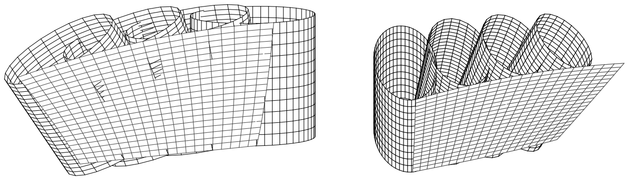
The Swept Surface of an Elliptic Cylinder

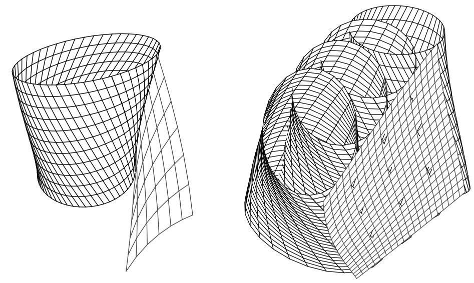
Stephen Mann
Computer Science Department
University of Waterloo

Sanjeev Bedi
Mechanical Engineering Department
University of Waterloo

David Roth
Mechanical Engineering Department
University of Waterloo



Elliptic Cylinder



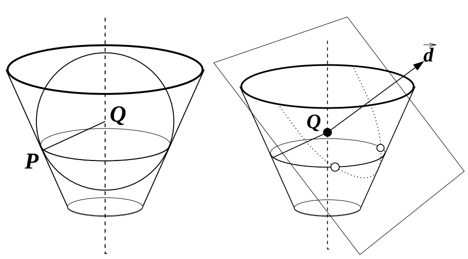
Twisted Elliptic Cylinder

Overview

- Swept surfaces required in many areas such as simulation of tool paths in NC-machining and intersection detection in robot trajectory planning.
- Traditionally, envelope theory which is computationally expensive and difficult to implement procedurally has been used for swept surface calculation.
- We previously developed a simple procedural method for surfaces of revolution [1]. Here we give an extension of our method to (twisted) elliptic cylinders.

Background: Surfaces of Revolution

- *Grazing point*: a point on a moving surface at which the direction of motion lies in the tangent plane.
- Construct piecewise linear approximation to grazing curve by computing grazing points for many circular slices.
- For surface of revolution, if we know motion of axis, then find grazing points on circular slice as follows:
  1. Choose any point P on circle
  2. Move along normal from P to point Q on axis having motion $\vec{d}$
  3. Intersect circle with plane through Q perpendicular to $\vec{d}$



Computing grazing points.

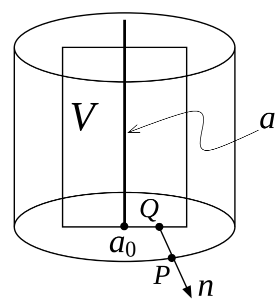
References

[1] D. Roth, S. Bedi, F. Ismail, and S. Mann. Surface swept by a toroidal cutter during 5-axis machining. *Computer Aided Design*, 33(1):57–63, 2001.

Extension to Elliptic Cylinder

- Motion of elliptic cylinder is described by
  - the motion of the bottom center of cylinder along a 3D trajectory $a_0(u)$;
  - the motion $a(u)$ of its axis orientation;
  - rotation of elliptic cylinder around its axis a.
- For fixed u, we can easily compute $a_0'(u)$, $a'(u)$, and the derivative of the rotation.

  Thus, we know the motion of this coordinate frame as a function of u, and the motion of every point on the elliptic cylinder is a linear function of the motion of this frame.



Elliptic Cylinder.

- First we fix u to determine the local reference frame and the motion of the focal plane V.

  Then we fix v to isolate one ellipse.
- Lines from a point on the ellipse in direction of the normal do not intersect cylinder axis, but they intersect major axis of ellipse.

  This allows us to express Q as function of P and therefore we can express Q as a function of θ. Thus, the velocity of points on V can be expressed as a function of θ.

  In addition, the normal is a function of θ only.
- To find grazing points on single elliptic slice, compute dot product of motion of V with equation for normal and solve for θ.

http://www.cgl.uwaterloo.ca/~smann/Papers/SM01.pdf