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APL NEWS

APL 79

The APL 79 Conference sponsored by STAPL will be held in Rochester, New York, May 30 - June 1, 1979. The program will feature a number of invited speakers and, according to the Call for Papers, "will be devoted to all aspects of APL including: Applications in all areas; The language; Implementations; APL in education; Interfaces with other software systems; Interfaces with special purpose hardware; APL system organization and management; Relations to Lisp and other languages".

Again quoting from the Call for Papers, "Further information may be obtained from either the Program Chairman (Professor Paul Penfield, Jr., Room 38-401, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, Telephone 617-253-4607) or from the General Chairman, Fletcher McTaggart, I.P. Sharp Associates, Suite 1150, 183 Main Street East, Rochester, New York 14604, Telephone 716-546-7270.

CORRECTIONS

Professor Mauldon has submitted the following corrections to his note on Ultimately Periodically Self-Reproducing Expressions which appeared in Newsletter 6:

1. The final quote in the last APL expression should be deleted.
2. If `2≤⍴A` the result of `⍕A` varies slightly from system to system. Replacing each occurrence of the symbol `⍕` in the functions *SRE* and *SRE*2 by `2 0⍕` removes this difficulty.

NEWLY AVAILABLE FROM APL PRESS

Proceedings of An APL Users Meeting, Sept. 18-20, 1978.
(see pages 3 and 4 for details)

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PROCEEDINGS OF AN APL USERS MEETING 381 pages

These proceedings of a conference sponsored by I. P. Sharp Associates in September, 1978, in Toronto, include several papers in each of the following sessions: Direction of APL Developments; Financial Planning; APL in Exposition; Applications in the Actuarial and Insurance Industry; In-house Migrations; Graphics and Plotting Applications in Business and Industry; Logistic Systems; Financial Applications in the Resource Industry; Applications in Banking and Finance; Quantitative Analysis and Statistical Techniques; APL Standards for Documentation and Programming.

RESISTIVE CIRCUIT THEORY R.L. Spence 279 pages

This is a freshman level introduction to circuit theory limited to resistive circuits (linear and nonlinear) and employing APL as the mathematical notation. The executability of APL permits a student to experiment with a wide variety of circuits, and thereby gain insight into circuit behavior and the structures of circuit theory. Circuit topology is described by the node-branch incidence matrix; the use of APL makes possible simple explicit expressions for conductance and impedance matrices, and for Kirchhoff's laws. CHAPTERS: Circuit Design; Measurement and Modelling of 2-Terminal Components; Component Interconnection; Sources and Power; Signals; Measurement and Modelling of 3-Terminal Components; Topology; Circuit Description; Linear Circuit Analaysis; Linear Circuit Properties; Nonlinear Circuit Analysis; Small Signal Behavior of Nonlinear Circuits.

APL AND INSIGHT P. Berry, G. Bartoli 89 pages
C. Dell'Aquila, V. Spadavecchia

This book discusses the use of APL and APL programs to represent concepts in teaching. It includes examples drawn from elementary physics and computer science, and discusses the programming style appropriate to use in teaching. The main themes are that the key concepts of serious disciplines can be represented as functions, that APL permits a readable formal definition of a function and a means of executing it, and that it is possible to write programs so that they correspond directly to the functional concepts of a discipline.

STARMAP P.C. Berry and J.C. Thorstensen 41 pages

This book provides a complete and concise statement, in APL functions, of a simple model of the solar system (ignoring planetary interactions). The functions are written so as to make clear the structure of the underlying model by providing formal definitions for a vocabulary of terms and concepts familiar in astronomy. Star tables are included so that the APL functions, together with standard plotting functions, permit a user at an APL terminal to produce a map of the sky as it should appear above any place on Earth, at any time of day or night, over a considerable range of dates.

MATHEMATICS

A coherent three-volume sequence in the central topics elementary mathematics, bridging the years from high school through first year college, is provided by:

ALGEBRA: an algorithmic treatment	K. E. Iverson	**361 pp**
ELEMENTARY ANALYSIS	K. E. Iverson	**218 pp**
CALCULUS in a new key	D. L. Orth	**286 pp**

APL is used as the exclusive mathematical notation throughout -- in exposition, in proofs, and in exercises. This notation is very close to conventional algebra but is simpler and more general, and is directly executable on an APL computer exactly as written. Although an APL computer is in no way essential, it is a useful adjunct, removing the tedium from some of the exercises and encouraging an experimental approach to the exploration of mathematical topics.

The simplicity and precision of the notation makes possible an algorithmic treatment -- every function introduced is defined in terms of at least one explicit construction. Even if a computer is not available, the algorithmic treatment presents the essentials of computer programming in a mathematical light, i.e., as the precise definition and application of functions.

The simplicity and precision of the notation have also made it possible to place a greater burden than normal on the exercises. This is reflected in their number and variety, and in the fact that interesting and non-trivial results are developed in them. The extensive exercises make it possible to foster a great deal of independent work on the part of the students. Finally, the exercises can be used effectively in courses devoted to teaching the APL language itself.

Because the notation applies to vectors and matrices (<u>lists</u> and <u>tables</u>, as they are called in the algebra text) in a simple and uniform manner, tables are used extensively to give a graphic view of functions by displaying the patterns produced by applying them to vectors. They are also used to clarify topics which use vectors directly, such as linear functions and polynomials. For example, the presentation of a scheme for determining the product of polynomials is shown at the right.

```
      C←3 1 4
      D←2 0 5 3
      C∘.×D
      6  0 15  9
      2  0  5  3
      8  0 20 12
6  2 23 14 23 12
```

The notation provides simple explicit means for function definition, and the notion of <u>function</u> is emphasized in all three texts. The Elementary Analysis and Calculus texts employ a particularly simple and effective form (that covers recursive definition) which is discussed in the first issue of APL News, available on request.

A 42-page solutions book for the algebra text is available; others are in preparation.

INTRODUCTIONS TO APL

Three introductory booklets by K.E. Iverson previously published by IBM Corp. are now available from APL Press:

INTRODUCING APL TO TEACHERS — 25 pages

In introducing the use of a computer to teachers it is desirable to start as soon as possible with material which they can see is relevant to their topic and their students, and to avoid digressions concerning the computer and computer language. This book presents such an introduction to APL for teachers of high school mathematics. Much of this material should also be suitable for teachers of other topics at other levels, although they would also benefit from auxiliary material specifically addressed to the topic of interest.

CONTENTS: Introduction; Experimentation; Systematic Experimentation; Multiplication and other Function Tables; Graphs and Bar Charts; Indexing and Characters; Exploring Functions of One Argument; Defining New Functions; Inverse Functions; Summation and Other Functions over a List; Factoring; Linear Expressions; Linear Equations; Tables and Graphs of Linear Functions; Polynomials; Generalizing a Function by Use of Patterns; The Positive Integers; Summation of Series; Power Series; Differencing a Function; Combinations and Binomial Coefficients; Iteration; References to other Topics; References; Summary of Notation.

AN INTRODUCTION TO APL FOR SCIENTISTS AND ENGINEERS — 26 pages

This is an introduction to APL addressed to the scientist or engineer and designed to exploit any previous acquaintance with the very similar notation of vector algebra. A careful study of these pages should bring the reader to the point where he can begin to make serious use of APL in some topic of interest to him. The use of an APL terminal in this study, while not absolutely essential, adds greatly to the depth and interest of the work.

The pleasure and efficiency of learning by experimentation is not sufficiently appreciated, and the first six pages are designed to encourage this type of use of a terminal in learning APL. Two pages are devoted a variety of identities and proofs expressed in APL.

APL IN EXPOSITION — 61 pages

This book illustrates the use of APL for exposition in the teaching of various topics. The first section presents the characteristics of the language, and each of the other sections illustrates its use in some discipline. The topics treated are: Elementary Algebra; Coordinate Geometry and Statics; Finite Differences and the Calculus; Logic; Sets; Electric Circuits; The Computer.

APL VADE MECUM: A complete summary of APLSV on a wallet-sized plastic card.

STILL MORE ON GRADE

Norman D. Thomson

The aim of this note is to describe the relationship between the grade functions ⍋ and ⍒ and the symmetries of the square.

As remarked by McPherson in APL News 6, a permutation vector *P* can be converted to and from a permutation matrix *M* by:

```
MAT:(⍳⍴P)∘.=P                    PERM:+⌿+⍀⊖M
```

A square possesses 8 symmetries in the sense that its shape is invariant under 8 plane transformations. Taking an origin O at the center of the square, and axes Ox, Oy parallel to its sides, we can denote these 8 transformations as follows:

- I : Identity
- R : Anti-clockwise rotation of 90 degrees about O
- H : Half-turn about O
- S : Clockwise rotation of 90 degrees about O
- X : Reflection in Ox
- Y : Reflection in Oy
- P : Reflection in diagonal axis x = y
- Q : Reflection in diagonal axis x = -y

Suppose now that W is a word from the alphabet which consists of the two symbols ⍋ and ⍒. It is reasonable to ask what transformation of *M* corresponds to the application to *P* of the function represented by the word W. John McPherson pointed out in APL News 6 that the words ⍒⍒⍒ and ⍒⍒ and ⍒ correspond to the transformations R, H, and S respectively. The picture can be completed as follows:

- I : empty word
- X : ⍋⍒
- Y : ⍒⍋
- P : ⍋⍒⍒ or ⍒⍒⍋
- Q : ⍋

If, as we must now suspect, there is a one-to-one correspondence between <u>distinct</u> words W (i.e., distinct in their effect on a permutation vector *P*) and the symmetries of the square, then we conclude that there are just 8 distinct words W made up from the function symbols ⍋ and ⍒.

The group of plane transformations I,H,X,Y,R,S,P,Q is an instance of what mathematicians call D_4 -- the dihedral group of order 4, that is, a group whose elements can be defined in terms of just 2 elements, one of order 4 and the other of order 2. (An element A is of order N if N is the smallest integer for which A*N = identity. Note that A represents a <u>function</u>, thereby demonstrating the desirability of a power <u>operator</u> in APL!)

For example, the group of plane transformations can be defined in terms of, say, R and X, since R*4 and X*2 both = identity, and further H=RR, S=RRR, P=XR, Y=XRR, and Q=XRRR. Moreover, we can choose which two elements generate the group elements provided one is of order 2 (but not H), and the other is of order 4. In particular, if we choose S and and Q as the generating elements, we have an exact correspondence (technically an isomorphism) between the elements S and Q defining plane transformations, and the functions ⍋ and ⍒ which define the essentially different words which transform any given permutation vector.

Of course internal symmetry in the permutations themselves will sometimes bring it about that these 8 words will not all generate different permutation vectors from a given original. For example, the permutation ι*ρP* has only two different forms, ι*ρP* and ⌽ι*ρP*. It is also clear that no permutation can belong to more than one such set of 8 or less, and so the effect of applying the 8 different words to <u>all</u> permutations of ι*ρP* is to partition the !*ρP* permutations into sets of 2, sets of 4, and sets of 8. The table below gives the number of such sets for different values of *ρP*.

ρP	Sets of 2	Sets of 4	Sets of 8	Totals
3	1	1		6
4	4	2	1	24
5	4	10	9	120
6	10	35	70	720

The interested reader should sketch the permutation matrices corresponding to one element (permutation) of each of the various sets.

For *P*←ι4, the permutations are partitioned as follows:

1234 4321
1324 4231
2143 3412
3142 2413
2134 3421 1243 4312
3214 2341 1432 4123
2314 2431 1423 4213 3241 4132 1342 3124

For *P*←ι5 and *P*←ι6, the sets of 2 are:

12345 54321   14325 52341   21354 45312   41352 25314

123456 654321   124356 653421   132546 645231   145236 632541
153426 624351   154326 623451   213465 564312   214365 563412
321654 456123   351624 426153