Relational Database Design Theory (I)

[additional exercises]

CS348 Spring 2023
Exercises for Attribute closure

• The closure of attributes $Z$ in a relation $R$ (denoted $Z^+$) with respect to a set of FDs, $\mathcal{F}$, is the set of all attributes $\{A_1, A_2, \ldots\}$ functionally determined by $Z$ (that is, $Z \rightarrow A_1A_2 \ldots$)

• Algorithm for computing the closure

  **Compute$Z^+(Z, \mathcal{F})$:**
  
  • Start with closure $= Z$
  • If $X \rightarrow Y$ is in $\mathcal{F}$ and $X$ is already in the closure, then also add $Y$ to the closure
  • Repeat until no new attributes can be added
Example for computing attribute closure

Consider the schema of a table EmpProj and the FDs:

\[ F \text{ includes:} \]
\[ SIN, PNum \rightarrow Hours \]
\[ SIN \rightarrow EName \]
\[ PNum \rightarrow PName, PLoc \]
\[ PLoc, Hours \rightarrow Allowance \]
Example for computing attribute closure

Compute $Z^+ \left( \{PNum, Hours\}, \mathcal{F} \right)$:

$\mathcal{F}$ includes:
- SIN, PNum $\rightarrow$ Hours
- SIN $\rightarrow$ EName
- PNum $\rightarrow$ PName, PLoc
- PLoc, Hours $\rightarrow$ Allowance

<table>
<thead>
<tr>
<th>FD</th>
<th>$Z^+$</th>
</tr>
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<tbody>
<tr>
<td>initial</td>
<td>PNum, Hours</td>
</tr>
<tr>
<td>PNum $\rightarrow$ PName, PLoc</td>
<td>PNum, Hours, PName, PLoc</td>
</tr>
<tr>
<td>PLoc, Hours $\rightarrow$ Allowance</td>
<td>PNum, Hours, PName, PLoc, Allowance</td>
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$PNum, Hours \rightarrow PName, PLoc, Allowance$
Using attribute closure

Given a relation $R$ and set of FD’s $\mathcal{F}$

• Does another FD $X \rightarrow Y$ follow from $\mathcal{F}$?
  • Compute $X^+$ with respect to $\mathcal{F}$
  • If $Y \subseteq X^+$, then $X \rightarrow Y$ follows from $\mathcal{F}$

• Is $K$ a key of $R$?
  • Compute $K^+$ with respect to $\mathcal{F}$
  • If $K^+$ contains all the attributes of $R$, $K$ is a super key
  • Still need to verify that $K$ is minimal (how?)
    • Hint: check the attribute closure of its proper subset.
    • i.e., Check that for no set $X$ formed by removing attributes from $K$ is $K^+$ the set of all attributes
Exercise I

Compute \( Z^+ (\{SIN, PNum\}, \mathcal{F}) \):

\[\mathcal{F} \text{ includes:}
\begin{align*}
SIN, PNum & \rightarrow \text{Hours} \\
SIN & \rightarrow \text{EName} \\
PNum & \rightarrow \text{PName}, PLoc \\
\text{PLoc}, \text{Hours} & \rightarrow \text{Allowance}
\end{align*}\]

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<tr>
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<td>( {SIN, PNum, \text{EName}} )</td>
</tr>
<tr>
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Compute \( Z^+ (\{SIN, PNum, Hours\}, \mathcal{F}) \)?

\( \{SIN, PNum, Hours, \text{EName}, \text{PName}, \text{PLoc}, \text{Allowance}\} \)

\( \rightarrow \text{A candidate key (why?)} \)
Exercise II

• R(A,B,C)
• F includes
  • FD1: A \rightarrow B
  • FD2: B \rightarrow C
  • FD3: A \rightarrow C
• ComputeZ^+({A}, F) = ?
  • {A,B,C}
• ComputeZ^+({B}, F) = ?
  • {B,C}
• ComputeZ^+({A,B,C}, F) = ?
  • {A,B,C}

• Super keys for R?
  • A, AB, AC, ABC
• Candidate keys for R?
  • A
Exercise III

• R(A,B,C)
• F includes
  • FD1: A \rightarrow B
• Compute Z^+({A}, F) = ?
  • \{A,B\}
• Super keys for R?
  • AC, ABC
• Candidate keys for R?
  • AC