Tools for Online Technical Collaboration

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Previous TRICS!

1. Computer Algebra's Dirty Little Secret
   - Stephen M. Watt
   - University of Western Ontario

2. The Mathematics of Mathematical Handwriting Recognition
   - Stephen M. Watt
   - University of Western Ontario

3. Dependent Types and Categorical Programming
   - or What can we learn from Aldor?
   - Stephen M. Watt
   - University of Western Ontario
And now for something completely different...
Menu

Appetizers
Collaboration
Technical Content vs Pictures

Mains
Digital Ink
Mathematical Handwriting

Deserts
Previous Software
Present Generation
Collaboration
Collaborative Software

- MS SharePoint
- Yahoo Messenger
- CmapTools knowledge modeling kit
- Slideshare
- Skype
- Campfire
- Dabbleboard
- Google Docs
- CollabNet
- Dropbox

Dabbleboard
The whiteboard reinvented
Visualize, explore, collaborate
Lots of Collaborative Software
Common Features

• Slide shows
• Whiteboarding
• Voice chat
• Video chat
• Image capture
Expected Enterprise Collaboration Features
Circa 2013

Content
- Text / Rich Media

Document & Files
- MS Office / PDF / XML / etc.

Conversation
- Social Depth / News Feeds / IM / UC

Apps
- Integrated Business Apps / App Stores

From http://zdnet.com/blog/hinchcliffe
Technical Collaboration

“I think you should be more explicit here in step two.”

from *What’s So Funny about Science?* by Sidney Harris (1977)
Technical Collaboration

Missing:
• Mathematics
• Diagrams, graphs
• Geometric figures
• Technical knowledge base
• Document markup

• Scientific software connections
  (Maple, Mathematica, GeoGebra, R,...)
Isn’t a shared whiteboard, with the ability to save images enough??
The Treachery of Images

(La trahison des images)
\[
D = \frac{1}{c^2} \frac{d}{dt} \frac{dl}{d\theta} = \frac{1}{c^2} \frac{dP}{d\theta}
\]

\[
D^2 = \frac{1}{P^2} \frac{P_0 - P}{P} \sim \frac{1}{P^2}
\]

(1a)

\[
D^2 = \frac{K}{3} \frac{P_0}{P_0} \sim \frac{1}{P^2}
\]

(2a)

\[
D^2 \sim 10^{-53}
\]

\[
\rho \sim 10^{-26}
\]

\[
\rho \sim 10^8 \text{ L} \cdot \text{J}
\]

\[
\alpha \sim 10^{10} (10^{11}) \text{ J}
\]
Einstein’s Blackboard

• Einstein to receive honorary doctorate at Oxford, May 1931.

• Lecture at Rhodes House.

• Board retrieved and preserved by Edmund ("Ted") Bowen.

• Nice to look at, but content is trapped.
Digital Ink

• Location, time information, sometimes also pressure and angles.
• Capture online pen strokes, *not* images.

• Suitable for
  • **Recognition** algorithms
  • **Semantic** grouping
  • **Annotation**
  • **Manipulation**: search, transformation, archival.

• Problem: Multiple vendor-specific formats.
Ink Markup Language (InkML)

W3C Recommendation 20 September 2011

This version:
http://www.w3.org/TR/2011/REC-InkML-20110920/

Latest version:
http://www.w3.org/TR/InkML

Previous version:
http://www.w3.org/TR/2011/PR-InkML-20110510/

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Pen-Based Math

- Input for CAS and document processing.
- 2D editing.
- Computer-assisted collaboration.
Pen-Based Math

• Different than natural language recognition:
  • 2-D layout is a combination of writing and drawing.
  • Many similar few-stroke characters.
  • Many alphabets, used idiosyncratically.
  • Many symbols, each person uses a subset.
  • No fixed dictionary for disambiguation.
Character Recognition

• A story about a UI proposal
• A story about three statisticians
• Concentrate on character recognition
• Several projects ignore this problem
Usual Character Reco. Methods

• Smooth and re-sample data  \textit{THEN}

• Match against \textit{N} models by sequence alignment \textit{OR}

• Identify “features”, such as
  • coordinate values of sample points, number of loops, cusps, writing direction at selected points, \textit{etc}

Use a classification method, such as
  • Nearest neighbour, Subspace projection, Cluster analysis, Support Vector Machine

\textit{THEN}

• Rank choices by consulting dictionary
Difficulties

- Having many similar characters (e.g. for math) means comparison against all possible symbol models is slow.

- Determining features from points
  - Requires many \textit{ad hoc} parameters.
  - Replaces measured points with interpolations
  - It is not clear how many points to keep, and most methods depend on number of points
  - Device dependent

- What to do since there is no dictionary?

- New ideas are needed!
What the Computer Sees
What the Computer Sees
Orthogonal Series Representation

• **Main idea:**
  Represent traces as curves, not discrete points and coordinate curves as truncated orthogonal series.
Orthogonal Series

• Start with inner product on a space of functions, e.g.

\[ \langle f, g \rangle = \int_{a}^{b} f(t)g(t)w(t)dt \]

• Functions \( \phi_i(t) \) give an orthogonal basis if we can write

\[ f(t) = \sum_{i=0}^{\infty} f_i \phi_i(t) \quad \text{and} \quad \langle \phi_i, \phi_j \rangle = 0 \text{ if } i \neq j \]

Then \( f_i = \langle f, \phi_i \rangle / \langle \phi_i, \phi_i \rangle \).

• If sum is truncated, \( f \) is approximated.

• Obtain orthogonal basis from any basis set, e.g. \( \{1, t, t^2, \ldots \} \), by Gram-Schmidt process.
Orthogonal Series Representation

• **Main idea:**
  Represent traces as curves, not discrete points and coordinate curves as truncated orthogonal series.

• **Advantages:**
  - *Compact* – few coefficients needed
  - *Geometric*
    – the truncation order is a property of the character set
    – gives a natural metric on the space of characters
  - *Algebraic*
    – properties of curves can be computed algebraically
      (instead of numerically using heuristic parameters)
  - *Device independent*
    – resolution of the device is not important
Distance Between Curves

• Elastic matching:
  • Approximate the variation between curves by some fn of distances between sample points.
  • May be coordinate curves or curves in a jet space.

• Sequence alignment
• Interpolation (“resampling”)

• Why not just calculate the area?
• This is very fast in ortho. series representation.
Distance Between Curves

\[ \bar{x}(t) = x(t) + \xi(t) \quad \xi(t) = \sum_{i=0}^{\infty} \xi_i \phi_i(t), \quad \phi_i \text{ ortho on } [a, b] \text{ with } w(t) = 1. \]

\[ \bar{y}(t) = y(t) + \eta(t) \quad \eta(t) = \sum_{i=0}^{\infty} \eta_i \phi_i(t) \]

\[ \rho^2(C, \tilde{C}) = \int_{a}^{b} \left[ (x(t) - \bar{x}(t))^2 + (y(t) - \bar{y}(t))^2 \right] dt = \int_{a}^{b} [\xi(t)^2 + \eta(t)^2] dt \]

\[ \approx \int_{a}^{b} \left[ \sum_{i=0}^{d} \xi_i^2 \phi_i^2(t) + \text{cross terms} + \sum_{i=0}^{d} \eta_i^2 \phi_i^2(t) + \text{cross terms} \right] dt \]

\[ = \sum_{i=0}^{d} \xi_i^2 + \sum_{i=0}^{d} \eta_i^2 \]

• Just as accurate as elastic matching. Much less expensive.

• Linear in \(d\), the degree of the approximation. < 3 \(d\) machine instructions (30ns) vs several thousand!
Problems

• Want fast response – how to work while trace is being captured.

• Low RMS does not mean similar shape.
Problem 1. On-Line Coefficients

• The main problem:  
  *In handwriting recognition, the human and the computer take turns thinking and sitting idle.*

• We ask:  
  *Can we do useful work while the user is writing and thereby get the answer faster after the user stops writing?*

• The answer is “Yes”!
Problem 1. On-Line Coefficients

• Use modified Legendre polynomials $P_i$ as basis on the interval $[0, 1]$, with weight function 1.

• Collect numerical values for $f(\lambda)$ on $[0, L]$.  
$\lambda = \text{arc length.}$  
$L$ is not known until the pen is lifted.

• As the sample points are collected, numerically integrate the moments $\int \lambda^i f(\lambda) d\lambda$.

• After last point, compute series coefficients for $f$ with domain and range scaled to $[0, 1]$.  
This uses a single linear transformation of the moments.
Problem 1. On-Line Coefficients

• Approach works for any inner product with linear weighting.

• This is the Hausdorff moment problem (1921), shown to be unstable by Talenti (1987).

• It is just fine, however, for the dimensions we need.
Problem 2. Shape vs Variation

• The corners are not in the right places.

• Work in jet space to force coords & derivs to be close.

• Legendre-Sobolev inner product.

\[ \langle f, g \rangle_{LS} = \int_{a}^{b} f(t)g(t)dt + \mu_1 \int_{a}^{b} f'(t)g'(t)dt + \mu_2 \int_{a}^{b} f''(t)g''(t)dt + \cdots \]

• 1\textsuperscript{st} jet space sufficient.
  • Choose $\mu_1$ experimentally to maximize reco rate.
  • Can be also done on-line. [Golubitsky + SMW 2008, 2009]
Legendre-Sobolev Basis
Life in an Inner Product Space

• With the Legendre-Sobolev inner product we have
  • Low dimensional rep for curves (10 + 10 + 1)
  • Compact rep of samples ~ 160 bits [G+W 2009]
  • >99% linear separability => convexity of classes
  • A useful notion of distance between curves
    that is very fast to compute
Linear Separability
Linear Separability
Comparison of Sample to Models

• Use Euclidean distance in the coefficient space.

• Can trace through SVM-induced cells incrementally.

• Normed space for characters gives other advantages.
The Joy of Convexity

• Convexity ⇒ Linear homotopies stay within a class

\[ C = (1 - t) A + t B \]

• Can compute distance of a sample to this line

• Distance to convex hull of nearest neighbors in class gives best recognition [Golubitsky+SMW 2009,2010]
Choosing between Alternatives

Red class or blue class?
Choosing between Alternatives

The nearest $k$ samples are blue.
Choosing between Alternatives

The nearest convex hull of neighbors is red.
Training

• Using CHKNN allows training with relatively few samples. (Dozens vs Thousands per class)
Error Rates as Fn of Distance

- Error rate as fn of distance gives confidence measure for classifiers [MKM – Golubitsky + SMW 2009]
Combining with Statistical Info

• Empirical confidence on classifiers allows geometric recognition of isolated symbols to be combined with statistical methods.

• Domain-specific $n$-gram information:
  • Research mathematics – 20,000 articles from arXiv [MKM -- So+SMW 2005]
  • 2$^{nd}$ year engineering math – most popular textbooks [DAS -- SMW 2008]
  • Inverse problem – identifying area via $n$-gram freq! [DML -- SMW 2008]
Baseline Estimation

• Figure out baseline from the characters, rather than the other way around, which is more usual.

• We can locate some important features by identifying special points.

We refer to a point such as this, that determines the height of a metric line, as a determining point.
Baseline Estimation

• Juxtaposition ambiguity

\[ \overline{Pq} \quad \overline{Pq} \quad \overline{Pq} \quad \overline{Pq} \]

\[ p9 \quad Pq \quad pq \quad p9 \]

• Handwriting neatening

\[ a_1 x^2 + a_2 \rightarrow a_1 x^2 + a_2 \]
The average symbol of a set of known samples for a class can be computed as the average point in the functional space,

\[ \bar{C} = \frac{1}{n} \sum_{i=1}^{n} C_i \]
Deriving from a Reference Symbol

Average Symbol

Sample-1-Initial

Sample-1-Derived

Sample-2-Initial

Sample-2-Derived

Optimization

Arc-length guess

Arc-length guess
Using Homotopy

• Some samples are far away from the reference symbol.

• We use a homotopy between the reference symbol and the target sample in a multi-step method.
Prior Generations of Software

• 2000 Cross Pad:
Prior Generations of Software

• 2002 Pocket PC:
Prior Generations of Software

• 2002-2008 Tablet PC:
Prior Generations of Software

• 2008-2013 Java Application:
InkChat (Java Version)

• Skype and GTalk add-on to the Java application.
Problems

• Requires installation:
  • Big hassle for someone to use only once in a while or on all their machines.

• Limited portability:
  • Users expect versions on Android, iOS, Windows, Mac OS X, Linux, etc...
  • Incompatible software bases
  • Flakey, moving APIs

• Need to support multiple devices.
  • Nowadays a single user will want to work across many devices.
Solution

• Use browser infrastructure.
Solution

• Use browser infrastructure.

• JavaScript is not a great language for large projects, but.....
  • It is ubiquitous: Telephones, tablets, laptops, ...
  • Libraries for many UI elements
  • Our new recognition algorithms are fast enough 😊

• Rapid development:
  • Prototype developed in 3 months by 3 students.
Current Generation

Desktop

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi = -\frac{\partial^2}{\partial z^2} \psi
\]

\[ a^2 + b^2 = c^2 \]

\[(a+b)^2 = 4 \left( \frac{1}{2} ab \right) + c^2 \]
\[ a^2 + 2ab + b^2 = 2ab + c^2 \]

Telephone

iPhone 5

\[ a^2 + b^2 = c^2 \]
Current Generation

Tablet

Nexus 10

\[ a^2 + b^2 = c^2 \]

InkML
Simple Interface with device-adapted menus

\[ \sqrt{a^2 + b^2} \]
Ink Controls
Collaboration:
Multiple Users Connected to Same URI

\[ a^2 + b^2 = c^2 \]
Collaboration:
Different Viewports from Different Devices
Collaboration:
Pointers for Discussions
Collaboration:
Document Annotation
Collaboration: Google Hangout Embedding
Cloud Integration

• Save or load files to cloud storage
  • DropBox
  • Google Keep
  • Others possible

• Previous work to store user profiles
  • Save cloud of ground-truth labelled symbols (corrected/accepted)

• Future work to store user-defined brushes
Architectural Direction

A

Control Bar

Popup Control

B

Control Layer

Presentation Layer

Background Layer
Architectural Direction
Architectural Direction

E

Touch/Trace Event

Platform-Specific Frame

Touch/Trace Event

Portable Application

F

1 - Request to create a control
2 - Callback to create control with local look and feel
3 - Touch/Trace Event
4 - Forward Touch/Trace event
5 - Forward Control Action
6 - Control Close Request

Read Events
1 - Notification of Participant event (object creation / deletion / movement
history navigation / page change / etc)

2 - Notification of Server Event (Passed on from another client or result of
server configuration)
1 - New stroke / event
2 - Distribute to recognizers for screen area
3 - Connect object space
4 - Return ranked results
5 - a Update presentation
   b Update server
Application Web Site
Conclusions

• Technical collaboration requires tools not found in the business setting.
• Drawing, mathematics and scientific documents are in the work flow.
• The treachery of images.
• Needed:
  • Math handwriting recognition.
  • Easy geometry and diagrams.
  • Document mark up.
  • APIs to scientific software.
• Even a little goes a long way....
• ... there is a lot of opportunity for future development.
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