Intro to the Relational Model

CS348 Spring 2024

Instructor: Sujaya Maiyya

Sections: **002 and 003 only**

Outline

- Part 1: Relational data model

- Part 2: Relational algebra

Relational data model

Modeling data as **relations** or **tables**, each storing logically related information together

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	A Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

relations (or tables)

Attributes

Group

gid	*name*
abc	A Book Club
gov	Student Government
dps	Dead Putting Society
…	…

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Member

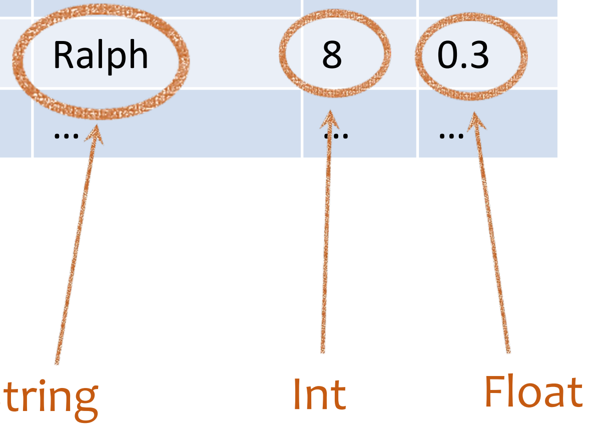
uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

attributes (or columns)

Domain

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



Group

gid	*name*
abc	A Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

domain (or type)

Tuples

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	A Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

tuples (or rows)

Duplicates (all attr. have same val) are not allowed

Ordering of rows doesn’t matter
(even though output can be ordered)

Set representation of tuples

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	A Book Club
gov	Student Government
edu	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

```
User: {⟨142, Bart, 10, 0.9⟩,
       ⟨857, Milhouse, 10, 0.2⟩, …}
Group: {⟨abc, A Book Club⟩,
        ⟨gov, Student Government⟩, …}
Member: {⟨142, dps⟩, ⟨123, gov⟩, …}
```

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a unique name and a domain (or type)
  - The domains are required to be **atomic**

Single, indivisible piece of information

- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

☞Simplicity is a virtue!

Schema vs. instance

- Schema (metadata)
  - Specifies the logical structure of data
  - Is defined at setup time, rarely changes

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

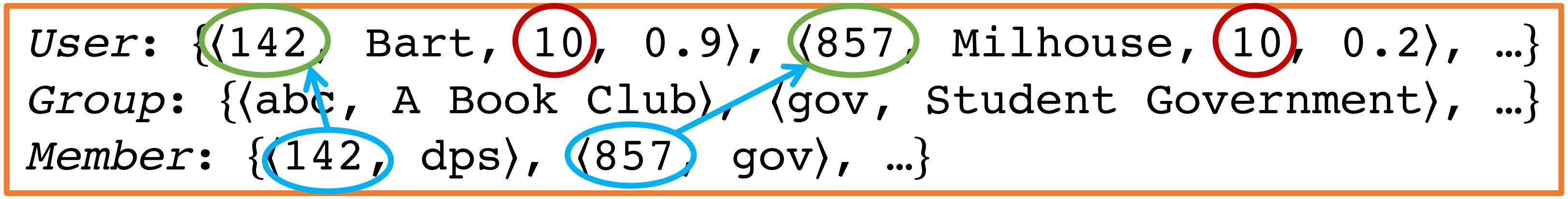
- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema
  - Typically has additional rules

User: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, …}
Group: {⟨abc, A Book Club⟩, ⟨gov, Student Government⟩, …}
Member: {⟨142, dps⟩, ⟨123, gov⟩, …}

Integrity constraints

- A set of rules that database instances should follow
- Example:
  - *age* cannot be negative
  - *uid* should be unique in the *User* relation
  - *uid* in *Member* must refer to a row in *User*

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)



User: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, …}
Group: {⟨abc, A Book Club⟩, ⟨gov, Student Government⟩, …}
Member: {⟨142, dps⟩, ⟨857, gov⟩, …}

Integrity constraints

- An instance is only valid if it follows the schema and satisfies all the integrity constraints.

- Reasons to use constraints:
  - Address consistency challenges
    (last class: duplicate entry for Bob)
  - Ensure data entry/modification respects to database design
  - Protect data from bugs in applications

Types of integrity constraints

- Tuple-level
  - Domain restrictions, attribute comparisons, etc.
    - E.g. *age* cannot be negative
    - E.g. for flights table, arrival time > take off time
- Relation-level
  - Key constraints (focus in this lecture)
    - E.g. *uid* should be unique in the *User* relation
  - Functional dependencies (week 5/6)
- Database-level
  - Referential integrity – foreign key (focus in this lecture)
    - *uid* in *Member* must refer to a row in *User* with the same *uid*

Key (Candidate Key)

Def: A set of attributes K for a relation R if

- Condition 1: In no instance of R will two different tuples agree on all attributes of K
  - That is, K can serve as a "tuple identifier"
- Condition 2: No proper subset of K satisfies the above condition
  - That is, K is minimal

- Example: *User* (*uid*, *name*, *age*, *pop*)
  - *uid* is a key of *User*
  - *age* is not a key (not an identifier)
  - {*uid*, *name*} is not a key (not minimal), but a superkey

Satisfies only Condition 1

Key (Candidate key)

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3

- Is *name* a key of *User*?
    - Yes? Seems reasonable for this instance
    - No! User names are not unique in general
- Key declarations are part of the schema

More examples of keys

- *Member (uid, gid)*

- Only **uid**?
  - No, because of repeated entries

- Only **gid**?
  - No, again due to repeated entries

- Use both!
  - *{uid, gid}*

  ☞**A key can contain multiple attributes**

Member

uid	*gid*
142	dps
123	gov
857	abc
456	gov
857	dps
256	gov
…	…

More examples of keys

- *Address* (*street_address*, *city*, *province*, *zip*)
  - Key 1: {*street_address*, *city*, *province*}
  - Key 2: {*street_address*, *zip*}

  ☞ A relation can have multiple keys!

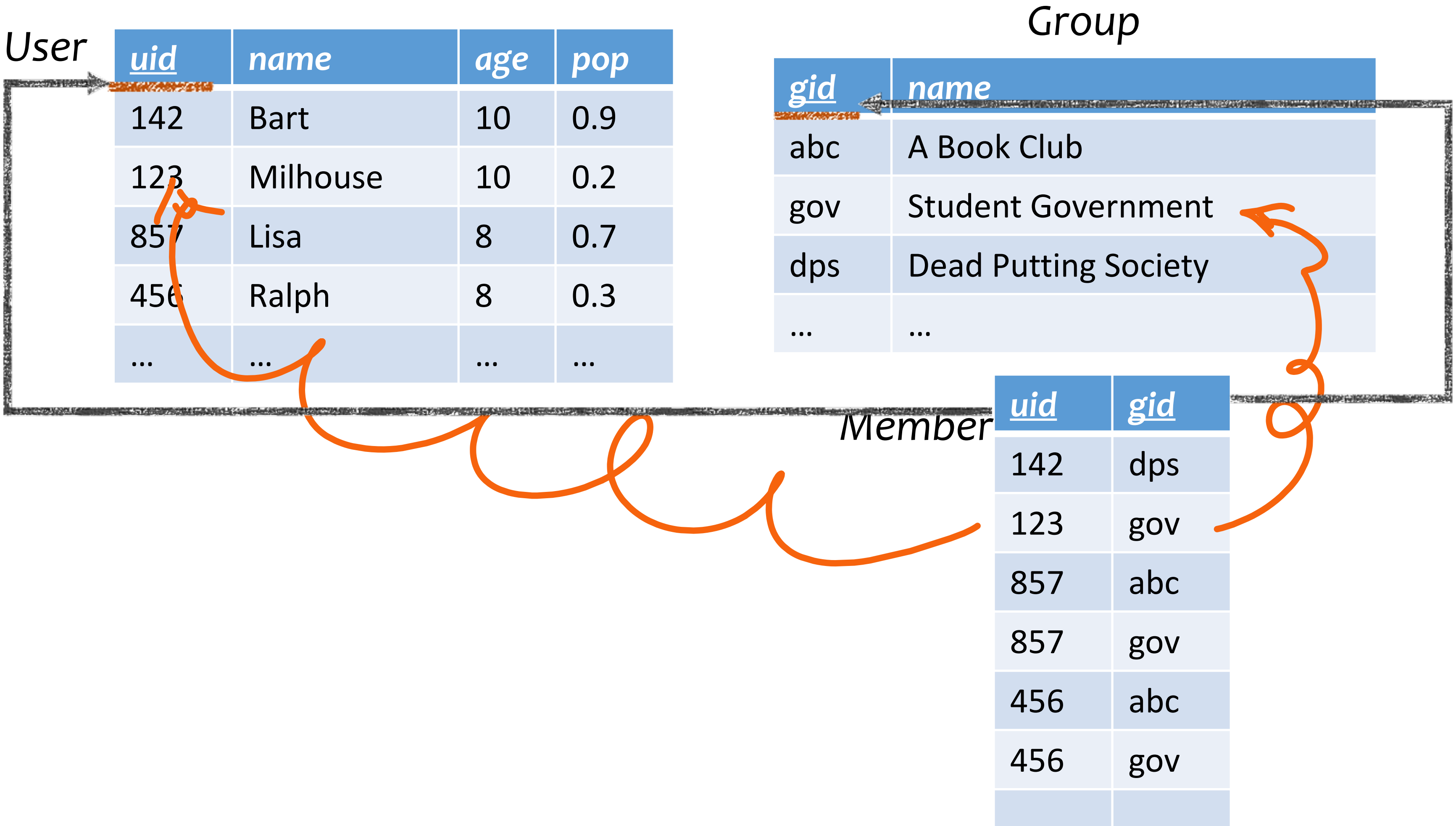
- Primary key: a designated candidate key in the schema declaration
  - Underline all its attributes, e.g., *Address* (*street_address*, *city*, *province*, *zip*)

Use of keys

- More constraints on data, fewer mistakes
- Look up a row by its key value
  - Many selection conditions are “key = value”
- “Pointers” to other rows (often across tables)

“Pointers” to other rows

- Foreign key: primary key of one relation appearing as attribute of another relation



User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

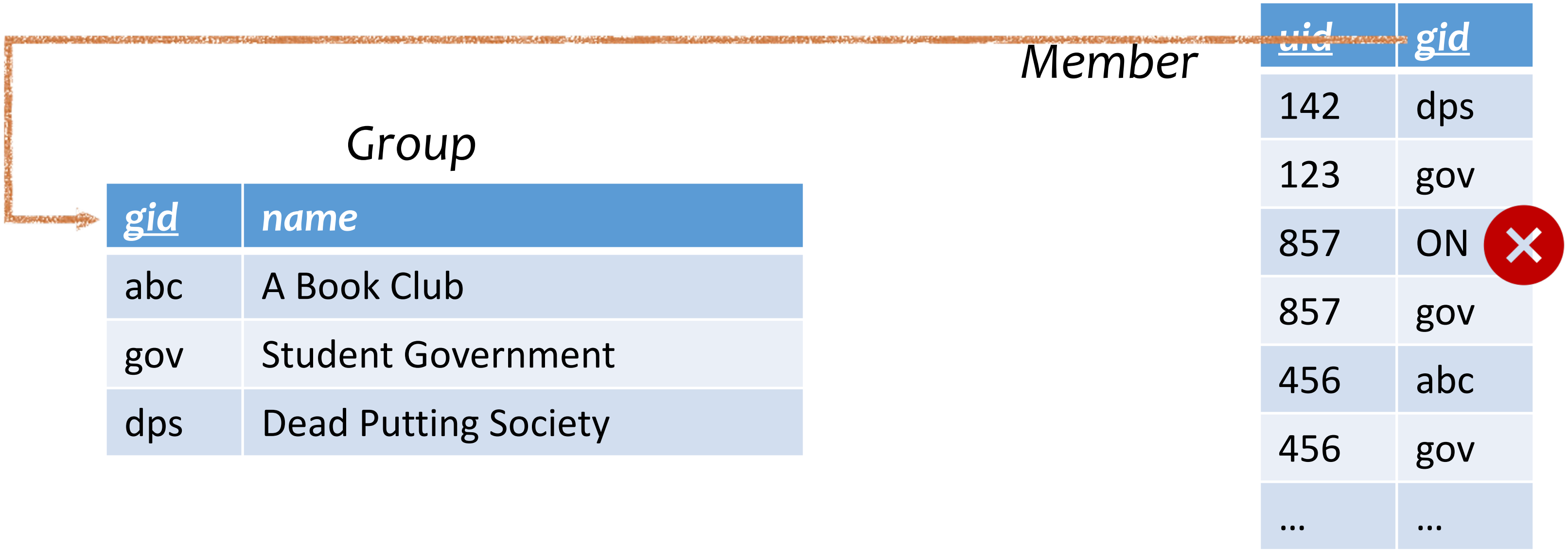
gid	*name*
abc	A Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov

"Pointers" to other rows

- Referential integrity: A tuple with a non-null value for a foreign key **must** match the primary key value of a tuple in the referenced relation



Group

gid	*name*
abc	A Book Club
gov	Student Government
dps	Dead Putting Society

Member

uid	gid
142	dps
123	gov
857	ON ✗
857	gov
456	abc
456	gov
…	…

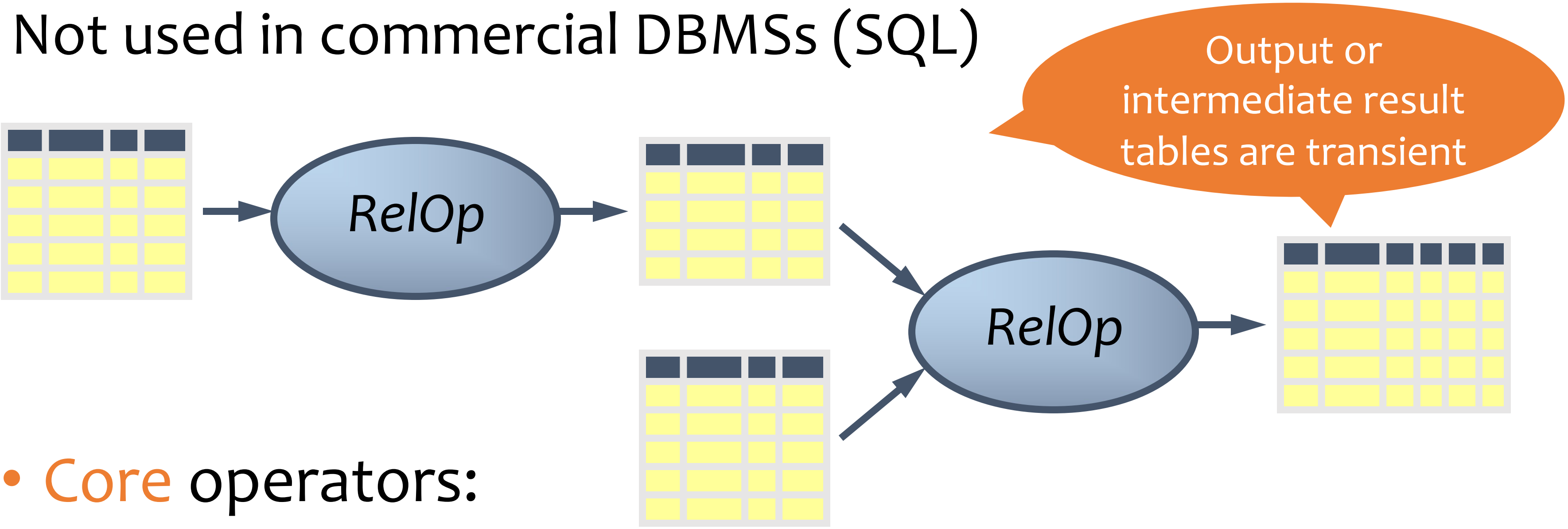
Referential integrity violation!

Outline

- Part 1: Relational data model
  - Data model
  - Database schema
  - Integrity constraints (keys)
  - Languages
    - Relational algebra (focus in this lecture)
    - SQL (next lecture)
    - Relational calculus (textbook, Ch. 27)
- Part 2: Relational algebra

Relational algebra

- A language for querying relational data based on "operators"
- Not used in commercial DBMSs (SQL)



Output or intermediate result tables are transient

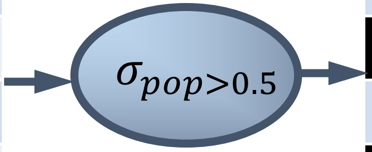
- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Core operator 1: Selection σ

- Example query: Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



uid	*name*	*age*	*pop*
142	Bart	10	0.9
[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7
[redacted]	[redacted]	[redacted]	[redacted]
…	…	…	…

Core operator 1: Selection

- Input: a table R
- Notation: $\sigma_p R$
  - p is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows of R that satisfy p

More on selection

- Selection condition can include any column of R, constants, comparison ($=$, $\leq$, etc.) and Boolean connectives (∧: and, ∨: or, ¬: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \land (age < 10 \lor age > 12)}\ User$$

- You must be able to evaluate the condition over each single row of the input table!
  - Example: the most popular user

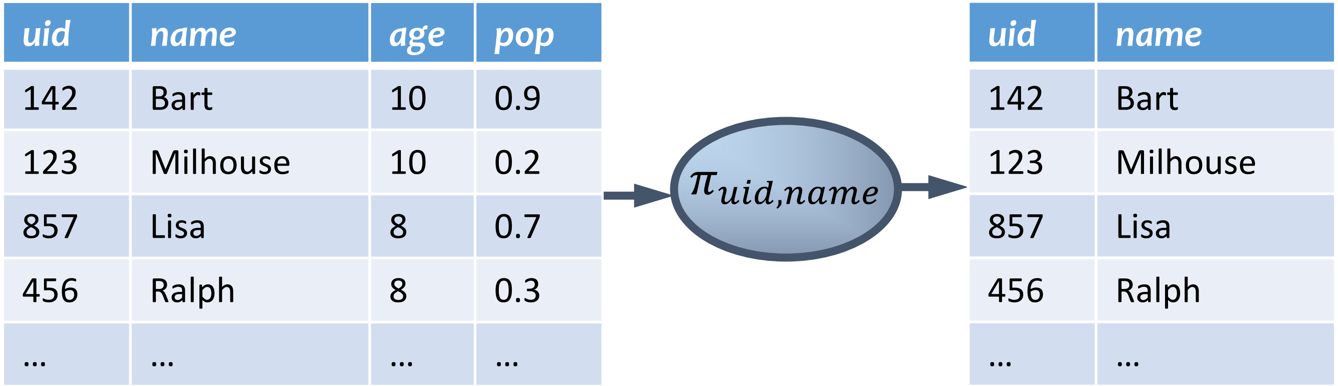
$$\sigma_{pop \geq every\ pop\ in\ User}\ User$$

WRONG!

Core operator 2: Projection π

- Example: IDs and names of all users

$$\pi_{uid,name}\ User$$



uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$\pi_{uid,name}$

uid	*name*
142	Bart
123	Milhouse
857	Lisa
456	Ralph
…	…

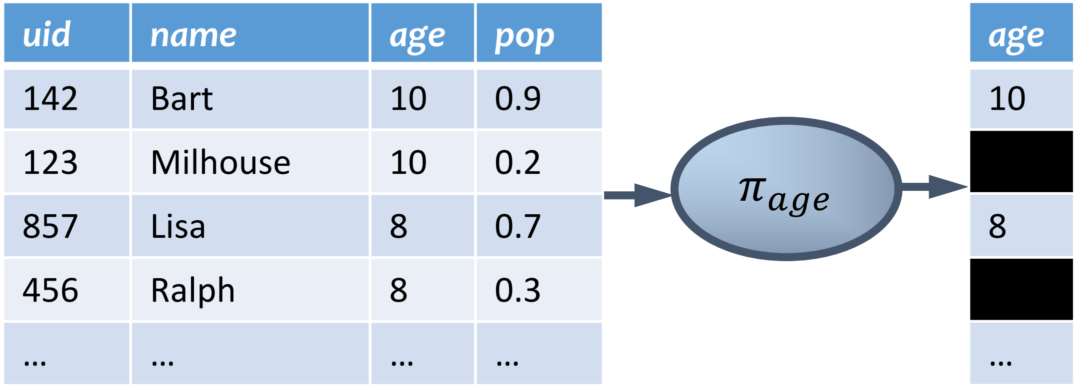
Core operator 2: Projection

- Input: a table R
- Notation: $\pi_L R$
  - L is a list of columns in R
- Purpose: output chosen columns
- Output: “same” rows, but only the columns in L

More on projection

- Duplicate output rows are removed (by definition)
    - Example: user ages

$$\pi_{age}\ User$$

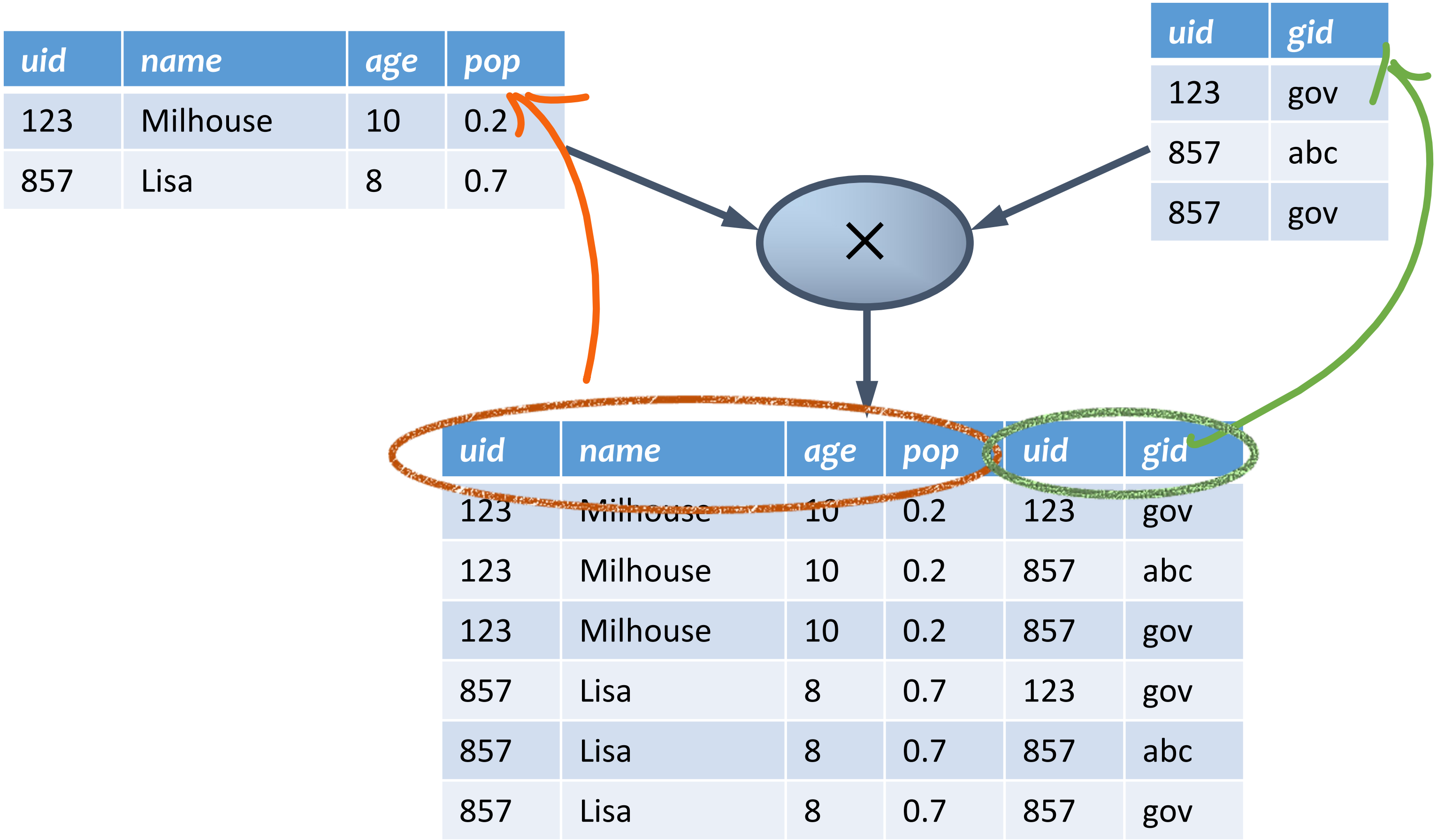


uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

age
10
■
8
■
…

Core operator 3: Cross product ×

$$User \times Member$$



uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7

uid	gid
123	gov
857	abc
857	gov

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov

Core operator 3: Cross product

- Input: two tables R and S
- Notation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each s in S, output a row rs (concatenation of r and s)

A note on column ordering

- Ordering of columns is unimportant as far as contents are concerned

uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

=

uid	*gid*	*uid*	*name*	*age*	*pop*
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
…	…	…	…	…	…

- So cross product is commutative, i.e., for any R and S, $R \times S = S \times R$ (up to the ordering of columns)

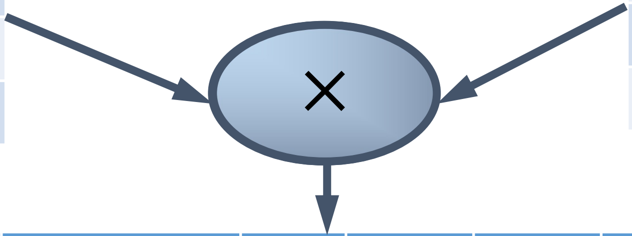
Derived operator 1: Join ⋈

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	*gid*
123	gov
857	abc
857	gov
…	…



uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

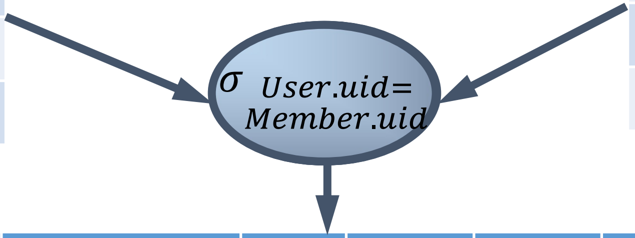
Derived operator 1: Join ⋈

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	*gid*
123	gov
857	abc
857	gov
…	…



$\sigma_{User.uid=Member.uid}$

uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

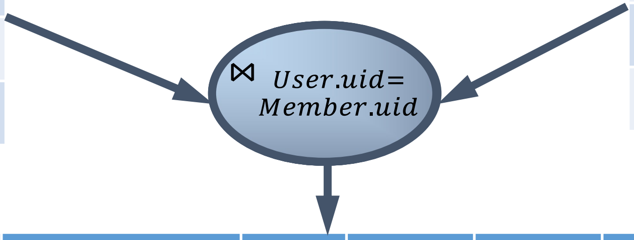
Derived operator 1: Join ⋈

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	*gid*
123	gov
857	abc
857	gov
…	…



Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

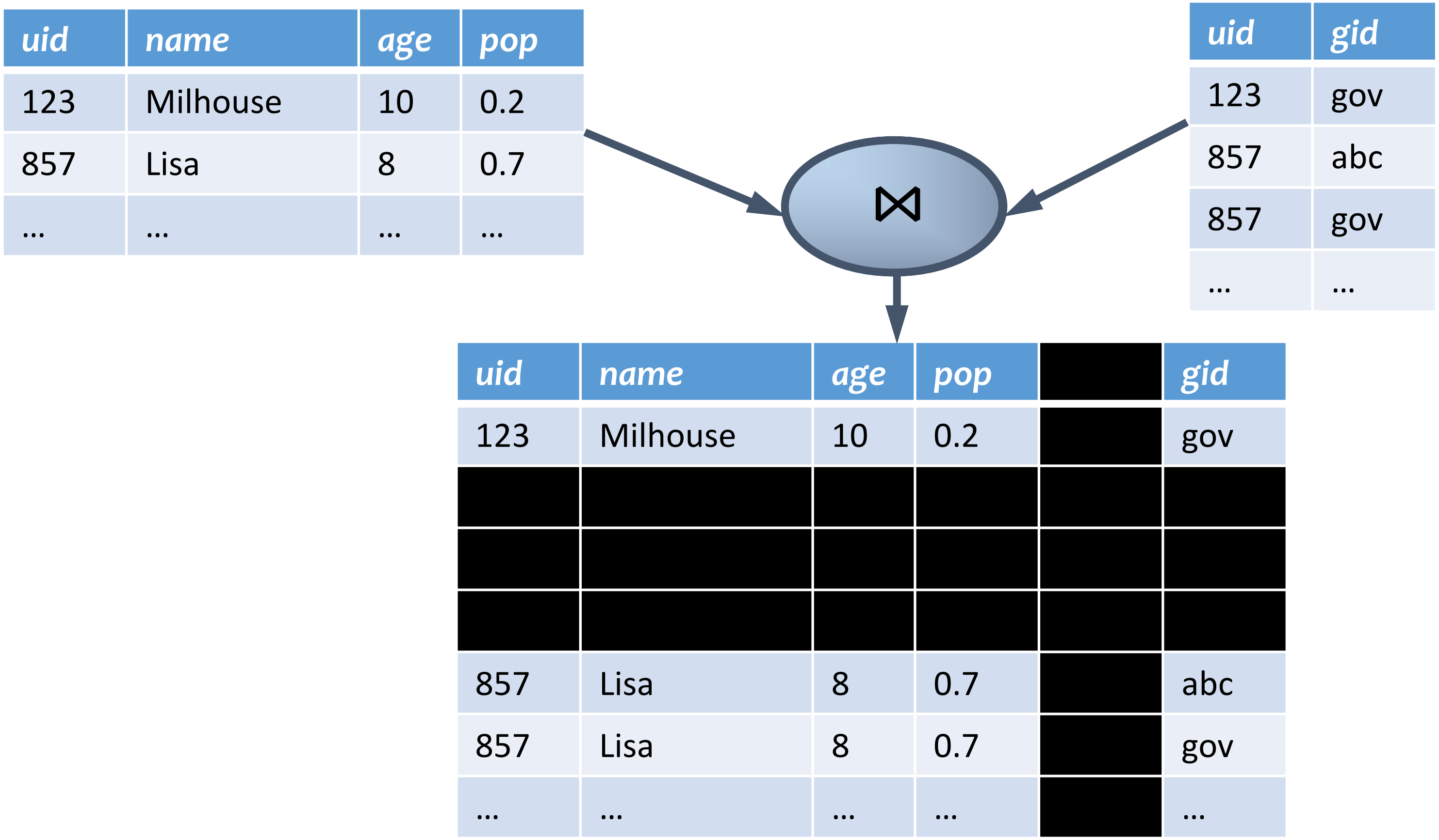
uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

Derived operator 1: Join

- Input: two tables R and S
- Notation: $R \bowtie_p S$
    - p is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$
- (A.k.a. “theta-join”)

Derived operator 2: Natural join

$$User \bowtie Member$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{\substack{User.uid=\\Member.uid}} Member\right)$$



uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	*gid*
123	gov
857	abc
857	gov
…	…

uid	*name*	*age*	*pop*	[redacted]	*gid*
123	Milhouse	10	0.2	[redacted]	gov
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7	[redacted]	abc
857	Lisa	8	0.7	[redacted]	gov
…	…	…	…	[redacted]	…

Derived operator 2: Natural join

- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L\left(R \bowtie_p S\right)$, where
  - p equates each pair of columns common to R and S
  - L is the union of column names from R and S (with duplicate columns removed)

Core operator 4: Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S (with duplicate rows removed)

uid	*gid*
123	gov
857	abc

$\cup$

uid	*gid*
123	gov
901	edf

=

uid	*gid*
123	gov
857	abc
901	edf

Core operator 5: Difference

- Input: two tables R and S
- Notation: $R - S$
    - R and S must have identical schema
- Output:
    - Has the same schema as R and S
    - Contains all rows in R that are not in S

uid	*gid*
123	gov
857	abc

−

uid	*gid*
123	gov
901	edf

=

uid	*gid*
857	abc

Derived operator 3: Intersection

- Input: two tables R and S
- Notation: $R \cap S$
    - R and S must have identical schema
- Output:
    - Has the same schema as R and S
    - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

1. Find tuples in R not in S
2. Remove those tuples from R

Core operator 6: Renaming

- Input: a table (or an expression) R
- Notation: $\rho_S\ R$, $\rho_{(A_1 \to A_1', \ldots)} R$, or $\rho_{S(A_1 \to A_1', \ldots)} R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as R, but called differently

Member

uid	*gid*
123	gov
857	abc

$$\rho_{M1(uid \to uid_1, gid \to gid_1)} Member$$

M1

uid1	*gid1*
123	gov
857	abc

9. Core operator: Renaming

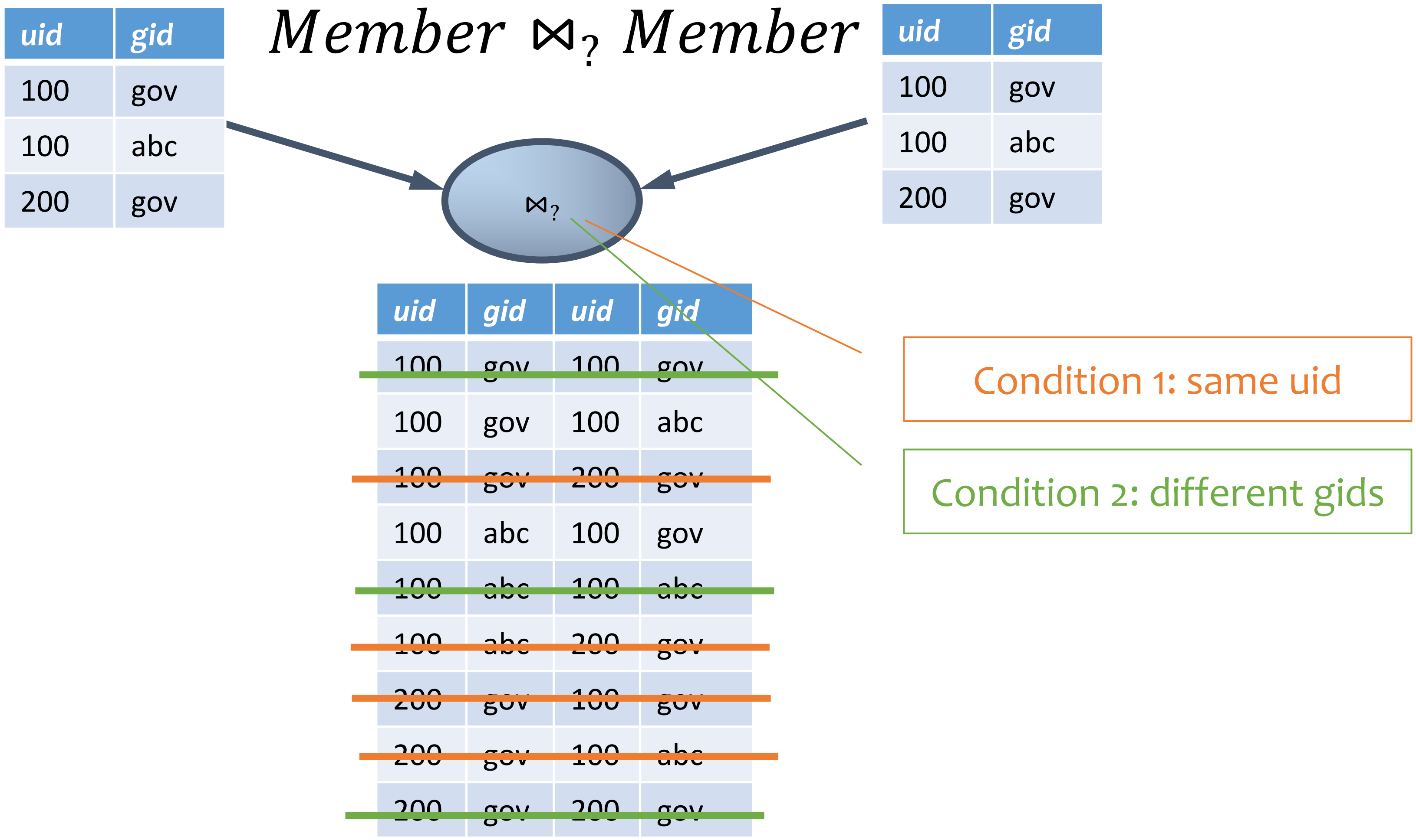
- As with all other relational operators, it doesn’t modify the database
  - Think of the renamed table as a copy of the original

- Used to: Avoid confusion caused by identical column names

9. Core operator: Renaming

- IDs of users who belong to at least two groups

$$Member \bowtie_? Member$$



uid	*gid*
100	gov
100	abc
200	gov

uid	*gid*
100	gov
100	abc
200	gov

uid	*gid*	*uid*	*gid*
~~100~~	~~gov~~	~~100~~	~~gov~~
100	gov	100	abc
~~100~~	~~gov~~	~~200~~	~~gov~~
100	abc	100	gov
~~100~~	~~abc~~	~~100~~	~~abc~~
~~100~~	~~abc~~	~~200~~	~~gov~~
~~200~~	~~gov~~	~~100~~	~~gov~~
~~200~~	~~gov~~	~~100~~	~~abc~~
~~200~~	~~gov~~	~~200~~	~~gov~~

Condition 1: same uid

Condition 2: different gids

Renaming example

- IDs of users who belong to at least two groups

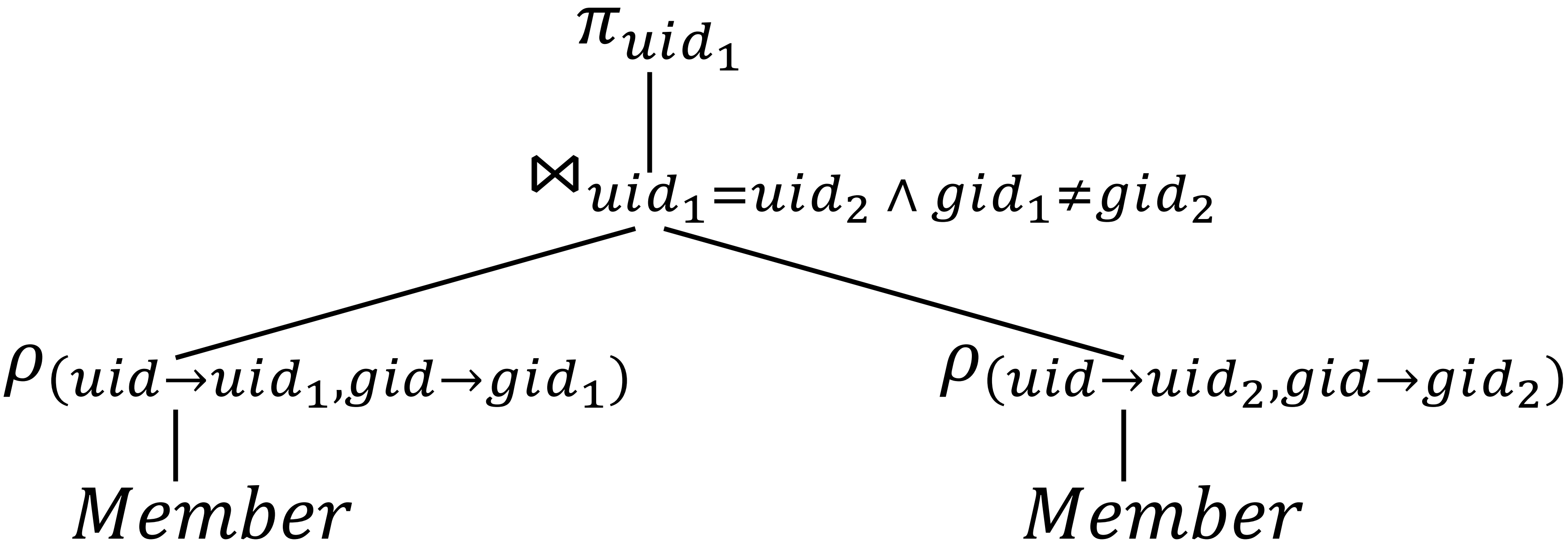
$$Member \bowtie_{?} Member$$

$$\pi_{uid}\left(Member \bowtie_{\substack{Member.uid=Member.uid \,\wedge \\ Member.gid \neq Member.gid}} Member\right)$$

WRONG!

$$\pi_{uid_1}\left(\begin{array}{c} \rho_{(uid\to uid_1, gid\to gid_1)} Member \\ \bowtie_{uid_1=uid_2 \,\wedge\, gid_1 \neq gid_2} \\ \rho_{(uid\to uid_2, gid\to gid_2)} Member \end{array}\right)$$

Expression tree notation



π_{uid_1}

$\bowtie_{uid_1=uid_2 \wedge gid_1 \neq gid_2}$

$\rho_{(uid \rightarrow uid_1, gid \rightarrow gid_1)}$

$\rho_{(uid \rightarrow uid_2, gid \rightarrow gid_2)}$

$Member$

$Member$

Take-home Exercises

- Exercise 1: IDs of groups who have at least 2 users?
- Exercise 2: IDs of users who belong to at least three groups?

Summary of operators

Core Operators

1. Selection: $\sigma_p R$
2. Projection: $\pi_L R$
3. Cross product: $R \times S$
4. Union: $R \cup S$
5. Difference: $R - S$
6. Renaming: $\rho_{S(A_1 \to A_1', A_2 \to A_2', \ldots)} R$

Derived Operators

1. Join: $R \bowtie_p S$
2. Natural join: $R \bowtie S$
3. Intersection: $R \cap S$

Note: Only use these operators for assignments & exams

More example

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- All groups (ids) that Lisa belongs to

More example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- All groups (ids) that Lisa belongs to

Writing a query bottom-up:

uid	name	age	pop
857	Lisa	8	0.7

Who’s Lisa? $\sigma_{name="Lisa"}$

$User$

$Member$

uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

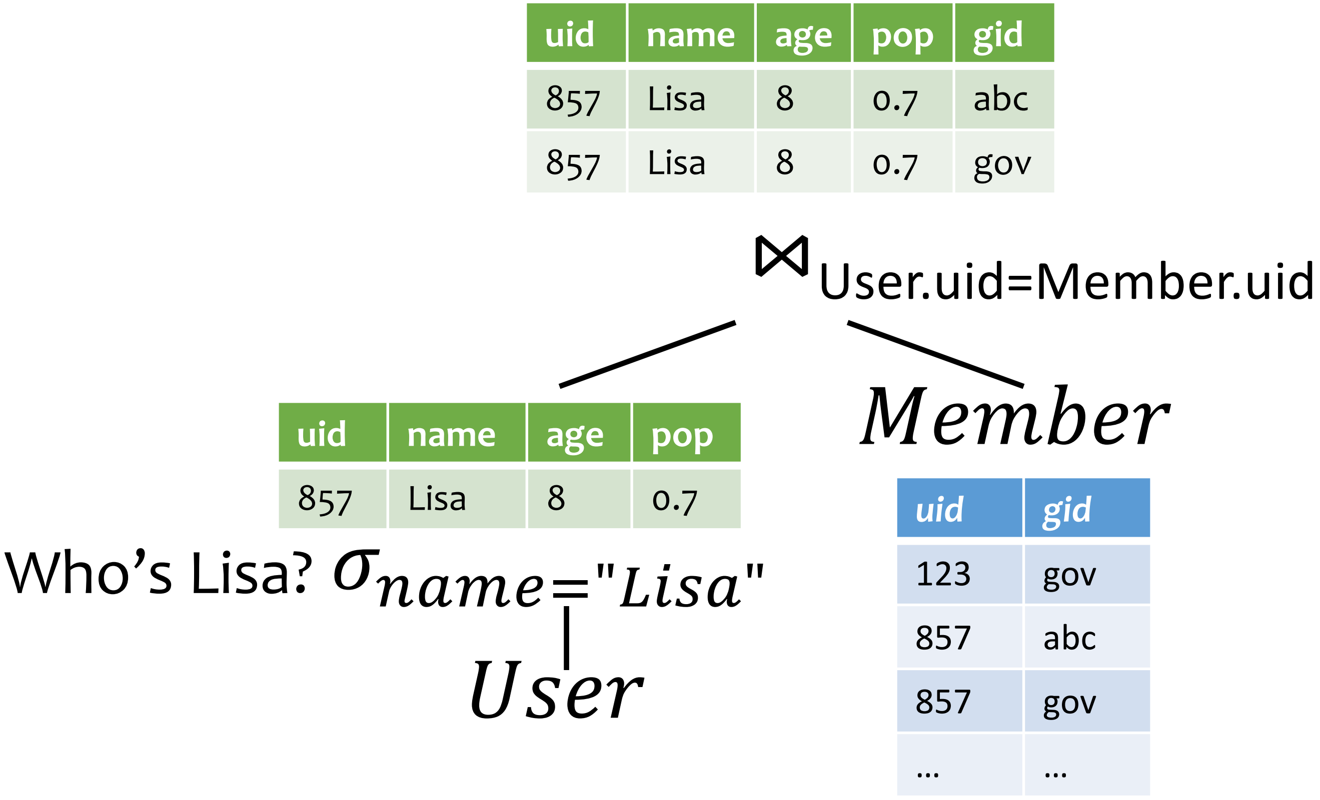
uid	*gid*
123	gov
857	abc
857	gov
…	…

More example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- All groups (ids) that Lisa belongs to

Writing a query bottom-up:



uid	name	age	pop	gid
857	Lisa	8	0.7	abc
857	Lisa	8	0.7	gov

$\bowtie$ User.uid=Member.uid

uid	name	age	pop
857	Lisa	8	0.7

Who's Lisa? $\sigma_{name="Lisa"}$

$User$

$Member$

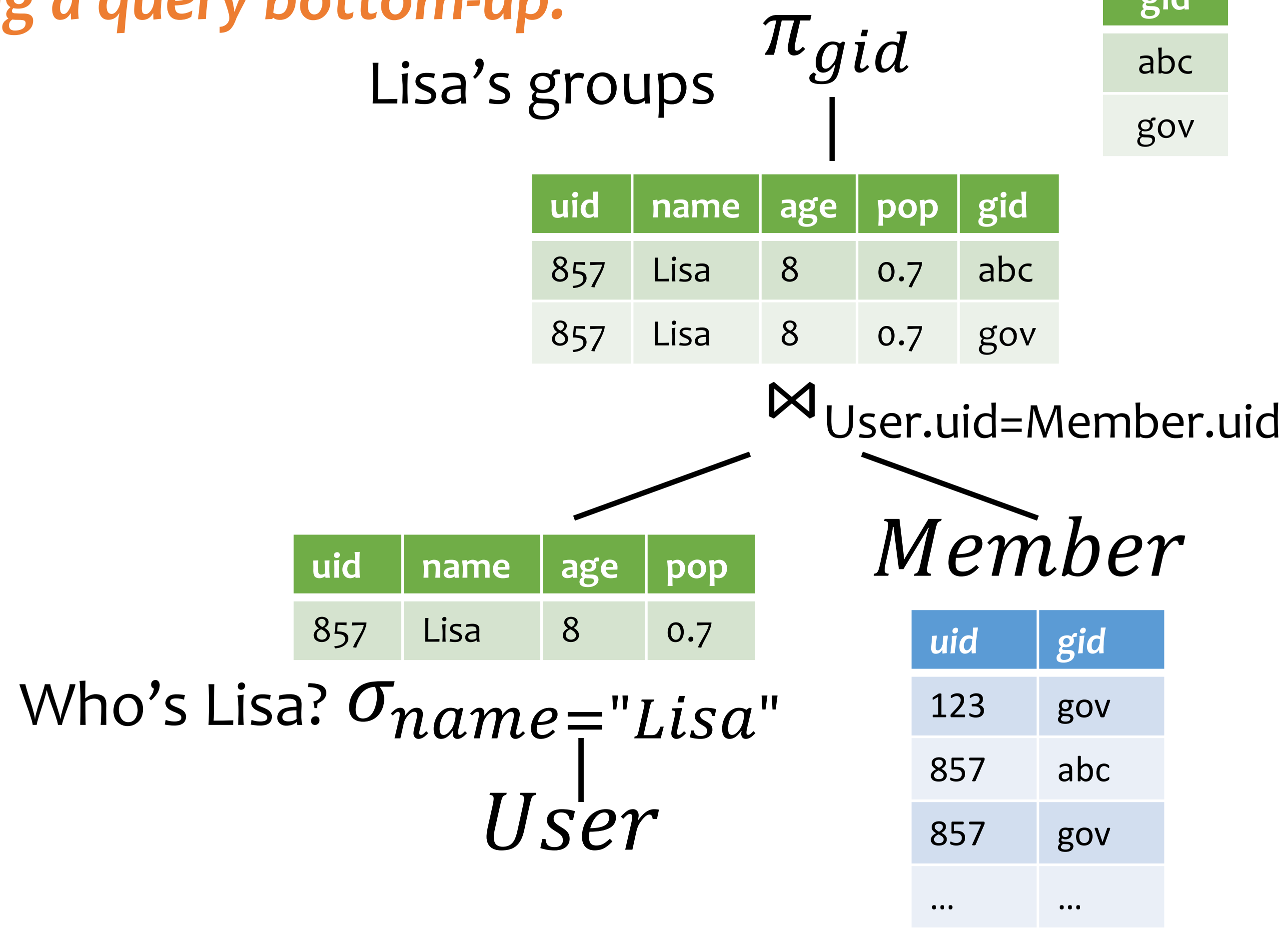
uid	gid
123	gov
857	abc
857	gov
…	…

More example

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- All groups (ids) that Lisa belongs to

Writing a query bottom-up:



Lisa's groups π_{gid}

gid
abc
gov

uid	name	age	pop	gid
857	Lisa	8	0.7	abc
857	Lisa	8	0.7	gov

$\bowtie$ User.uid=Member.uid

uid	name	age	pop
857	Lisa	8	0.7

Who's Lisa? $\sigma_{name="Lisa"}$

$User$

$Member$

uid	*gid*
123	gov
857	abc
857	gov
…	…

Take home ex.

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- All groups ~~(ids)~~ that Lisa belongs to
  names?

Summary

- Part 1: Relational data model
  - Data model
  - Database schema
  - Integrity constraints (keys)
  - Languages (relational algebra, relational calculus, SQL)

- Part 2: Relational algebra – basic language
  - Core operators & derived operators (how to write a query)

- What's next?
  - More examples in RA
  - Relational calculus
  - SQL