Surface Swept by a Toroidal Cutter during 5-Axis Machining

Submitted to CAD

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Abstract

This paper presents a method of determining the shape of the surface swept by a tool that follows a 5-axis tool path for machining curved surfaces. The method is based on discretizing the tool into pseudo-inserts and identifying imprint points using a modified principle of silhouettes. An imprint point exists for each pseudo-insert, and the piecewise linear curve connecting them forms an imprint curve for one tool position. A collection of imprint curves is joined to approximate the swept surface. This method is simple to implement and executes rapidly. The method has been verified by comparing predicted results of a 3-axis tool path with analytical results and of a 5-axis tool path with measurements of a part made with the same tool path.

Keywords: 5-axis machining, imprint curves, toolpath verification

1 Introduction

Automation of the manufacturing process from the nominal part geometry on a CAD system to the final machined part offers the opportunity for huge gains in productivity and cost savings. The advent of 5-axis machining and methods for generating NC tool paths has already offered the opportunity to reduce machining time by up to 85% [1]. However, this added flexibility also brings added complexity. Research efforts have concentrated on generating interference free NC tool paths that also produce machined parts free from excessive gouging or undercutting. Central to these ideas is the generation of the swept volume of the tool along its programmed NC tool path, and the ideas of the simulation, verification and correction of NC tool path programs [2].

General methods for determining the volume swept by a tool undergoing 5-axis motion, namely, envelope theory [3, 4] and the SEDE [5], are computationally complex and difficult to implement. Vector methods offering an approximate approach generally fall into three classes: view based, z-map based or normal vector based. These methods are based on either envelope theory or the SEDE or are based on static instances of the tool at various locations and ignore the movement of the tool [6, 7, 8]. While the tool movement can be approximated by using many instances of the tool at intermediately interpolated positions, the computational cost is prohibitive.

To overcome these problems, a few other methods have been proposed for 3-axis and 4-axis generation of swept volumes, namely using silhouettes and generating curves. The use of silhouette curves in 3-axis machining verification has yielded exact analytical results for the swept volume of a linear tool movement [9]. For 3-axis machining, the silhouette curve represents the imprint of the tool on the machined surface. Sheltami et al. [1] assumed that there exists a curve on the cutting surface of the tool that leaves its imprint behind as the tool moves forward during 4-axis machining. This curve, called a generating curve, is

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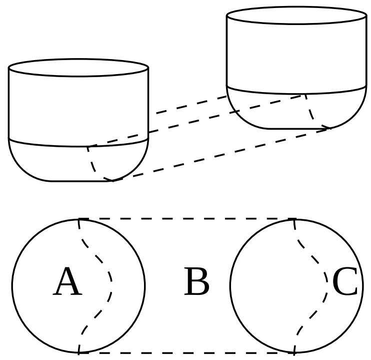

Figure 1: Silhouette surface.

approximated by a circle. This crucial assumption is valid only along a portion of the generating curve, and would be an especially poor assumption for 5-axis motion.

This paper addresses the need for a method to generate the surface swept by a moving tool in 5-axis machining. The concept of identifying the curve that leaves its imprint behind as the tool moves from one programmed location to the next is extended to 5-axis machining. A piecewise linear approximation of this imprint curve is made at each tool position by finding imprint points on pseudo-inserts of the tool, and adjacent imprint curves are connected with triangles. This gives a piecewise linear approximation to the swept surface. While the flat end mill and the ball end mill are both commonly used in surface machining, this paper concentrates on the toroidal tool since this tool is a generalisation of both the aforementioned tool types [10].

The next section describes silhouette and imprint curves. Section 4 describes our procedure for computing the swept path of a 5-axis, toroidal cutter. Section 5 focuses on verification of the developed method.

2 Silhouette and Imprint Curves

In NC machining, a swept surface is the set of points on the moving cutter that lie on the machined surface. For this purpose, the tool can be considered a solid (in this paper, a torus), and the points on the cutter that also lie on the swept surface will be those points on the cutter where the direction of motion of each point lies in the tangent plane of the cutter at that point. One way to simplify the calculation of the swept surface is to compute the portion of the swept surface generated at each tool position, and connect the pieces generated for adjacent tool positions.

Chung, Park, Shin and Choi [9] present a method for determining the surface swept by a generalised APT cutter for 3-axis machining. As stated by Chung et al., the swept surface, "for a linear NC-code block consists of three regions: the cutter bottom surface (CBS) at the start position, a ruled surface formed by sweeping the cutter, and the CBS at the end position." Figure 1 shows these regions designated A, B and C, respectively. The ruled surface, B, is formed by two silhouette curves on the CBS at the start and end positions. (In computer graphics, a silhouette curve is a curve that separates visible faces from invisible faces of an object from a given viewing direction [11].)

In 3-axis NC machining, the silhouette curve is a collection of those points at which a set of parallel rays tangentially intersects the surface of the cutting tool (Figure 2). The direction of this set of rays is coincident with the direction of movement of the tool, which is simply the vector joining one cutter location to the next. For every programmed tool location there exists a silhouette curve. The surface swept by a cutter during any machining motion is the unification of the silhouette curves for all tool locations along its programmed path.

The generalisation of Chung et al.'s method to 5-axis machining is non-trivial. For all but the simplest 5-axis motions, the silhouette curve does not represent the imprint of the tool on the stock material because

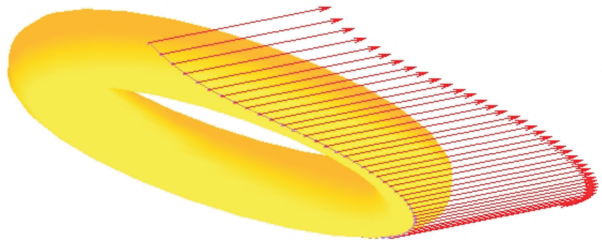

Figure 2: Silhouette curve on toroidal tool.

the cutting tool is undergoing both translation and rotation. As a first step, the idea of a silhouette curve must be generalised.

We define an *imprint curve* in 5-axis machining as the set of points on the rotating tool surface at which the direction of motion lies in the tangent plane of the cutter. Each imprint curve will be a curve on the swept surface. The silhouette curve used in 3-axis machining is a special case of an imprint curve where there is no rotation. An illustration of an imprint curve on the toroidal tool would look similar to the silhouette curve of Figure 2, although the direction vectors will not be parallel.

The idea of this paper is to quickly find an approximation to the imprint curve at each tool position, and connect these imprint curves to form the swept surface.

3 The Imprint Method for Generating a Swept Surface

The method of this paper computes a piecewise linear approximation to the swept surface created by a 5-axis toroidal cutter. There are four parts to the algorithm:

1. Construct pseudo-inserts for the toroidal cutter.
2. At each cutter location, find an imprint point on each pseudo-insert.
3. Approximate the swept surface as the piecewise linear interpolant between imprint curves on neighboring cutter locations.
4. Merge multiple tool passes to find the final interpolant.

Each step is described in its own subsection. For this discussion, we will assume that there are P tool positions, and each tool position $\mathbf{T}_p$ represents the centre of the toroidal cutter, with the vector $\vec{\mathbf{k}}_p$ representing the tool orientation.

3.1 Toroidal Tool and Pseudo-Inserts

This paper deals with the surface swept by a toroidal tool undergoing 5-axis motion. A typical 4-insert toroidal tool is shown on the left in Figure 3. Geometrically, the toroidal endmill can be modelled as a radiused cylinder, since the rotational speed of the tool is much greater than the feed rate. Therefore, the endmill appears as a solid torus to the stock material. The parameters of this torus such as the major radius R and the minor radius r are indicated in the centre of Figure 3. The radius r is also the radius of the tool inserts.

Our method discretizes the torus using a finite number of sectional planes that contain the axis of revolution to create circular cross sections, as shown on the right in Figure 3. These sections may be thought of as *pseudo-inserts*, each of which will contribute to the final shape of the machined part. The number of actual inserts is typically small, for example from two to four, whereas the number of pseudo-inserts is selectable and typically would be large; for example, in the testing discussed in Section 4, 40 pseudo-inserts were used. The number of pseudo-inserts would dictate the accuracy of the generated imprint curves; a larger number leads to higher accuracy at the expense of computational processing time. For the remainder

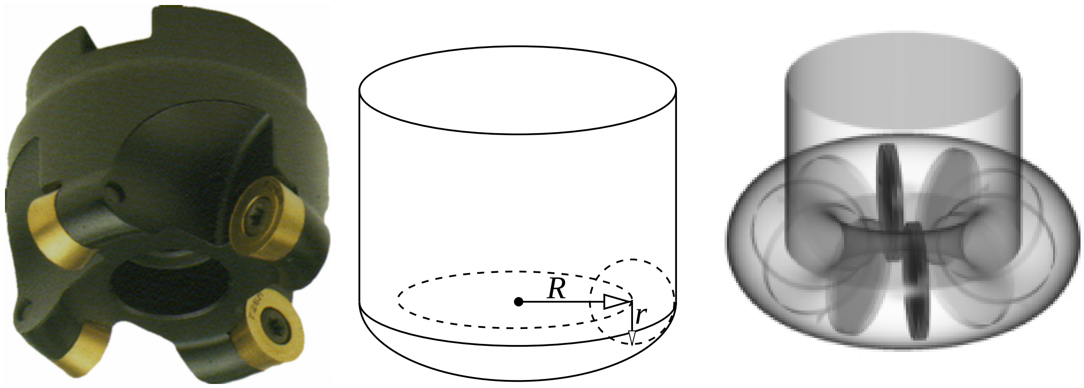

Figure 3: Toroidal tool: actual, mathematical, pseudo-inserts

of this paper, Q will denote the number of pseudo-inserts, with the centre of every pseudo-insert is given by the point $\mathbf{T}_{p,q}$. Associated with each pseudo-insert is $\widehat{\mathbf{n}}_{p,q}$, the unit normal to the plane of the insert.

One of the key observations in this work is that except for one special case, there is at most one point on each imprint curve that lies on the swept surface. This observation and the special case are both discussed further in the next section.

3.2 Numerical Method for Developing the Silhouette and Imprint Curves

Chung et al. found an analytic expression for the silhouette curve on a toroidal 3-axis machine, which involves finding the roots of the fourth order equation for a torus. We developed the following numerical method as an alternative for identifying points on the silhouette for 3-axis motion of a toroidal cutter. At each tool position, for each pseudo-insert, compute the cross product of the tool direction ray $\Delta\vec{\mathbf{S}}_p = \mathbf{T}_{p+1} - \mathbf{T}_p$ with the normal to the plane containing the pseudo-insert. This yields a second ray, which is rescaled to lie on the pseudo-insert boundary, giving a point on the silhouette curve:

$$\mathbf{m}_{p,q} = \mathbf{T}_{p,q} + r\frac{\Delta\vec{\mathbf{S}}_p \otimes \widehat{\mathbf{n}}_{p,q}}{\left|\Delta\vec{\mathbf{S}}_p \otimes \widehat{\mathbf{n}}_{p,q}\right|}. \tag{1}$$

A point computed in this manner is on the silhouette since by construction the tool direction will lie in the tangent plane of the torus at the this point. Some additional details on this method are given below in the generalization of this approach to imprint curves.

In 5-axis motion the direction rays emanating from the centre of the pseudo-inserts are a function of both the tool translation and rotation and are not in general parallel. Thus, the numerical method of identifying silhouette curves must be modified to find the imprint of the tool as it moves from one point to the next. For a fixed sequence of tool positions, the actual rotation and tool path will depend on the particular NC machine. However, if the steps between tool positions are small (as is common with 5-axis machines), then the tool paths produced by different machines will be approximately the same. With this in mind, we designed a method that will compute the approximate direction of motion for points on the cutter.

Our method is based on determining the direction ray separately for each pseudo-insert. All points on a pseudo-insert circumference are moving along a torus (due to the tool rotation) and along a tabulated cylinder (due to translation along the direction ray). Only those points on the pseudo-insert at which the direction of motion lies in the tangent to the torus will leave a mark behind. All other points on the pseudo-insert have either been previously removed or will be removed by the motion of pseudo-inserts that follow.

Figure 4 shows a pseudo-insert q in two subsequent tool positions p and $p+1$. The direction of tool motion is given by $\Delta\vec{\mathbf{S}}_p$. The direction ray vector we use to compute the imprint point is $\Delta\vec{\mathbf{T}}_{p,q}$, the vector that joins the centre of insert q at the two locations p and $p+1$.

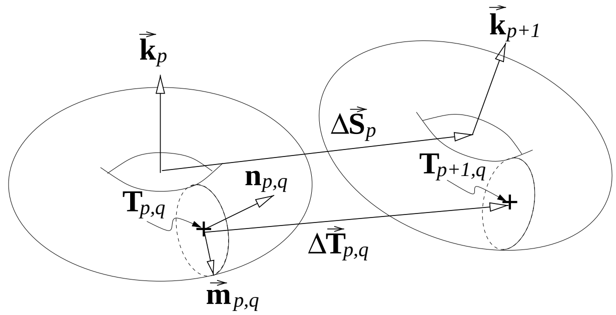

Figure 4: The position of a pseudo-insert at two tool positions.

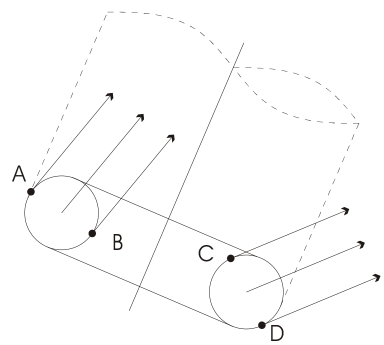

Figure 5: Sectional view of a toroidal cutter showing two pseudo-inserts.

There are two points on each pseudo-imprint curve that satisfy the condition of the direction $\Delta\vec{\mathbf{T}}_{p,q}$ lying in the tangent plane of the torus at that point (e.g., points C and D in Figure 5). Only one of these points (point D in the figure) will lie on the outside (cutting) surface of the torus, and the other point need not be considered. As in the case of the silhouette curve, the point $\mathbf{m}_{p,q}$ on the pseudo-insert responsible for leaving its imprint on the workpiece surface can be determined from vector algebra as

$$\mathbf{m}_{p,q} = \mathbf{T}_{p,q} + r\frac{\Delta\vec{\mathbf{T}}_{p,q} \otimes \widehat{\mathbf{n}}_{p,q}}{\left|\Delta\vec{\mathbf{T}}_{p,q} \otimes \widehat{\mathbf{n}}_{p,q}\right|}, \tag{2}$$

where r is the radius of the insert.

As a further optimization, we assume that the cutter is tilted slightly in the direction of motion. This means that only half the pseudo-inserts need be considered at a single tool position, since the other half of the inserts will not be milling the surface (e.g., the pseudo-insert containing points A and B in Figure 5 will not be tangent to the machined surface at this tool position). Thus, at a location p, the points $\mathbf{m}_{p,q}$, $q = 1 \ldots Q/2{+}1$ are joined to form the imprint curve at location p. This process is repeated at all P positions (the selection of $\mathbf{m}_{p,1}$ is discussed in the next section). The assumption of a tilted tool also simplifies the construction of the swept surface. While relying on this assumption may mean our algorithm will miss "back-gouging," we can readily detect back-gouging by computing a second swept surface, running the tool motion in the opposite direction and seeing if the back surface intersects the front surface.

It should be mentioned that when the direction vector $\Delta\vec{\mathbf{T}}_{p,q}$ becomes parallel to the normal $\widehat{\mathbf{n}}_{p,q}$, Equation 2 does not yield a unique solution (with a zero over zero appearing in Equation 2). This special case will occur for motions where the tool direction vector is perpendicular to the tool orientation $\vec{\mathbf{k}}_p$. In this case, the entire side profile of the tool will produce the workpiece surface. This case is easily recognised and treated accordingly. A similar problem occurs with Equation 1 when $\Delta\vec{\mathbf{S}}_p$ is parallel to $\widehat{\mathbf{n}}_{p,q}$.

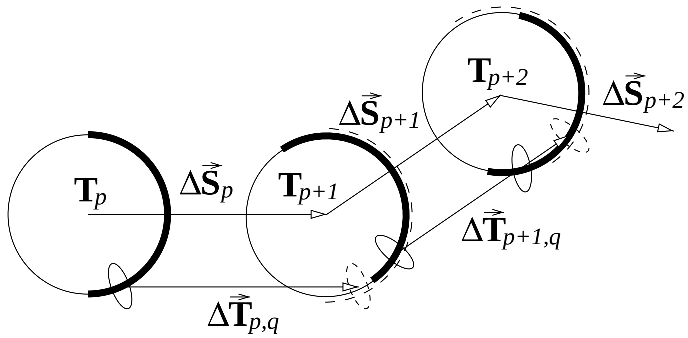

Figure 6: Three cutter locations

3.2.1 Demarcation Point

The direction ray for each pseudo-insert is determined by joining its centre at two consecutive locations. As mentioned earlier, the basic algorithm described herein is dependent on discretizing the toroidal cutter into Q pseudo-inserts. However, since we assume that our tool is tilted, only the front half of the tool generates the surface. From the standpoint of tool motion, the *demarcation point* separates the front half of the torus from the back half. Therefore, what is considered the front half of the torus is conditional on the direction of motion of the torus. In machining a surface with linear movements, the tool arrives at a cutter location from one direction and leaves in another direction. This introduces two demarcation points at every cutter location, except for the first and last.

Figure 6 shows the cutter in three subsequent positions, p, $p+1$, and $p+2$. For purposes of illustration, the movement is two dimensional. The large circles are the circles of radius R that generate the torus. The front half of the outgoing tool motion is shown by a thicker line in this figure; the front half of the incoming tool motion is shown by a dashed line. The small ellipses represent one pseudo-insert in various positions.

As indicated in the centre and right cutter locations, there are two positions for each pseudo-insert at each tool location: one for the incoming tool path (the dashed insert), and one for the outgoing tool path (the solid insert). We are primarily interested in the outgoing pseudo-insert, as that is the one used by our algorithm for computing the imprint curve at a tool location. However, the incoming insert at $p+1$ is used to compute $\Delta\vec{\mathbf{T}}_{p,q}$.

3.3 The Swept Surface

The imprint curves generated at the programmed tool locations are combined to generate the swept surface. This process of combining can be done with linear segments or with higher order blending. In this work, the imprint curves were connected linearly to generate the swept surface. Two complications occur. First, adjacent tool locations may generate imprint points on different sets of pseudo-inserts. And second, at each tool location, one imprint curve should be created by the incoming tool, and a second one for the outgoing tool. The first problem is handled by noting that each imprint curve has the same number of imprint points. Two adjacent imprint curves can be triangulated by connecting corresponding imprint points on the front of each tool position starting with the demarcation points, which generates a set of non-planar quadrilaterals. These quadrilaterals are then split into two triangles to form the piecewise linear approximation of the swept surface.

For the second problem, we generate one imprint curve for each tool position and use it for both the incoming and outgoing imprint curve. If the tool positions are spaced closely together, this should be a good approximation to the actual swept surface. If a more accurate representation is desired, then one could generate both imprint curves for each tool position, connect them to the corresponding imprint curves on adjacent tool positions, and add as a third surface the shape of the tool itself in the tool position.

3.4 Merging Swept Surfaces

The piecewise linear approximations generated for the swept tool path may overlap and need to be merged. Any technique could be used to merge the piecewise linear sweeps; to test our method, we used the normal vector approach [8]. The normal vector approach triangulates the design surface and "grows" vectors from each of the vertices of the triangulation. For testing, we used a triangulation with a maximum triangle edge length of $s = \sqrt{6re}$, where r is the radius of the pseudo-insert and e is the maximum depth a sphere of radius r could penetrate an equilateral triangle with side length s. After running the simulation, a check of the length of the vectors shows how much undercutting and gouging occurred.

4 Experimental Verification

This section highlights the testing of the method presented in this paper. For more details and additional tests, see [12].

4.1 3-Axis Machining

The form of the silhouette curve obtained by Chung et al. [9] for a radiused end mill is seen in Figure 1. A simulation was run to prove the compatibility of the methodology presented in this paper to the algebraic results of Chung et al. For this simulation, the toroidal cutter was discretized into $Q = 40$ pseudo-inserts, and only the front half of the tool was considered. Similar to the cutter used by Chung et al., the tool had a major radius of $R = 1$ unit and a minor radius of $r = 1$ unit. The linear 3-axis motion had a slope of 0.2. The imprint curve obtained is identical in form (except for some discretization) to the analytical solution of Figure 1.

The imprint curves were also back-substituted into the analytical equation for a linear 3-axis motion as developed by Chung et al. For completeness, this analytical expression is reproduced here:

$$N(u,v) \cdot D = \sqrt{t^2 - u^2} \cdot (t - f)/t - s\sqrt{r^2 - (t-f)^2} = 0, \tag{3}$$

where N is the surface normal of the cutter bottom surface, D is the cutter movement direction, $t = \sqrt{u^2 + v^2}$, $f = R$, $s =$ slope, $u = y$ and $v = x$. The generated imprint points from Equation 2 satisfied Equation 3 exactly.

Chung et al. also describe the computational complexity of the analytical solution in terms of the number of mathematical operations required to compute the silhouette curve. As a best case for a rounded endmill, the 3-axis analytical solution requires 27 multiplications, 24 additions and 7 square root evaluations per silhouette curve point computed, with the typical case costing only one or two additional multiplications and additions. The worst case (which is also the typical case) for the 5-axis method presented in this paper requires 27 multiplications, 23 additions and 1 square root evaluation, which includes the cost of computing $\mathbf{T}_{p,q}$, $\widehat{\mathbf{n}}_{p,q}$, $\Delta\vec{\mathbf{T}}_{p,q}$. Thus, in addition to being applicable to 5-axis machining, the silhouette/imprint curve method described in this paper is slightly faster than the Chung et al. method.

4.2 5-Axis Machining (Turbine Blade)

In this section, results from the developed algorithm will be compared with actual cutting test data of a turbine blade. The test workpiece is shown in Figure 7.

The size of the turbine blade is approximately $300mm \times 150mm$. The blade was machined on a Rambaudi 5-axis milling machine in wax using a toroidal cutter with tool dimensions $R = 16mm$ and $r = 3mm$. The swept surface was computed using the method described in this article; a linear interpolation step size of approximately $1.5mm$ was used. The cutter was discretized into 40 pseudo-inserts with only the 21 front-half pseudo-inserts being used to generate the imprint curves. For the normal vector approach, the length of the triangle edges was computed as described in Section 3.4 with $e = 0.005mm$.

From the machined surface, one pass was measured on a Mitutoyo BHN305 coordinate measuring machine. The measurements were obtained using a $1mm$ diameter probe.

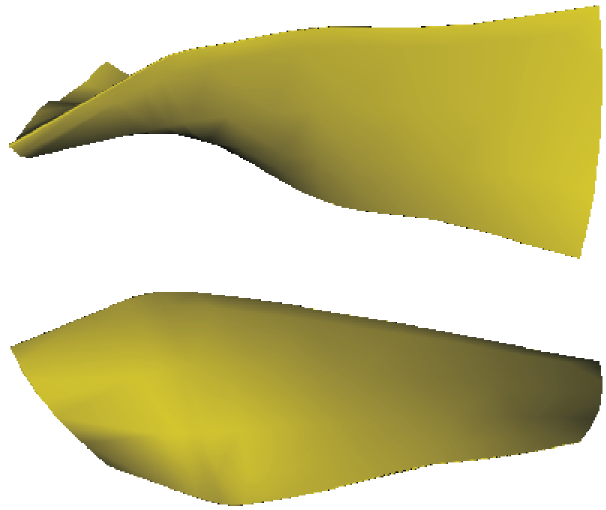

Figure 7: Two rendered views of the turbine blade.

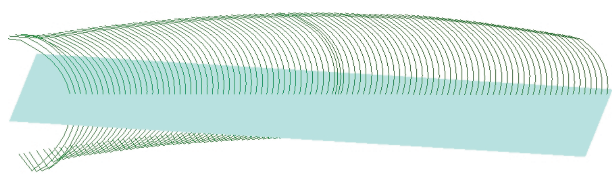

Figure 8: Planar slice of simulated swept surface.

The CMM probe measured along a straight pass close to the centre of a tool pass. The corresponding straight path along the bottom swept surface was approximately found by sectioning the imprint curves with a plane as shown in Figure 8. The differences between the measured data and the simulation results are shown in Figure 9. They were obtained by calculating the distance between the linear segments and the measured CMM data points. The error is bounded by a maximum error of $25\mu m$ and a minimum error of $-24\mu m$. The average error is $0.788\mu m$ with a standard deviation of $11.8\mu m$. Notice that the variations follow a bow shape that results, based on past experience with the Rambaudi, from set-up error. This set-up error was estimated around $\pm 12\mu m$. Overall, the simulation was accurate to within measurement tolerance.

5 Conclusions

A method to determine the imprint curve of a tool as it executes a 5-axis tool movement has been presented. This method is an improvement over previous methods as it computes a faceted approximation to the surface swept by the tool as it executes a 5-axis tool path. The swept surface can be used for both verification and correction of the tool path.

The present method was assessed by conducting two tests. In the first test, the method was compared against a 3-axis analytical solution. The method agrees exactly with the analytical solution. In the second test, the method was used to verify one pass, randomly selected, from a 5-axis CLDATA file for a turbine blade, consisting of 100 tool locations. The result compare well when compared to the set-up error.

The method has only been tested on finely spaced tool positions because this is the common way to produce curved surfaces on 5-axis machines without risking cutting errors due to machine kinematics. In a tool path with large moves, the large moves can be divided into smaller moves using linear interpolation.

Overall, the method presented is simple to implement, accurate and fast to execute. It is general and applies to all types of 5-axis tool movements.

The method presented in this paper makes an approximations to 5-axis machine motion by assuming

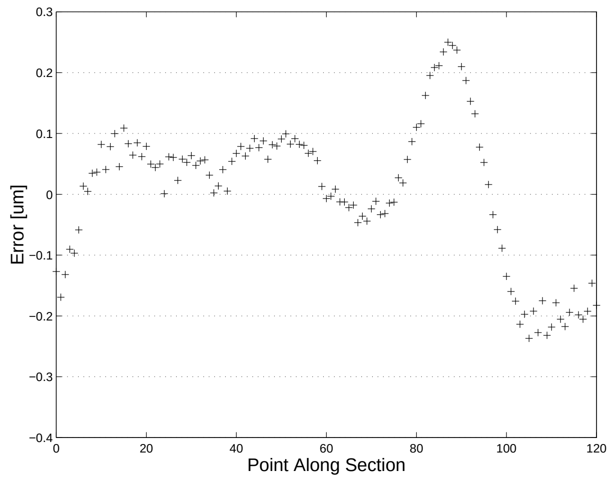

Figure 9: 5-axis results.

piecewise linear motion of the tool. The accuracy of the imprint method could be improved if it were tailored to the behavior of each particular milling machine. In addition to accounting for a machine's interpolation of position and orientation, the imprint model could be modified to account for any blending of linear motions used to smooth the piecewise linear toolpath. Such improvements might be achievable by using non-linear connecting pieces between pairs of imprint curves.

Further, while the details of the presented method are given for a toroidal cutter, the method for computing imprint points should generalize to other cutters as long as there is some form of pseudo-insert for the cutter.

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