On the Boundary of Regular Languages

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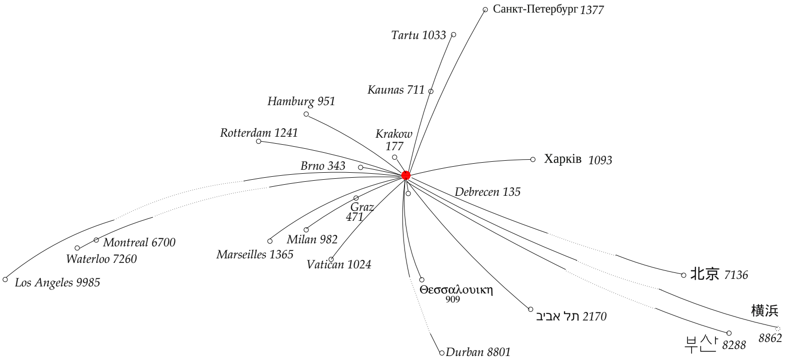

Outline

- Motivation and history
- Two problems by JS 2010
  1. L^{*c*}
  2. $\mathrm{bd}(L) = L^* \cap (L^c)^*$
- Known results
- Our results
  1. tight bounds for $\mathrm{bd}(L)$
  2. 5-letter alphabet
  3. optimal size (?)
- Applications

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Motivation I

J. Brzozowski, E. Grant, J. Shallit:
Closures in formal languages
and Kuratowski's theorem
[DLT 09, IJFCS 11]

- concepts of "open" and "closed"
- $L \subseteq \Sigma^*$ is closed if $L = L^*$
- L is open if L^c closed
- natural analogues of classical THMs

In point-set topology:

$\mathrm{bd}(S) = \mathrm{closure}(S) \cap \mathrm{closure}(S^c)$

- $S = \{(x,y)\colon x^2 + y^2 \leq 1\} \Rightarrow \mathrm{bd}(S) = \{(x,y)\colon x^2 + y^2 = 1\}$

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Motivation II

A. Salomaa, K. Salomaa, S. Yu:
State complexity
of combined operations [TCS 07]

operation	composition	complexity
$(K \cap L)^*$	$3/4 \cdot 2^{mn}$	$3/4 \cdot 2^{mn}$
$(K \cup L)^*$	$3/4 \cdot 2^{mn}$	$\leq 3/4 \cdot 2^{m+n}$

Combined operations [SSY 07, ...]

comb. operations	complexity	
without c and $\cap$	$2^{O(m+n)}$	
without c	$2^{poly(mn)}$	
L^{*c*}	$2^{\Theta(n \log n)}$	[JS 12]

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Motivation III (for me:)

An article by Horák about Paul Erdös:

- A pop-singer needs crowds;
  the larger, the better...
- A researcher needs
  to be acknowledged by 5 people;
  he knows them by name.
- Horak's fives:
  Erdös, Erdös, Erdös, Erdös, Erdös.
- My fives?
  ...

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Known results

operation	complexity	
L^c	n	[folklore]
$K \cap L$	mn	[RS 59, Ma 70]
L^*	$3/4 \cdot 2^n$	[YZS 94]

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Trivial upper bound on sc of $\mathrm{bd}(L)$:

operation	complexity
L^*	$3/4 \cdot 2^n$
L^{c*}	$3/4 \cdot 2^n$
$\mathrm{bd}(L) = L^* \cap L^{c*}$	$9/16 \cdot 4^n$

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Triv. upper bound for $\mathrm{bd}(L)$ is $9/16 \cdot 4^n$

Question: Is it attainable???

Answer: Almost!!!

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Triv. upper bound for $\mathrm{bd}(L)$ is $9/16 \cdot 4^n$

Question: Is it attainable???

Answer: Almost!!!

If L is accepted by an n-state DFA with k final states, then

$$\begin{aligned}\mathrm{sc}(L^* \cap L^{c*}) \le 2 &+ 2^{n-k}\, 2^{n-1} - 3^{n-k}\, 2^{k-1} \\ &+ 2^{k-1}\, 2^{n-1} - 3^{k-1}\, 2^{n-k} \\ &+ 4^{n-1} - \binom{n-1}{k-1},\end{aligned}$$

which is maximal if $k = 2$, and it equals $3/8 \cdot 4^n + 2^{n-2} - 2 \cdot 3^{n-2} - n + 2$.

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This upper bound is tight!!! ($|\Sigma| \geq 5$)

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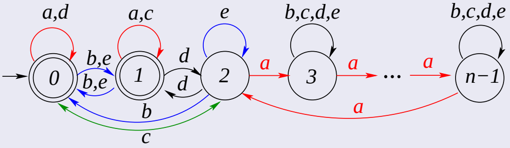
Worst-case example over $\{a, b, c, d, e\}$



a:	0 −> 0	1 −> 1	cycle (2,3,4,..., n−1)
b:	0 <−> 1	2 −> 0	i −> i
c:	0 <−> 2		i −> i
d:	1 <−> 2		i −> i
e:	0 <−> 1		i −> i

meets $3/8 \cdot 4^n + 2^{n-2} - 2 \cdot 3^{n-2} - n + 2$

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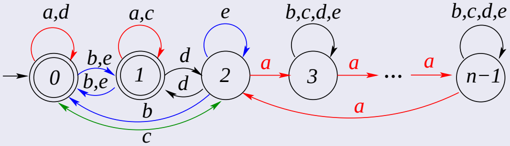

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Quaternary case:

The DFA restrited to $\{a, b, c, d\}$ meets $3/8 \cdot 4^n + 2^{n-2} - 2 \cdot 3^{n-2} - n + 1$

- this lower bound is tight if $|\Sigma| = 4$

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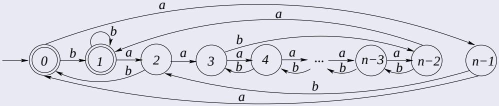
Binary case:

L :



- $\mathrm{sc}(\mathrm{bd}(L)) \geq 1/256 \cdot 4^n$
- asymptotically tight bound $\Theta(4^n)$ if $|\Sigma| = 2, 3$

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Binary case:

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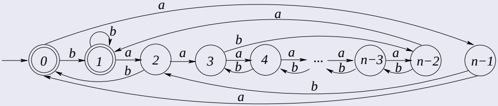

- $\mathrm{sc}(\mathrm{bd}(L)) \geq 1/256 \cdot 4^n$
- asymptotically tight bound $\Theta(4^n)$ if $|\Sigma| = 2, 3$

Unary case:

- $\mathrm{bd}(L) = L^*$ or $\mathrm{bd}(L) = L^{c*}$
- $(n-1)^2 + 1$ [Yu, Zhuang, Salomaa 94]

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Summary

$\lvert\Sigma\rvert$	state complexity of boundary
≥ 5	$3/8 \cdot 4^n + 2^{n-2} - 2 \cdot 3^{n-2} - n + 2$
$= 4$	$3/8 \cdot 4^n + 2^{n-2} - 2 \cdot 3^{n-2} - n + 1$
$= 3$	$\Theta(4^n)$
$= 2$	$\Theta(4^n)$
$= 1$	$(n-1)^2 + 1$

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Applications

- Euclid and his student
- Osuský about Poincaré conjecture:

"I agree with the view
of the ancient Indian
religious culture
that the *commercial thinking*
is a thinking of animals,
and that only thanks
to the ingenious creativity,
we've got to today's level..."

Thank You for Your Attention

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