Relational Database Design Theory (II)

CS348 Spring 2024

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Sections: **002 & 003 only**

Announcements

- Milestone 1: Due on June 20th
  - Up to 2 days extension with 25% penalty per day

- Assignment 2: Due **June 29th**
  - Up to 2 days extension with 5% penalty per day
  - Accessibility/Short term absence: **still ONLY 2 days** extension but no penalty

- Must email Sylvie for accessibility/short term absence! It's your responsibility to email Sylvie since I do not check/track them.

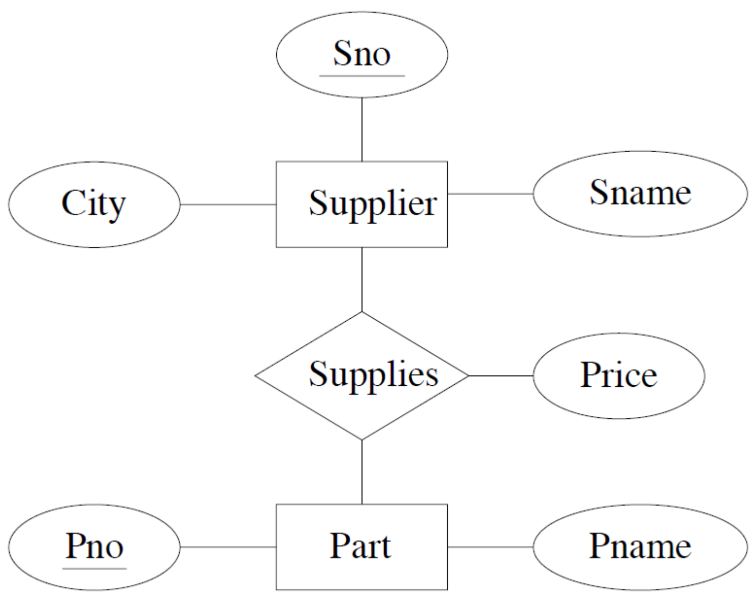
Outline For Today

1. Application Constraints and Decompositions
2. Functional Dependencies
3. Boyce-Codd Normal Form (BCNF) & BCNF Decomposition Alg.
4. Dependency Preservation and 3rd Normal Form

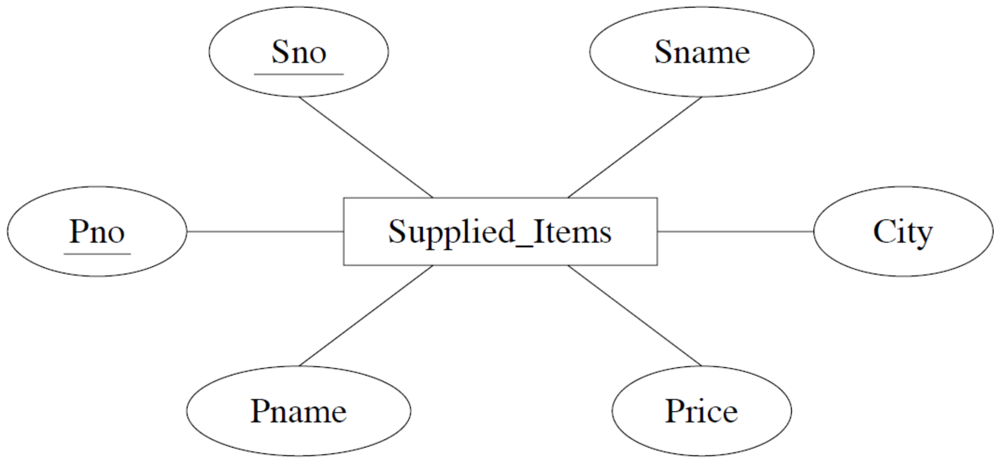
This lecture

A Parts/Suppliers database example

- Each type of part has a name and an identifying number and may be supplied by zero or more suppliers.
- Each supplier has an identifying number, a name, and a contact location for ordering parts.
- Each supplier may offer the part at a different price.



Single table?



Supplied_Items

Sno	Sname	City	Pno	Pname	Price
S1	Apple	K-W	P1	A	$25
S1	Apple	K-W	P2	B	$34
S2	BBuy	Lon	P3	A	$5
S2	BBuy	Lon	…	…	…

Or decomposed tables?

- An instance

Suppliers

Sno	Sname	City
S1	Apple	K-W
S2	BBuy	Lon

Parts

Pno	Pname	Price
P1	A	$25
P2	B	$34
P3	A	$5

Supplies

Sno	Pno	Price
S1	P1	$25
S1	P2	$34
S2	P3	$5
S2	…	…

Schema decomposition

- Let R be a relation schema (= set of attributes).
- The collection $\{R_1, \ldots, R_n\}$ of relations is a decomposition of R if $R = R_1 \cup \cdots \cup R_n$

Supplied_Items

R: Sno	Sname	City	Pno	Pname	Price
S1	Apple	K-W	P1	A	$25
S1	Apple	K-W	P2	B	$34
S2	BBuy	Lon	P3	A	$5
S2	BBuy	Lon	…	…	…

R1: Sno	Sname	City
S1	Apple	K-W
S2	BBuy	Lon

R2: Pno	Pname	Price
P1	A	$25
P2	B	$34
P3	A	$5

R3: Sno	Pno	Price
S1	P1	$25
S1	P2	$34
S2	P3	$5
S2	…	…

- What is a good decomposition?

Is this a good decomposition?

- Example 1

R

Sno	Sname	City	Pno	Pname	Price
S1	Apple	K-W	P1	A	$25
S1	Apple	K-W	P2	B	$34
S2	BBuy	Lon	P3	A	$5
S2	BBuy	Lon	…	…	…

R1

Sno	Sname	Pname
S1	Apple	A
S1	Apple	B
S2	BBuy	A
S2	BBuy	…

R2

Pname	City	Pno	Price
A	K-W	P1	$25
B	K-W	P2	$34
A	Lon	P3	$5
…	Lon	…	…

Natural Join

Sno	Sname	City	Pno	Pname	Price
S1	Apple	K-W	P1	A	$25
S1	Apple	K-W	P2	B	$34
S1	Apple	Lon	P3	A	$5
S2	BBuy	K-W	P1	A	$25
S2	BBuy	Lon	…	…	…

But computing the natural join of R1 and R2, we get extra data (spurious tuples).

We would therefore lose information if we were to replace R by R1 and R2

"Good" Schema Decomposition

- Lossless-join decompositions
  - We should be able to construct the instance of the original table from the instances of the tables in the decomposition

A decomposition $\{R_1, R_2\}$ of R is lossless iff the common attributes of R_1 and R_2 form a superkey for either schema,

$$R_1 \cap R_2 \rightarrow R_1 \text{ or } R_1 \cap R_2 \rightarrow R_2$$

**If X is a superkey of R, then $X \rightarrow R$ (all the attributes) [last lecture]*

Is this a lossless join decomposition?

- Example 1
  - $R = \{Sno, Sname, City, Pno, Pname, Price, PType\}$

Sno	Sname	City	Pno	Pname	Price
S1	Apple	K-W	P1	A	$25
S1	Apple	K-W	P2	B	$34
S2	BBuy	Lon	P3	A	$5
S2	BBuy	Lon	…	…	…

  - $R_1 = \{Sno, Sname, Pname\}, R_2 = \{Pname, City, Pno, Price\}$

$R_1 \cap R_2 = \{Pname\}$ is not a superkey of either R_1 or R_2

→ This decomposition is lossy

Which one is a better decomposition?

- Example 2: a table for a company database
  - $R = \{Proj, Dept, Div\}$

$\mathcal{F}$ includes:
FD1: $Proj \rightarrow Dept$ FD2: $Dept \rightarrow Div$ FD3: $Proj \rightarrow Div$

- Consider 2 decompositions

$$D_1 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Dept, Div\} \end{Bmatrix} \qquad D_2 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Proj, Div\} \end{Bmatrix}$$

- Both are lossless. (Why?) $R_1 \cap R_2 \rightarrow R_1$ or R_2

- However, testing FDs is easier on one of them. (Which?)

Testing FDs

- Example 2: a table for a company database
  - $R = \{Proj, Dept, Div\}$

$\mathcal{F}$ includes:

FD1: $Proj \rightarrow Dept$ FD2: $Dept \rightarrow Div$ FD3: $Proj \rightarrow Div$

  - Consider 2 decompositions

$$D_1 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Dept, Div\} \end{Bmatrix}$$

$$D_2 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Proj, Div\} \end{Bmatrix}$$

- FD1 (in R1)
- FD2 (in R2)
- FD3 (join R1 and R2?)
- → No need, if FD1 and FD2 hold, then FD3 hold

Testing FDs

- Example 2: a table for a company database
  - $R = \{Proj, Dept, Div\}$

$\mathcal{F}$ includes:

FD1: $Proj \rightarrow Dept$ FD2: $Dept \rightarrow Div$ FD3: $Proj \rightarrow Div$

- Consider 2 decompositions

$$D_1 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Dept, Div\} \end{Bmatrix}$$

- FD1 (in R1)
- FD2 (in R2)
- FD3 (join R1 and R2?)
- → No need, if FD1 and FD2 hold, then FD3 hold

$$D_2 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Proj, Div\} \end{Bmatrix}$$

interrelational

- FD1 (in R1)
- FD3 (in R2)
- FD2 (join R1 and R2?)

→ Yes. FD1 and FD3 are not sufficient to guarantee FD2

Testing FDs

- Example 2: a table for a company database
  - $R = \{Proj, Dept, Div\}$

$\mathcal{F}$ includes:

FD1: $Proj \rightarrow Dept$ FD2: $Dept \rightarrow Div$ FD3: $Proj \rightarrow Div$

- Consider 2 decompositions

$$D_1 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Dept, Div\} \end{Bmatrix} \qquad D_2 = \begin{Bmatrix} R_1\{Proj, Dept\}, \\ R_2\{Proj, Div\} \end{Bmatrix}$$

- FD1 (in R1)
- FD2 (in R2)
- FD3 (join R1 and R2?)
- → No need, if FD1 and FD2 hold, then FD3 hold

(i) Equivalent to $\mathcal{F}$
(ii) Not interrelational

FD1 (in R1)
FD3 (in R2)
- FD2 (join R1 and R2?)

→ Yes. FD1 and FD3 are not sufficient to guarantee FD2

interrelational

"Good" Schema Decomposition

- Lossless-join decompositions
- Dependency-preserving decompositions

> Given a schema R and a set of FDs $\mathcal{F}$,
> decomposition of R is dependency preserving
> if there is an equivalent set of FDs $\mathcal{F}'$,
> none of which is interrelational in the decomposition.

- Next, how to obtain such decompositions?
  - BCNF → guaranteed to be a lossless join decomposition!

Boyce-Codd Normal Form (BCNF)

- A relation R is in BCNF iff whenever $(X \rightarrow Y) \in \mathcal{F}^+$ and $XY \subseteq R$, then either
  - $(X \rightarrow Y)$ is trivial (i.e., $Y \subseteq X$), or
  - X is a super key of R (i.e., $X \rightarrow R$)
    - That is, all non-trivial FDs follow from "key → other attributes"

- Example: $R = \{Sno, Sname, City, Pno, Pname, Price\}$

$\mathcal{F}$ includes:

FD1: $Sno \rightarrow Sname, City$ FD2: $Pno \rightarrow Pname$ FD3: $Sno, Pno \rightarrow Price$

  - The schema is not in BCNF because, for example, *Sno* determines *Sname,City*, is non-trivial but is not a superkey of R

BCNF decomposition algorithm

- Find a BCNF violation
  - That is, a non-trivial FD $X \rightarrow Y$ in $\mathcal{F}^+$ of R where X is not a super key of R
    - Example: $R = \{Sno, Sname, City, Pno, Pname, Price\}$

$\mathcal{F}$ includes:
FD1: $Sno \rightarrow Sname, City$ FD2: $Pno \rightarrow Pname$ FD3: $Sno, Pno \rightarrow Price$

- Decompose R into R_1 and R_2, where
  - R_1 has attributes $X \cup Y$;
  - R_2 has attributes $X \cup Z$, where Z contains all attributes of R that are in neither X nor Y

$R = \{Sno, Sname, City, Pno, Pname, Price\}$

BCNF violation: $Sno \rightarrow Sname, City$

R2{Sno,Pno,Pname,Price} R1{Sno,Sname,City}

- Repeat (till all are in BCNF)

BCNF decomposition example

- R = {Sno,Sname,City,Pno,Pname,Price}

$\mathcal{F}$ includes:

FD1: $Sno \rightarrow Sname, City$ FD2: $Pno \rightarrow Pname$ FD3: $Sno, Pno \rightarrow Price$

{Sno,Sname,City,Pno,Pname,Price}

BCNF violation: $Sno \rightarrow Sname, City$

R2{Sno,Pno,Pname,Price}

$Pno \rightarrow Pname$ $Sno, Pno \rightarrow Price$

BCNF violation: $Pno \rightarrow Pname$

R2b{Sno,Pno,Price}

BCNF: $Sno, Pno \rightarrow Price$

R2a{Pno,Pname}

BCNF: $Pno \rightarrow Pname$

R1{Sno,Sname,City}

BCNF: $Sno \rightarrow Sname, City$

{SNo}$^+$={Sno, Sname, City}
→ a superkey of R1

BCNF helps remove redundancy

Sno	Sname	City	Pno	Pname	Price
S1	Apple	K-W	P1	A	$25
S1	Apple	K-W	P2	B	$34
S1	Apple	K-W	P3	A	$20
S2	BBuy	London	…	…	…

BCNF violation: $Sno \rightarrow Sname, City$

Sno	Pno	Pname	Price
S1	P1	A	$25
S1	P2	B	$34
S1	P3	A	$20
S2	…	…	…

Sno	Sname	City
S1	Apple	K-W
S2	BBuy	London
..	…	…

Another example

$\mathcal{F}$ includes:

uid → *uname*, *twittered*

twitterid → *uid*

uid, *gid* → *fromDate*

UserJoinsGroup (*uid*, *uname*, *twitterid*, *gid*, *fromDate*)

Another example

$\mathcal{F}$ includes:

uid → *uname*, *twitterid*
twitterid → *uid*
uid, *gid* → *fromDate*

UserJoinsGroup (*uid*, *uname*, *twitterid*, *gid*, *fromDate*)

BCNF violation: *uid* → *uname*, *twitterid*

{uid}+={uid, uname, twitterid}

User (*uid*, *uname*, *twitterid*)

uid → *uname*, *twitterid*
twitterid → *uid*

BCNF

{uid}+={uid, uname, twitterid}
{twitterid} +={uid, uname, twitterid}

Member (*uid*, *gid*, *fromDate*)

uid, *gid* → *fromDate*

BCNF

{uid,gid}+={uid,gid ,fromeDate}

Alt. solution

$\mathcal{F}$ includes:
- *uid → uname, twitterid*
- *twitterid → uid*
- *uid, gid → fromDate*

UserJoinsGroup (uid, uname, twitterid, gid, fromDate)

BCNF violation: *twitterid → uid*

UserId (twitterid, uid) — *twitterid → uid*

BCNF

No FDs in $\mathcal{F}$ violate BCNF here! (as uid is missing in this relation)

UserJoinsGroup (twitterid, uname, gid, fromDate)

But we need to check all the FDs in $\mathcal{F}^+$!!

twitterid → uname
twitterid, gid → fromDate

BCNF violation: *twitterid → uname*

UserName (twitterid, uname)

BCNF

Member (twitterid, gid, fromDate)

BCNF

"Good" Schema Decomposition

- Lossless-join decompositions
- Dependency-preserving decompositions
- BCNF → guaranteed to be a lossless join decomposition!
    - Depends on the on the sequence of FDs for decomposition
    - Not necessarily dependency preserving

Example: consider R={A, B, C}

$\mathcal{F}$ includes: FD1: $AB \rightarrow C$ FD2: C $\rightarrow B$

BCNF violation: C $\rightarrow B$

{A, C} {C, B}

$AB \rightarrow C$ is interrelational and cannot be tested directly

"Good" Schema Decomposition

- Lossless-join decompositions
- Dependency-preserving decompositions
- BCNF → guaranteed to be a lossless join decomposition!
    - Depend on the on the sequence of FDs for decomposition
    - Not necessarily dependency preserving
- 3NF → both lossless join and dependency preserving

Third normal form (3NF)

- A relation R is in 3NF iff whenever $(X \rightarrow Y) \in \mathcal{F}^+$ and $XY \subseteq R$, then either
  - $(X \rightarrow Y)$ is trivial (i.e., $Y \subseteq X$), or
  - X is a super key of R (i.e., $X \rightarrow R$) or
  - **Each attribute in $\mathbf{Y} - \boldsymbol{X}$ is contained in a candidate key of $\boldsymbol{R}$**

  - Example: consider R={A, B, C}
    - Satisfies 3NF, but not BCNF

$\mathcal{F}$ includes: FD1: $AB \rightarrow C$ FD2: $C \rightarrow B$

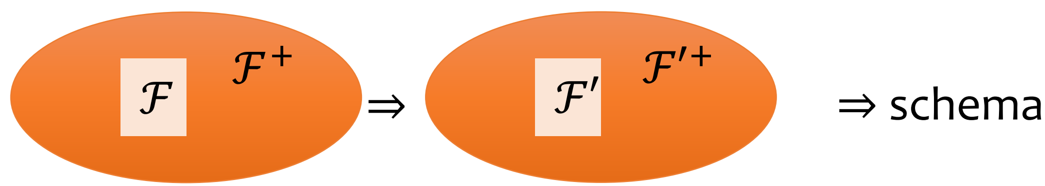
{B}-{C} = {B} is part of the key {AB}

- 3NF is looser than BCNF → Allows more redundancy

How to find a 3NF relation schemas?

- Find a way for lossless-join, dependency-preserving decomposition

  - Step 1: Finding the minimal cover of the FD set $\mathcal{F}$



  Given a set of FDs $\mathcal{F}$, we say $\mathcal{F}'$ is equivalent to $\mathcal{F}$ if their closures are the same: $\mathcal{F}^+ = \mathcal{F}'^+$.

  - Step 2: Decompose based on the minimal cover (i.e., $\mathcal{F}'$ is minimal).

Minimal cover

- A set of FDs $\mathcal{F}$ is minimal if

1. every right-hand side of a FD in $\mathcal{F}$ is a single attribute

- Example: $R = \{Sno, Sname, City, Pno, Pname, Price, PType\}$

$\mathcal{F}$: FD1: $Sno \rightarrow Sname, City$

FD2: $Pno \rightarrow Pname$

FD3: $Sno, Pno \rightarrow Price$

FD4: $Sno, Pname \rightarrow Price$

FD5: $Pno, Pname \rightarrow Ptype$

Fail condition 1

Minimal cover

No redundant attributes in X

- A set of FDs $\mathcal{F}$ is minimal if

1. every right-hand side of a FD in $\mathcal{F}$ is a single attribute
2. there does not exist $X \rightarrow A$, and Z a proper subset of X, such that the set $(\mathcal{F} - \{X \rightarrow A\}) \cup \{Z \rightarrow A\}$ is equivalent to F,
   English: no extraneous (redundant) attributes in the left-hand side of an FD in F

- Example: $R = \{Sno, Sname, City, Pno, Pname, Price, PType\}$

$\mathcal{F}$: FD1: $Sno \rightarrow Sname, City$
FD2: $Pno \rightarrow Pname$
FD3: $Sno, Pno \rightarrow Price$
FD4: $Sno, Pname \rightarrow Price$
FD5: $Pno, Pname \rightarrow Ptype$

Fail condition 2: replace by
FD5': Pno → Ptype
$(\mathcal{F} - \{FD5\} + \{FD5'\})$ is equiv. to $\mathcal{F}$

$computeX^{+}(\{Pno\}, \{FD1, FD2, FD3, FD4, FD5\}) = \{\ldots, Ptype, \ldots\}$
[visit last lecture for how to compute closure]

Minimal cover

- A set of FDs $\mathcal{F}$ is minimal if

1. Every right-hand side of a FD in $\mathcal{F}$ is a single attribute
2. There does not exist X → A and Z a proper subset of X, such th
   $(\mathcal{F} - \{X \rightarrow A\}) \cup \{Z \rightarrow A\}$ is equivalent to F,
   English: no extraneous (redundant) attributes in the left-hand side of a FD in F
3. There does not exist $X \rightarrow A$ in $\mathcal{F}$, such that $\mathcal{F} - \{X \rightarrow A\}$ equivalent to $\mathcal{F}$

No redundant FD in $\mathcal{F}$

Example: $R = \{Sno, Sname, City, Pno, Pname, Price, PType\}$

$\mathcal{F}$: FD1: $Sno \rightarrow Sname, City$
FD2: $Pno \rightarrow Pname$
FD3: $Sno, Pno \rightarrow Price$
FD4: $Sno, Pname \rightarrow Price$
FD5: $Pno, Pname \rightarrow Ptype$

Fail condition 3: FD2+FD4 can give FD3
$(\mathcal{F} - \{\text{FD3}\})$ is equiv. to $\mathcal{F}$

$computeX^{+}$({Sno, Pno}, {FD1,FD2,FD4,FD5})
= {… , Price, … }

Finding minimal cover

- A minimal cover for $\mathcal{F}$ can be computed in 3 steps.
  1. Replace $X \to YZ$ with the pair $X \to Y$ and $X \to Z$
  2. Remove A from the left-hand side of $X \to B$ in $\mathcal{F}$ if B $\in$ $computeX^{+}(X - \{A\}, \mathcal{F})$
  3. Remove $X \to A$ from $\mathcal{F}$ if $A \in computeX^{+}(X, \mathcal{F} - \{X \to A\})$
  - Note that each step must be repeated until it no longer succeeds in updating $\mathcal{F}$.

- Example: $R = \{Sno, Sname, City, Pno, Pname, Price, PType\}$

$\mathcal{F}$: FD1: $Sno \to Sname, City$
FD2: $Pno \to Pname$
FD3: $Sno, Pno \to Price$
FD4: $Sno, Pname \to Price$
FD5: $Pno, Pname \to Ptype$

FD1 → $Sno \to Sname$, $Sno \to City$

FD3 → Remove FD3

FD5 → $Pno \to Ptype$

Computing 3NF decomposition

Efficient algorithm for computing a 3NF decomposition of R with FDs $\mathcal{F}$:

1. Initialize the decomposition with empty set
2. Find a minimal cover for $\mathcal{F}$, let it be $\mathcal{F}^*$
3. For every $(\mathrm{X} \rightarrow \mathrm{Y}) \in \mathcal{F}^*$, add a relation {XY} to the decomposition
4. If no relation contains a candidate key for R, then compute a candidate key K for R, and add relation {K} to the decomposition.

Example for 3NF decomposition

- $R = \{Sno, Sname, City, Pno, Pname, Price\}$

$\mathcal{F}$: FD1: $Sno \rightarrow Sname, City$
FD2: $Pno \rightarrow Pname$
FD3: $Sno, Pno \rightarrow Price$
FD4: $Sno, Pname \rightarrow Price$

- Minimal cover $\mathcal{F}^*$

Exercise

$\mathcal{F}^*$: FD1a: $Sno \rightarrow Sname$
FD1b: $Sno \rightarrow City$
FD2: $Pno \rightarrow Pname$
FD4: $Sno, Pname \rightarrow Price$

R1a(Sno, Sname)
R1b(Sno, City)
R2(Pno, Pname)
R4(Sno,Pname,Price)

R5(Sno,Pno)

Exercise

- Add relation for candidate key
- Optimization for this example: combine relations R1a and R1b

Summary

- Functional dependencies: provide clues towards elimination of (some) redundancies in a schema.
    - Closure of FDs (rules, e.g. Armstrong's axioms)
    - Compute attribute closure
- Schema decomposition
    - Lossless join decompositions
    - Dependency preserving decompositions
    - Normal forms based on FDs
        - BCNF → lossless join decompositions
        - 3rd NF → lossless join and dependency-preserving decompositions with more redundancy