Review[4] of

Biscuits of Number Theory

Editors of book: Arthur T. Benjamin and Ezra Brown

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Review by Jeffrey Shallit[5]

Art Benjamin and Ezra Brown, editors of *Biscuits of Number Theory*, describe this book as follows: "an assortment of articles and notes on number theory, where each item is not too big, easily digested, and makes you feel all warm and fuzzy when you're through."

Benjamin teaches at Harvey Mudd College, and was named "America's best math whiz" by *Reader's Digest* in May 2005. He's also a professional magician who has appeared on many TV shows and National Public Radio. Brown teaches mathematics at Virginia Tech, and according to his web page, likes to bake his students actual biscuits.

Biscuits of Number Theory consists of 40 short articles copied from journals such as *Math Horizons*, *Mathematics Magazine*, *Mathematics Teacher*, and the *American Mathematical Monthly*. And when I say "copied", I mean it: some of the articles appear in exactly the same font as their original source — which means that typographically, the book feels a bit disorganized. Whether copied or not, errors from the original works seem to be faithfully preserved: read 257 for 251 on page 85, and replace U by $\bigcup$ on page 223 (twice).

The authors represented include some of the best expositors of elementary number theory: Peter Borwein, Stan Wagon, Carl Pomerance, Ivan Niven, Edward Burger, Ross Honsberger, and Martin Gardner, just to name a few. The articles are classified into seven different parts: arithmetic, primes, irrationality and continued fractions, sums of squares and polygonal numbers, Fibonacci numbers, number-theoretic functions, and elliptic curves and Fermat's last theorem.

Many of the chapters will be accessible to high school students or even bright junior high students. For example, chapter 3, entitled "Reducing the Sum of Two Fractions", explores the following simple question: sometimes when we add two fractions by putting them both over the same denominator l (the least common multiple of the two denominators), the resulting fraction is in lowest terms, and sometimes it isn't. For example, $\frac{4}{21} + \frac{7}{15} = \frac{69}{105}$ is not in lowest terms, but $\frac{7}{10} + \frac{11}{12}$ is. Can we characterize those pairs of denominators according to their behavior upon addition? Harris Shultz and Ray Shiflett show the answer is yes, and depends on the exponents of the prime factors dividing the denominators.

Some of the chapters are truly excellent. I particularly liked Henry Cohn's article, "A short proof of the simple continued fraction of e". Here Cohn shows how to derive the expansion

$$e = [2, 1, 2, 1, 1, 4, 1, 1, 6, \ldots, 1, 1, 2n, \ldots]$$

using nothing more than some simple integrals and the product rule for derivatives.

Other chapters will likely be very mysterious even for beginning graduate students. Furstenberg's topological proof of the infinitude of the primes (Chapter 8) will likely be incomprehensible for many students, as will the last article, about Fermat's last theorem. But that doesn't matter; it's *good* when a book has *some* content above the level of the typical reader, because this will

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intrigue some readers sufficiently that they'll feel the need to learn the required material. The challenge is to have the right amount, and my feeling is that this book has a good balance of material.

This book wouldn't make a good textbook, but could certainly be used as a supplement to an introductory undergraduate (or even high school) course on number theory.

Review[6] of

Combinatorial Geometry and Its Algorithmic Applications: The Alcalá Lectures
Author: János Pach and Micha Sharir
Publisher: American Mathematical Society, 2009
Mathematical Surveys and Monographs, Volume 152

Reviewer: Sergio Cabello

1 Overview

Elementary geometric structures, like points, lines, and planes give rise to a large number of fascinating combinatorial problems. Several of these problems naturally appear in the study of algorithms handling geometric data, and thus are motivated by algorithmic applications. Others are just intriguing: easy to state, but apparently hard to solve. Each of the nine chapters of this book constitutes a survey of some area of combinatorial geometry.

2 Summary of Contents

Chapter 1 is dedicated to combinatorial problems concerning points and the set of lines passing through at least two of them. The chapter starts with a historic perspective on Sylvester's problem: is it true that for any finite set of points in the plane, not all on a line, there exists a line that contains precisely two of them? Different extensions of this problem are then considered, namely how many such lines through precisely two points exist, how many different directions are determined by those lines, as well as versions where the points have two colors. A part of the chapter is dedicated to an extension of the problem studying the set of directions determined in 3-space.

Chapter 2 is dedicated to arrangements of surfaces and hypersurfaces, and it spans about 30% of the book. The arrangement of a set of geometric objects is the decomposition of the space into cells, where each cell is a maximal connected subset of points contained in the intersection of some subset of the objects. Although one has to read the definition a few times to understand it, it pays off: it is a fundamental structure used in most papers in computational geometry. The chapter starts with some motivating examples, followed by formal definitions and assumptions used in the study of arrangements. An arrangement has several important substructures: single cells, zones, levels, etc. A section of the chapter is devoted to each of these substructures. The chapter then describes computational aspects of constructing, storing, and manipulating arrangements and its substructures. The chapter concludes as it started, listing several applications, which well could be considered motivation.