rappaport, Salomaa 2015)

The prefix distance of $x$ and $y$ counts the number of symbols which do not belong to the longest common prefix of $x$ and $y$.

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suffix

$$d_s(x, y) = d_p(x^R, y^R)$$

### Theorem (NRS2015)

*Let $k \geq 0$ and $L$ be a regular language recognized by an NFA with $n$ states. Then there exists an NFA recognizing $E(L, d_s, k)$ with at most $n + k$ states.*

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Given a DFA $A = (Q, \Sigma, \delta, q_0, F)$, we want to construct a DFA that recognizes $E(L, d_s, k)$.

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For each state $q \in Q$, we define the function

$$\psi_A(q) = \min_{w \in \Sigma^*} \{|w| \mid \delta(q_0, w) = q\}.$$

States are of the form

$$(i, P)$$

where $i = 0, \ldots, k$ and $P \subseteq Q$.

The initial state is

$$(0, \{q \in Q \mid \psi_A(q) \leq k - 1\}).$$

For $0 \leq i < k$ and $a \in \Sigma$,

$$\delta((i, P), a) = (i + 1, X),$$

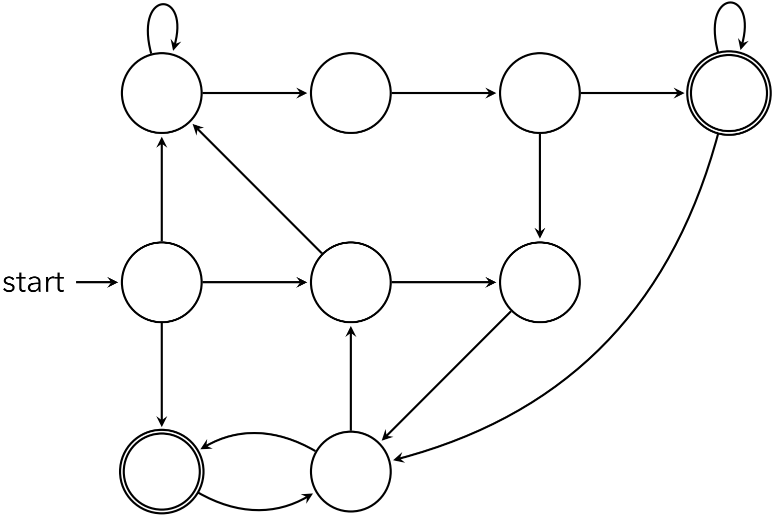
where $X = \{\delta(p, a) \mid p \in P\} \cup \{q \in Q \mid \psi_A(q) \leq k - (i + 1)\}$.

For $i = k$ and $a \in \Sigma$,

$$\delta((k, P), a) = (k, \{\delta(p, a) \mid p \in P\}).$$

Then a word is accepted when it reaches a final state

$$(i, \{P \subseteq Q \mid P \cap F \neq \emptyset\}).$$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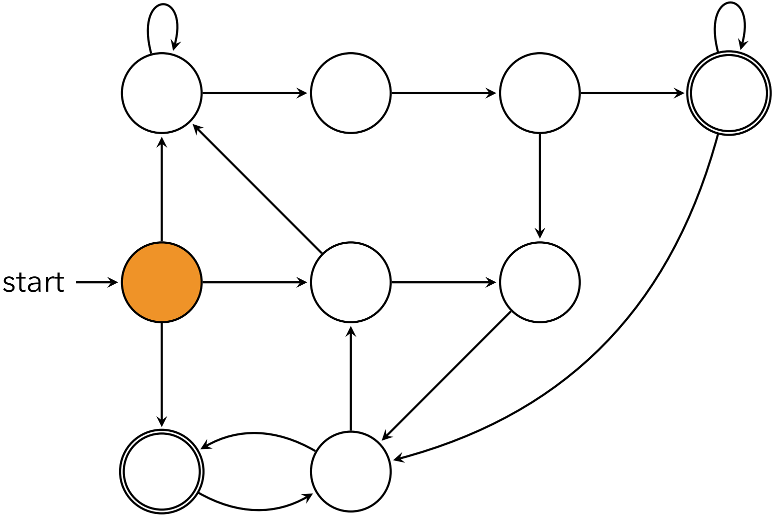
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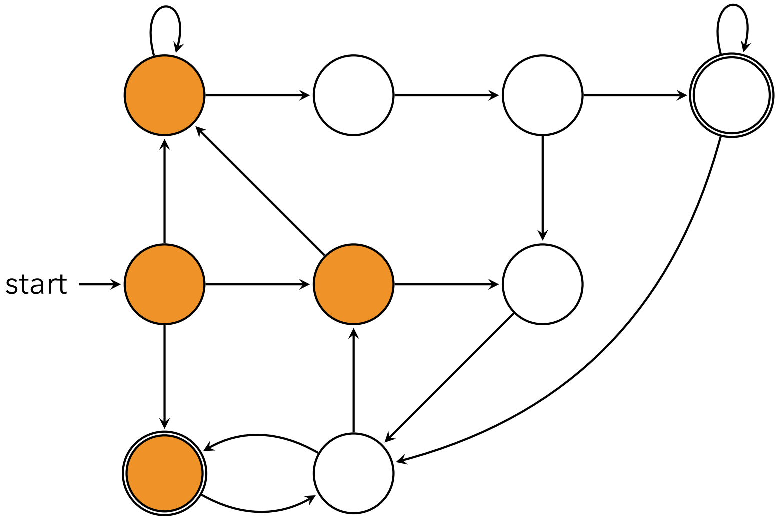
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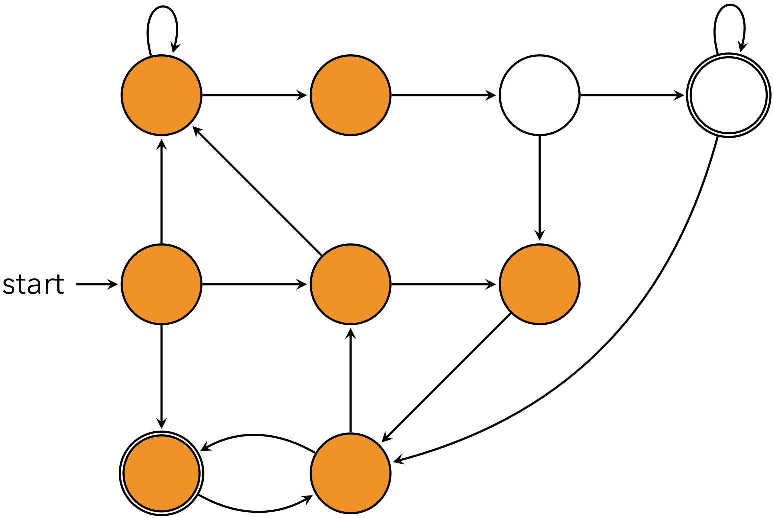
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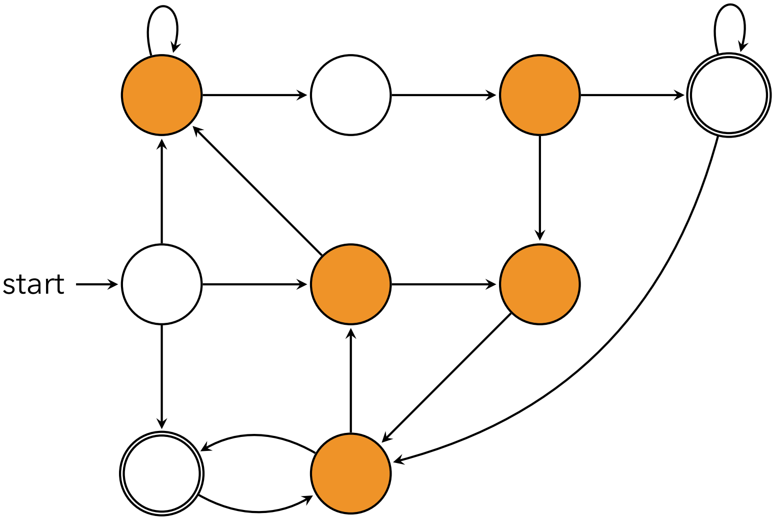
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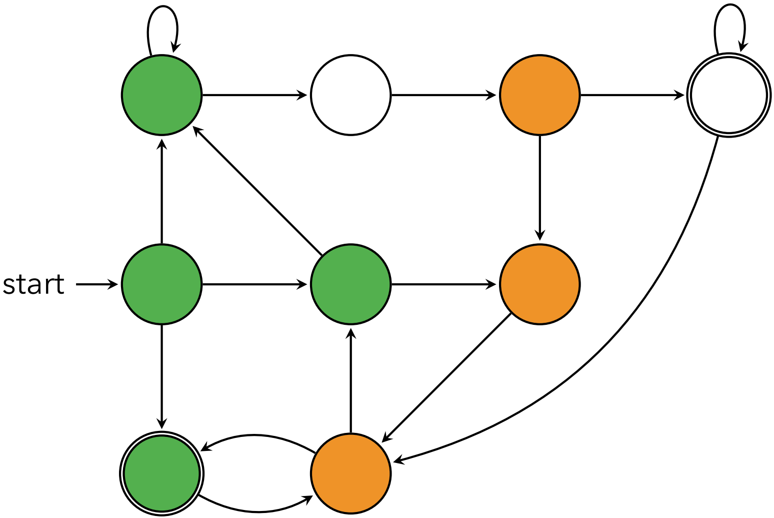
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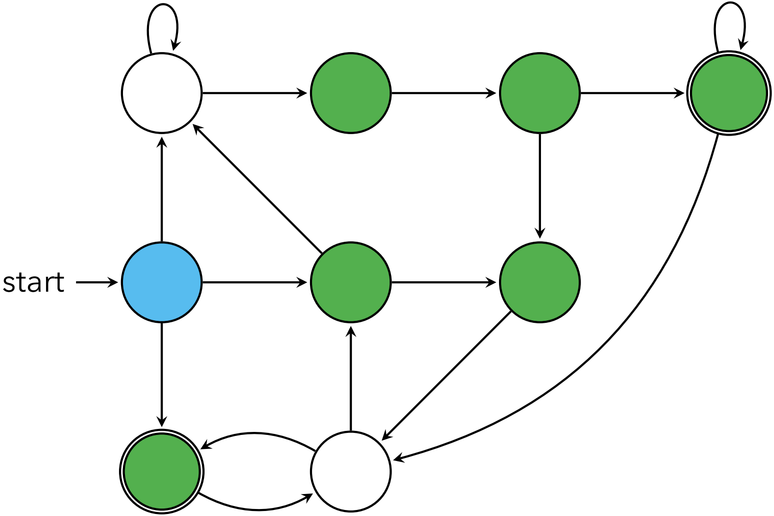
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In total, there are at most

$$\frac{|\Sigma|^k - 1}{|\Sigma| - 1} + 2^n - 1$$

reachable states.

The state complexity of suffix distance neighbourhoods of radius $k$ given a DFA with $n$ states is

| | $n > k$ | $k \geq n$ |
|---|---|---|
| Regular | $\frac{\lvert\Sigma\rvert^k - 1}{\lvert\Sigma\rvert - 1} + 2^n - 1$ | $(k-n) + 2^{n+1} - 2$ |
| Finite | $2^k + k \cdot 2^{\lfloor \frac{n}{2} \rfloor} - 1$ | $(k-n) + 2^{n+1} - 2$ |
| Suffix-closed | $n + k + 1$ | |

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Given a DFA with $n$ states, there exists a DFA that recognizes subword distance neighbourhoods of radius $k$ with at most $\frac{\lvert\Sigma\rvert^k - 1}{\lvert\Sigma\rvert - 1} + (k+2) \cdot 2^{n \cdot (k+1)}$ states.

Future work

- Lower bounds for suffix distance neighbourhoods that don't depend on the alphabet or radius
- Lower bounds for subword distance neighbourhoods
- State complexity of suffix distance neighbourhoods for suffix-closed, -free, -convex languages