The Limits of Computation

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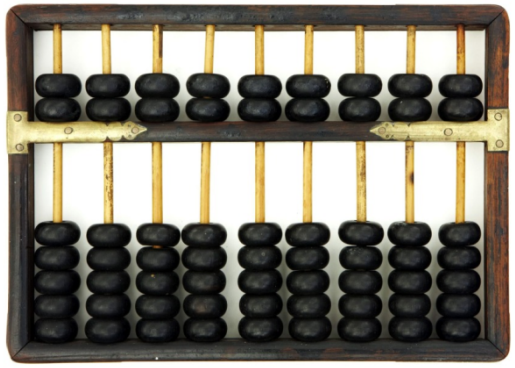
16 April 2013

What Can Computers Ultimately Do?

1940s Code Breaking
1960s Man on the Moon
1970s Mainframes and the Rise of IBM
1980s Office Workflow and Microsoft
1990s The Internet
2000s Smartphones
2010s Grandparents on Facebook
2020s ????

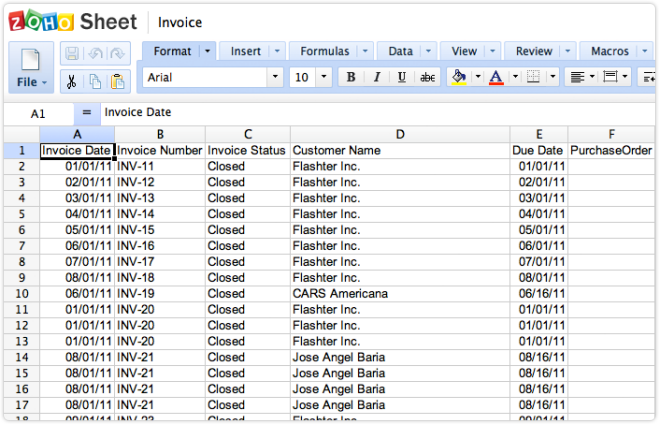
What is a Computer?



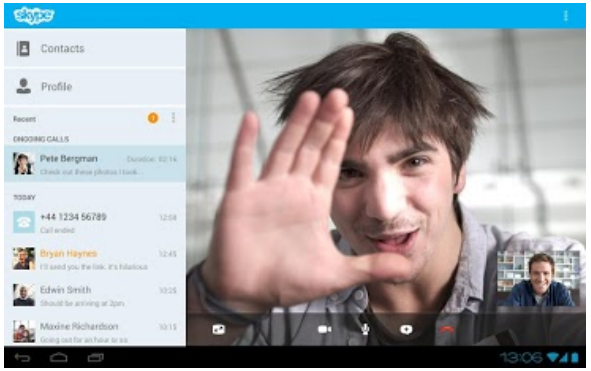
27 + 15 = 42

6.28318530717959

What is a Computer?



	A	B	C	D	E	F
1	Invoice Date	Invoice Number	Invoice Status	Customer Name	Due Date	PurchaseOrder
2	01/01/11	INV-11	Closed	Flashter Inc.	01/01/11	
3	02/01/11	INV-12	Closed	Flashter Inc.	02/01/11	
4	03/01/11	INV-13	Closed	Flashter Inc.	03/01/11	
5	04/01/11	INV-14	Closed	Flashter Inc.	04/01/11	
6	05/01/11	INV-15	Closed	Flashter Inc.	05/01/11	
7	06/01/11	INV-16	Closed	Flashter Inc.	06/01/11	
8	07/01/11	INV-17	Closed	Flashter Inc.	07/01/11	
9	08/01/11	INV-18	Closed	Flashter Inc.	08/01/11	
10	06/01/11	INV-19	Closed	CARS Americana	06/16/11	
11	01/01/11	INV-20	Closed	Flashter Inc.	01/01/11	
12	01/01/11	INV-20	Closed	Flashter Inc.	01/01/11	
13	01/01/11	INV-20	Closed	Flashter Inc.	01/01/11	
14	08/01/11	INV-21	Closed	Jose Angel Baria	08/16/11	
15	08/01/11	INV-21	Closed	Jose Angel Baria	08/16/11	
16	08/01/11	INV-21	Closed	Jose Angel Baria	08/16/11	
17	08/01/11	INV-21	Closed	Jose Angel Baria	08/16/11	



What is a Computer?

What is a Computer?

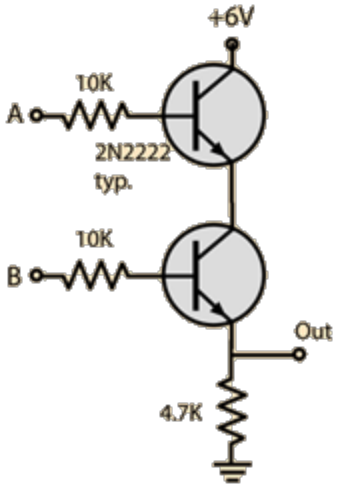
How Do Computers **Work**?

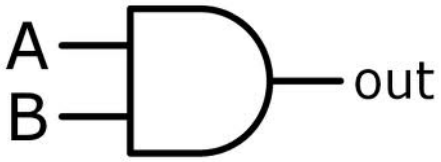
- Very basic operations, but **billions** of them!

Logical Operations

- Use different voltages to mean true and false.
- Design circuits for AND, OR, etc.

A	B	A and B	A or B
F	F	F	F
T	F	F	T
F	T	F	T
T	T	T	T





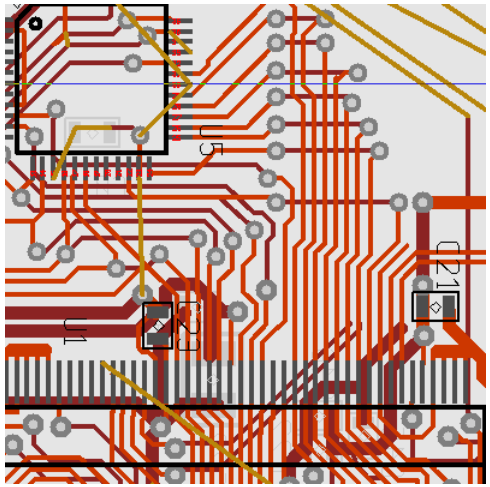
Data

- Write numbers, letters, pictures, music,
  – **everything –** in terms of “true” and “false”.

$$42 = 32 + 8 + 2$$

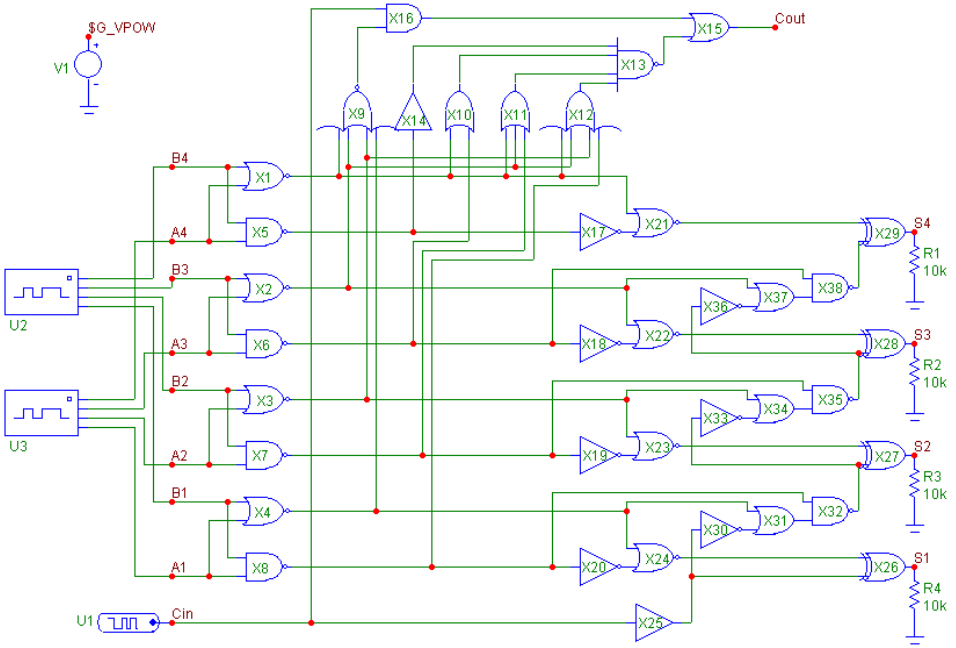
$$= 1 \times 32 + 0 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1$$

$$= 101010 \text{ “base 2”}$$



Arithmetic

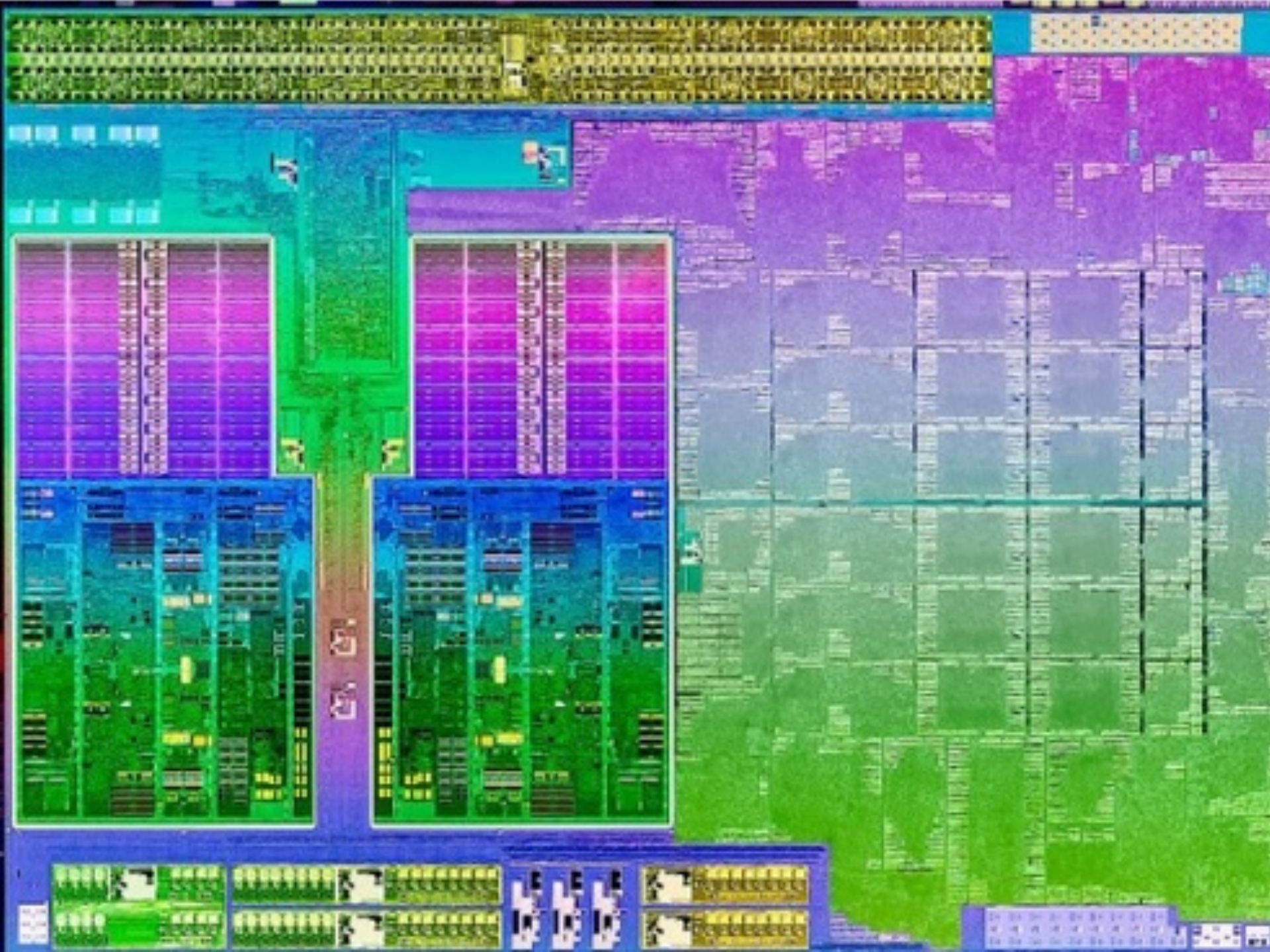
- Arithmetic operations are done as logical operations (combinations of AND and OR) on the zeros and ones making up the numbers.

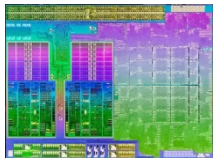


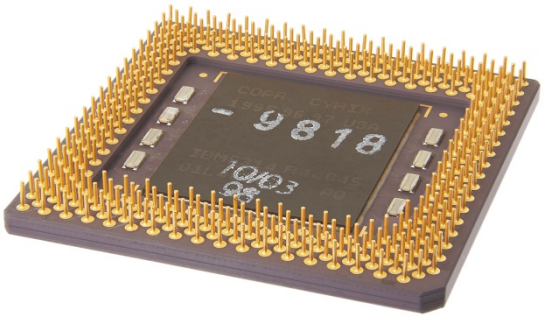
Computer Chip

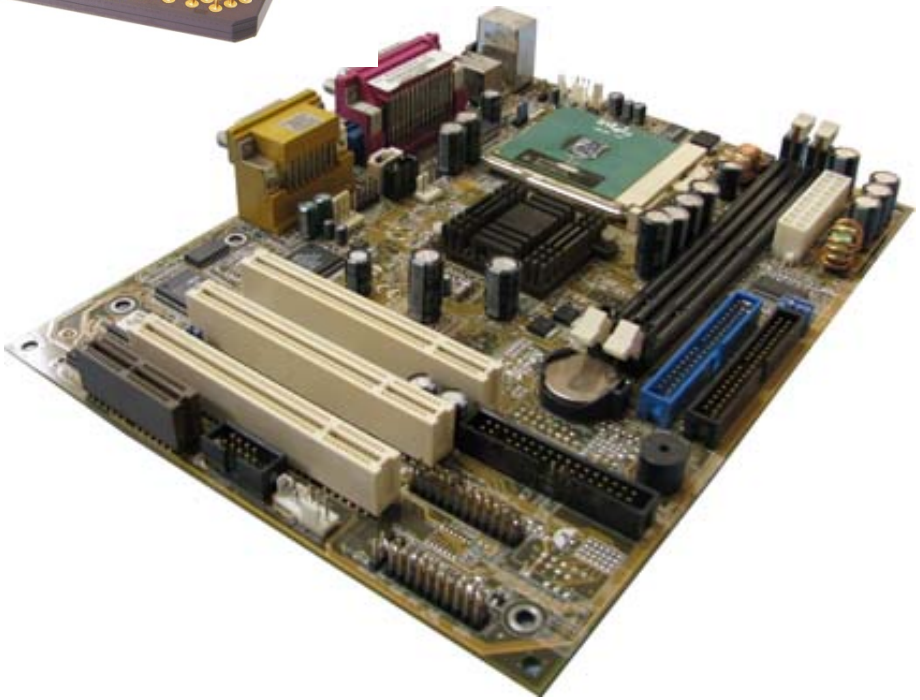
- Lots of circuits like this to do different things: add, multiply, test equality.

- Which circuit is activated at any moment is determined by an “instruction word”, which is just a bunch of 0s and 1s that pick the circuit to use.






9818




COOLER MASTER

Programming

- Creating a set of instructions to do something.

1. Start with a general idea of what is to be done.
2. Figure out a way to do it –
   an "algorithm".
3. Convert it into a set of precise instructions –
   a "program".

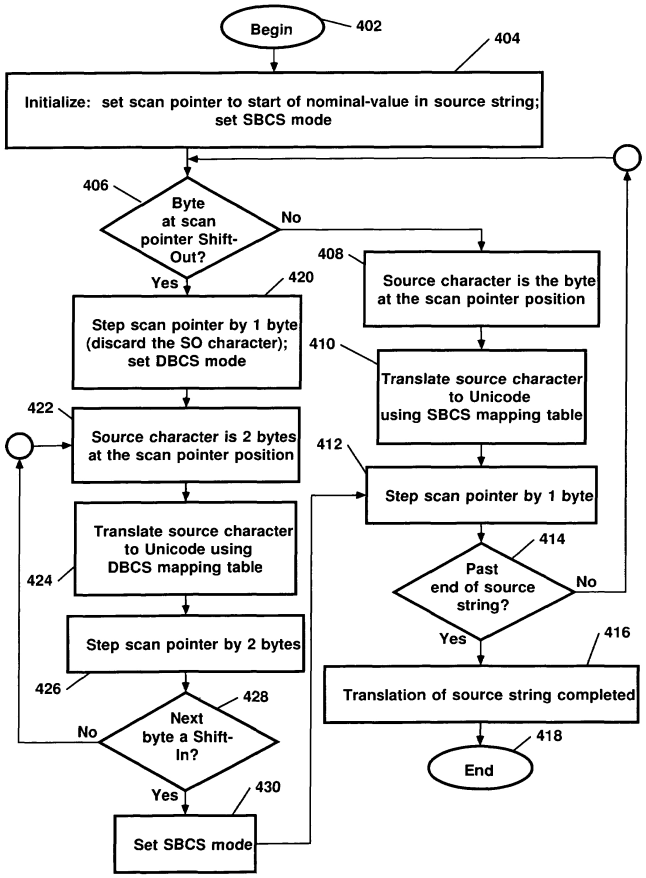
Algorithms

CORNBREAD

Colvin Run Mill Corn Bread

1 cup cornmeal
1 cup flour
½ teaspoon salt
4 teaspoons baking powder
3 tablespoons sugar
1 egg
1 cup milk
¼ cup shortening (soft) or vegetable oil

Mix together the dry ingredients. Beat together the egg, milk and shortening/oil. Add the liquids to the dry ingredients. Mix quickly by hand. Pour into greased 8x8 or 9x9 baking pan. Bake at 425 degrees for 20-25 minutes.



Algorithm to Add 2-Digit Numbers

1. We can add 1 digit numbers using a table.

Algorithm to Add 2-Digit Numbers

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2. AB + CD

   Call the tens and ones digits of the first number A and B.
   Call the tens and ones digits of the second number C and D.

   E.g. 36 + 45 has A=3, B=6 and C=4, D=5.

Algorithm to Add 2-Digit Numbers

1. We can add 1 digit numbers using a table.
2. Call the tens and ones digits of the 1st # A and B.
   Call the tens and ones digits of the 2nd # C and D.

36+45

A ← 3, B ←6

C ← 4, D ←5

3. Add together B and D according to table to get E.

E ← 11

4. If E is 10 or more
   Let Y = ones digit of E and K = 1.
   If E is 9 or less
   Let Y = E and K = 0.

Y ← 1, K ←1

5. Let X = A + C + K.

X ← 3 + 4 + 1 = 8

6. Answer has tens digit X and ones digit Y.

81

Programming Languages

- Artificial languages for giving instructions that people can understand.
- Examples: JAVA, C, C++, C#, FORTRAN, COBOL
- There are 1000s.

A Program in Java

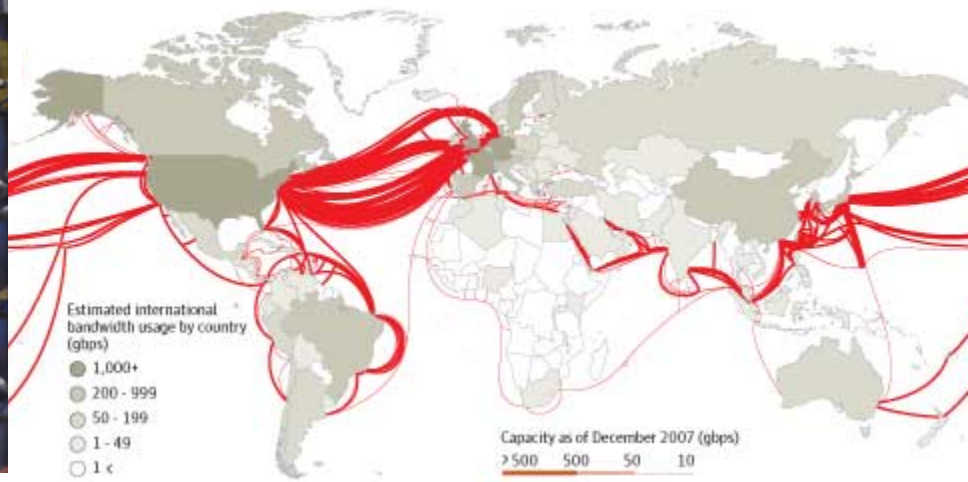
```
int X, Y;

void addNums(int A, int B, int C, int D) {
        int E, K;

        E = B + D;

        if (E >= 10)
                { Y = E % 10; K = 1; }
        else
                { Y = E; K = 0; }

        X = A + C + K;
}
```

Compilers

- "Compilers" convert
  sequences of instructions in a
  programming language (that people can read)
  to
  sequences of
  instruction words (that control a computer chip).

Computer Systems Today

- Billions of circuits
- Millions or billions of lines of computer code.
- Powerful, complex systems, spanning the planet.



Right Now

- Do you play competitive Tic-Tac-Toe?

- Computers best at chess in world.
- Checkers (8x8) now completely solved.

- Expert systems competitive with expert humans in many areas.

Right Now

- 2011 Watson wins Jeopardy.
- 2013 Watson used to manage lung cancer treatment.

Easy Projections

- Better, faster, smarter.
  - Richer media
    - high res displays, sound fields, wave front cameras, …
  - Photo-realistic real-time games
  - Verbal instructions, Conversation with computers
  - Computer vision
    - Smart alarm systems, self-driving trucks, recycling, …
  - More intelligent digital assistants
    - Smart telephone reception systems, E-mail secretary, …
  - Real time decision making based on mass data.
    - Traffic routing, public health decision making, …

Easy Projections

- Bigger – larger storage.
  - No more BluRays, DVDs, etc.
  - Books as artwork.
  - Handwriting becoming a lost art.

Easy Projections

- Smaller.
  - More power in phones, watches, glasses.
    - Intelligent and watching back.
  - Also jewelry, clothes, paint.
- Pervasive.
  - Everywhere and in everything.
- Custom materials and objects
  - Advanced modelling and fast processing => game-changing 3D printing

Ethical and Legal Choices

- Just because we *can,* does it mean we *should?*

- Privacy concerns.
  - Facebook? CSIS? Oppressive regime?
  - Analytic deductions from aggregated information.

- Change in the nature of property.
  - From owned to licensed.

- Change in the nature of work.
- Change in the nature of money.

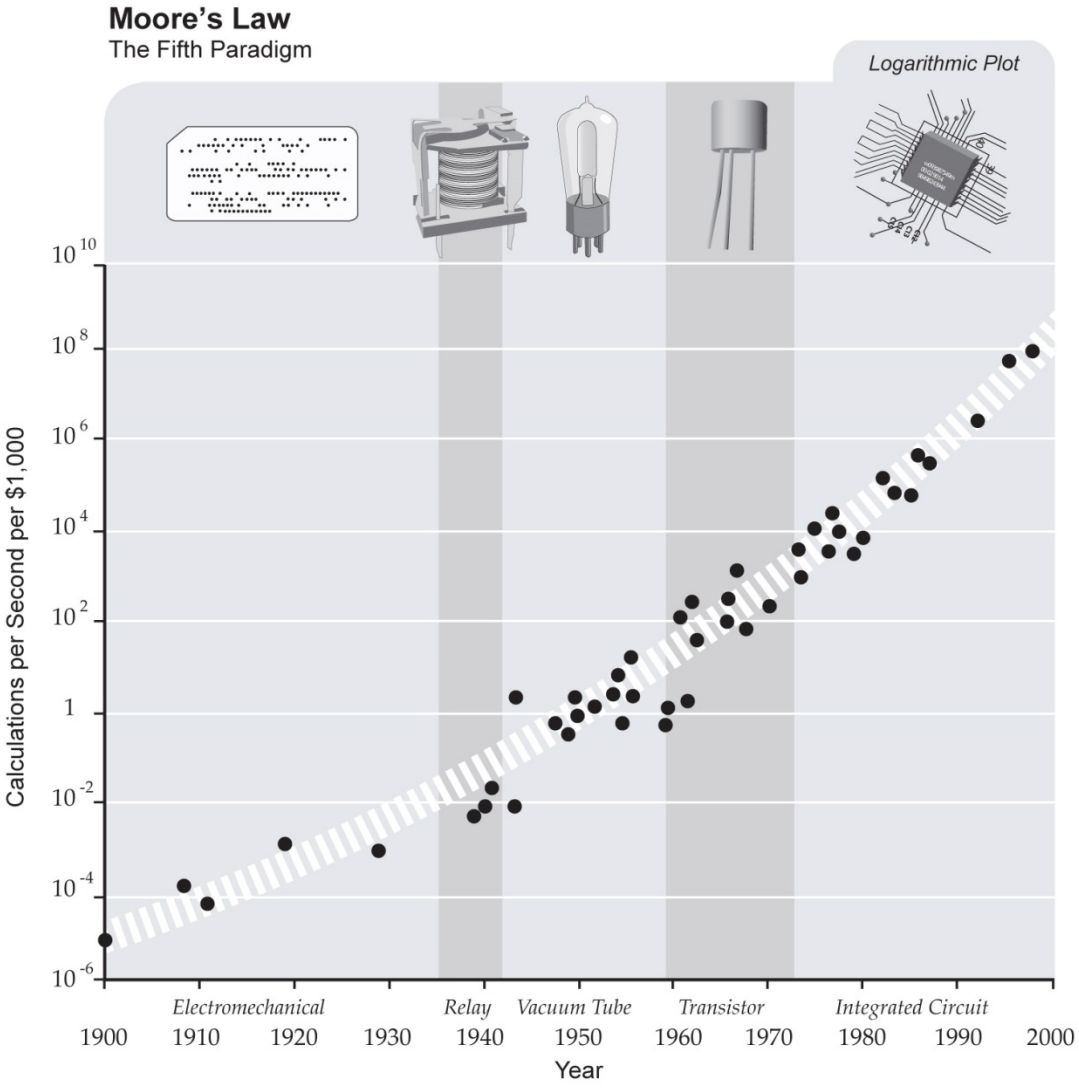
Moore's Law

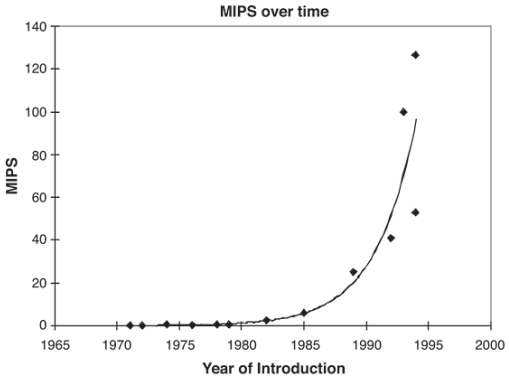
- Gordon E. Moore noticed in 1965 that integrated circuit speeds had been doubling every year for the past many years.

- This has remained essentially true to date: computer processor speed has doubled about once every year to 18 months since the 1950s.

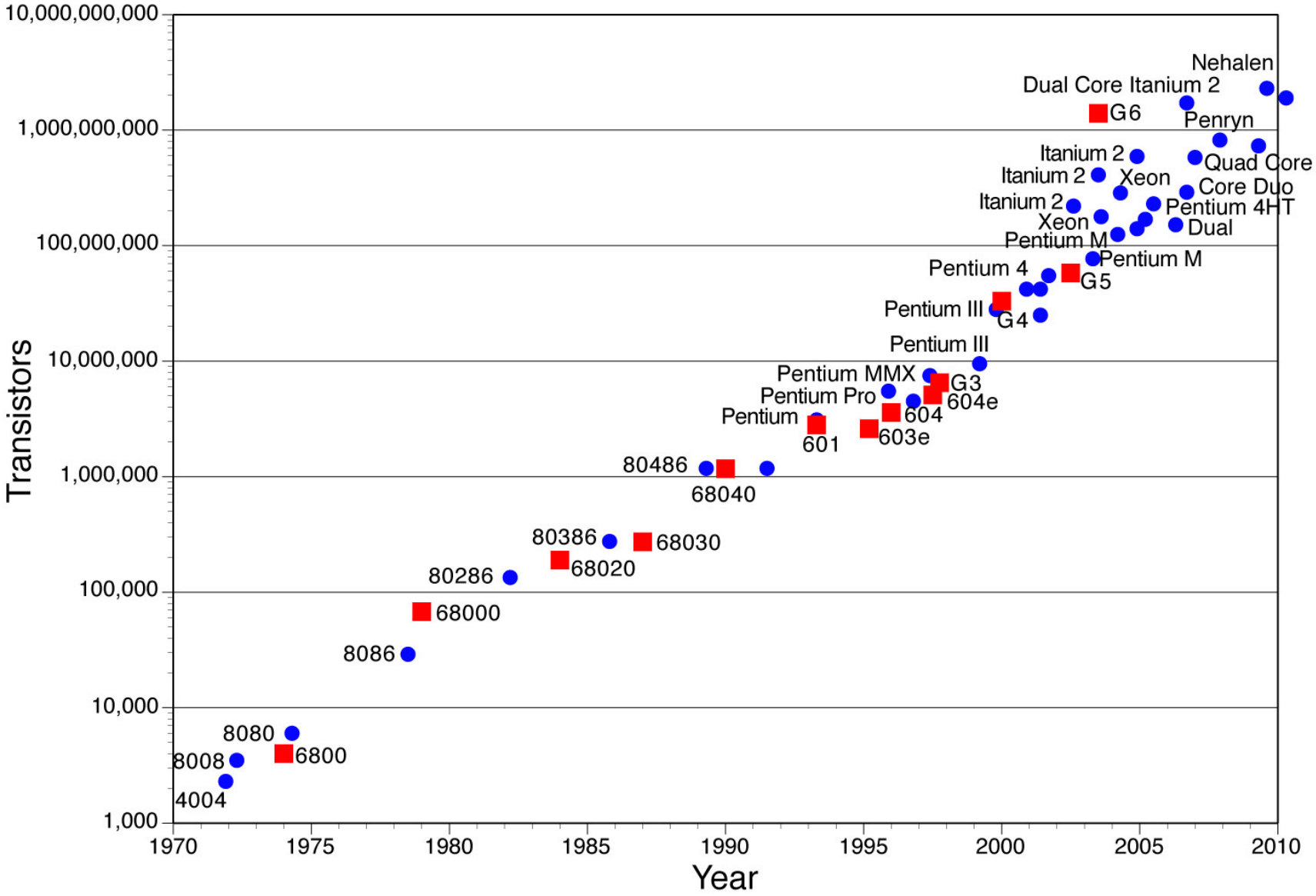
- Has become self-fulfilling prophesy.

Moore's Law – Speed





Moore's Law – Circuit Complexity

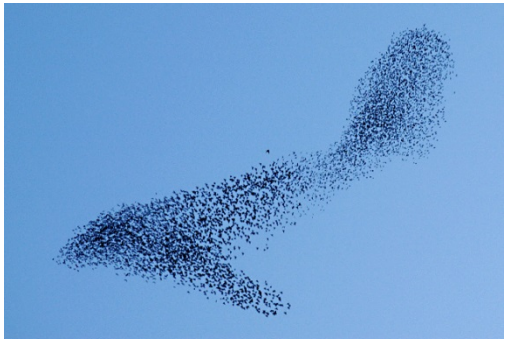


Moore’s Law

- Continued doubling of capacity into the future.
- Because we can.

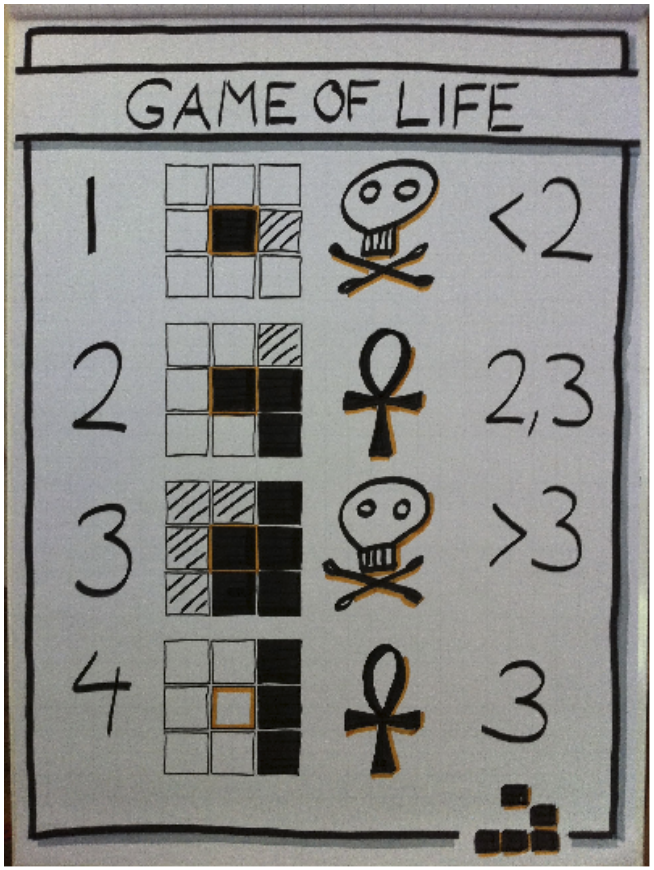
Emergent Phenomena

- Many simple parts yielding complex systems.

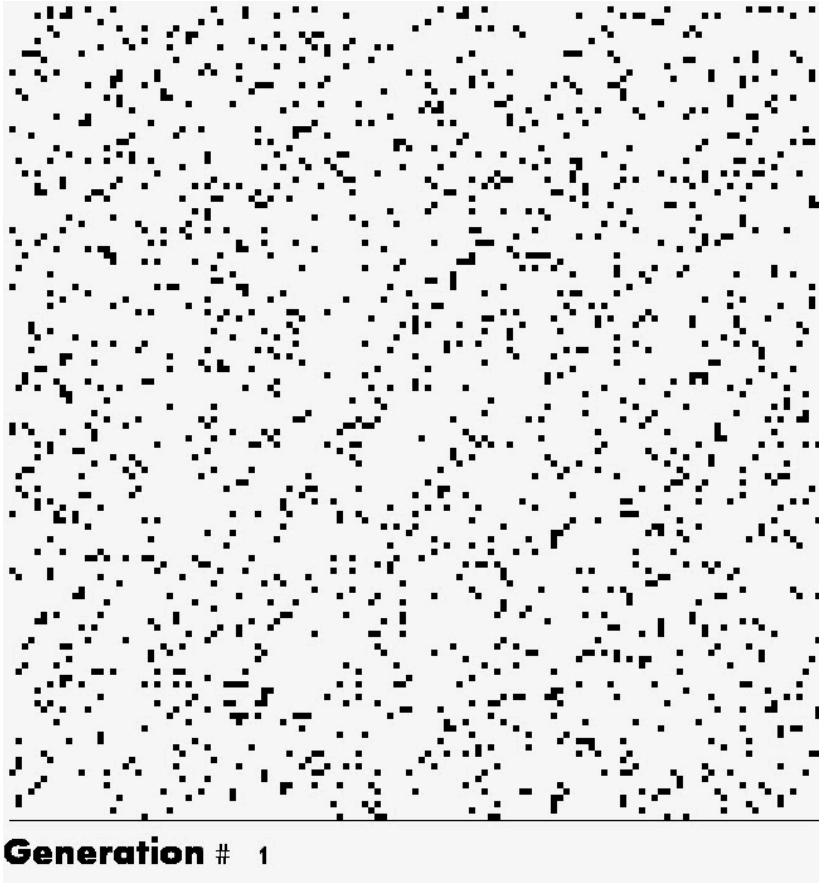


Conway's Game of Life

- Played on a grid.
- Time steps.
- Black squares are alive.
- Too few or too many neighbors => die.
- Empty cell can give birth.

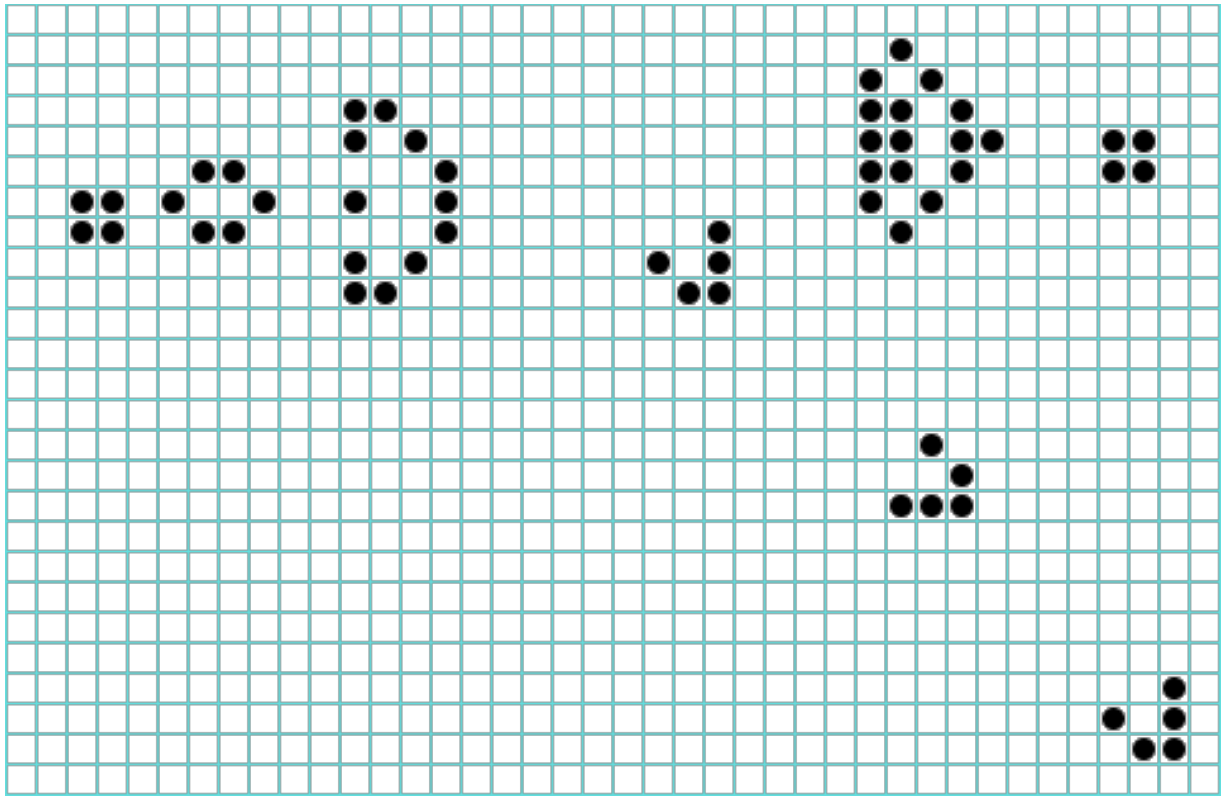


Conway's Game of Life

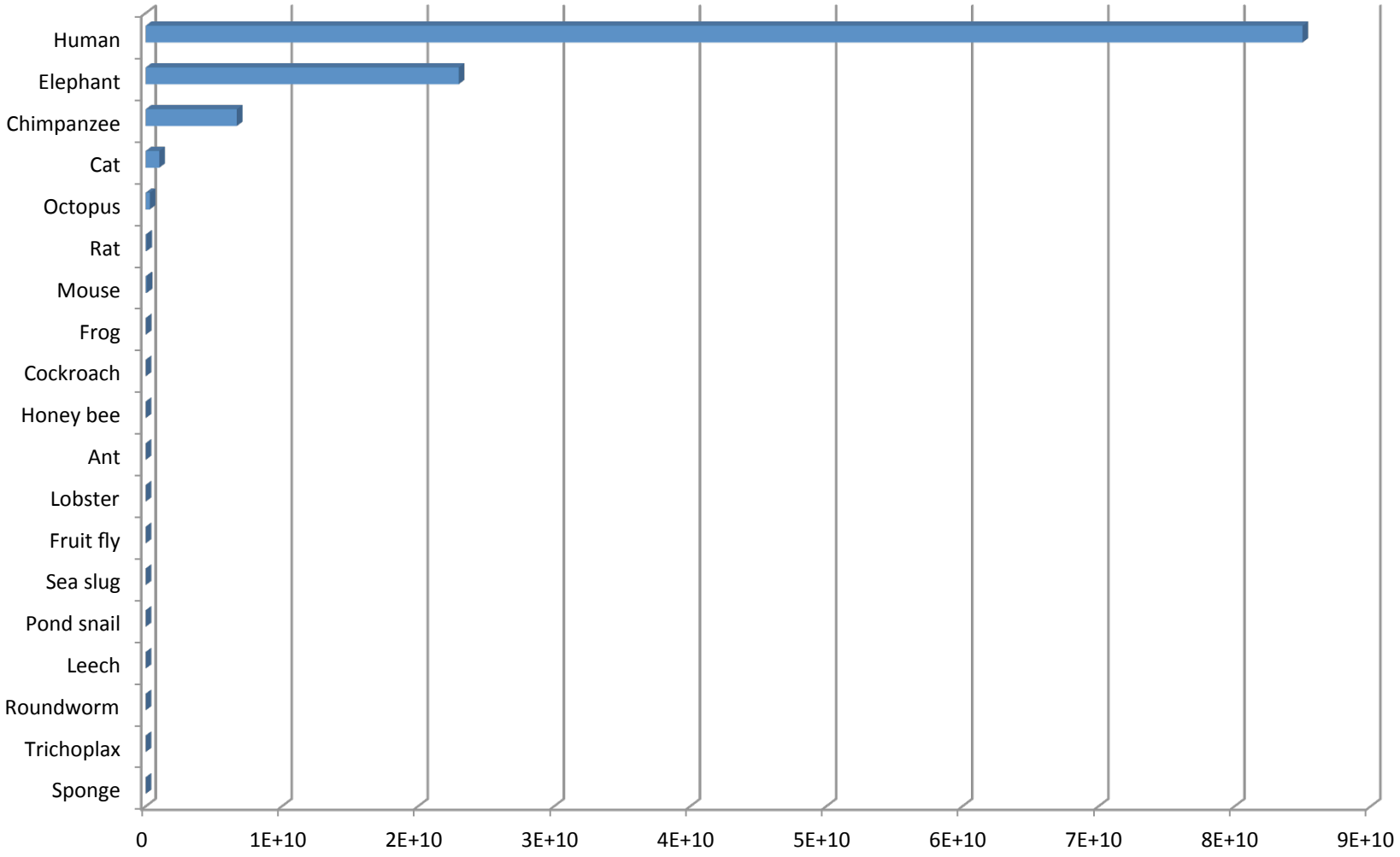


Conway's Game of Life

- Glider factory



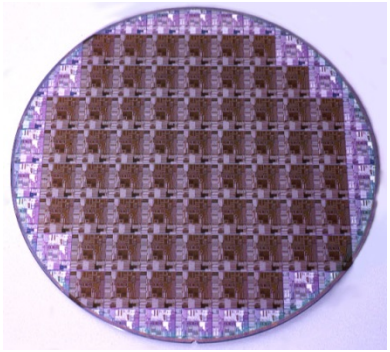
Size of Nervous System



Comparison

- Human cortex:
  20,000,000,000 neurons

- Current processor technology:
  2,000,000,000,000 transistors



- 100 × the number of neurons in the cortex.

Coming In Our Lifetimes?
(science fiction warning)

- Simulated people
  - As computer interfaces
  - For business
  - In the arts

- Super intelligent systems
  - Read and understand all of research mathematics?
  - Read and understand all of Wikipedia?
  - Super intelligent systems accelerating other areas.

New Ethical Questions (science fiction warning)

- What should people be entitled to do?
- Relationship between natural persons and digital entities.
- Rights or obligations to limit synthetic intelligence.

Limits of Computation

- Does this mean computers will be unlimited in their capabilities?

- No, there are some things they will never be able to do.

Limits of Computation

- "Imponderable" questions:
  - Will a computer ever be able to know God?
  - Will a computer ever be able to speak with the dead?

Limits of Computation

- "Soft" questions:
  - Will a computer ever be able to compose an expressive piece of music?
  - Will a computer ever be able to feel love?

Limits of Computation

- "Exact" questions:
  - Will a computer ever be able to simulate a financial system with one million participants?
  - Will a computer ever be able to calculate the effects of global warming?
  - Can a computer prove the Pythagorean Theorem?

Decidability

- In logic, a set of questions is “decidable” if there is an algorithm that correctly returns true or false for all questions in the set.

Undecidable Problems

- Proven there is no algorithm.
- E.g.
  - Given an arbitrary program and a finite input, decide whether the program finishes running or will run forever. (Turing, 1936).
  - Determine whether a particular kind of equation has a solution or not. (Hilbert's 10^{th} problem, 1900).
  - Given two grammars, determine whether they describe the same language.
  - Decide whether a mathematical expression is identically zero.

Undecidable Problems

- These are *proven* to not be computable.
- No computer, no mater how fast or how big can ever solve these problems, even if it uses all the matter in the universe and runs for billions of years.

The Limits of Computation

- Computers are amazing objects.
- Complex systems built of billions of simple parts.
- Will act intelligent within our lifetimes.
- Cannot solve certain simple problems.