Targeted Pandemic Containment
Through
Identifying Local Contact Network Bottlenecks

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Network epidemic modelling and control strategies

• Networks are a powerful tool for modelling epidemic dynamics

• Previous models of infection control mostly focused on node-level interventions, e.g., targeted vaccination
Network epidemic modelling and control strategies

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  “… in networks with strong community structure, immunization interventions targeted at individuals bridging communities are more effective …” (Salathe and Jones, 2010)
Network epidemic modelling and control strategies

- In this work we look at edge-level interventions, e.g., contact reduction, physical distancing, quarantine

  - For county-level networks, selectively closing roads or quarantining towns and cities

  - For individual-level networks, enforce or encourage physical distancing by providing incentives
In this work we look at edge-level interventions, e.g., contact reduction, physical distancing, quarantine.

How to identify important edges for intervention strategies?

Network epidemic modelling and control strategies

- Shortest-path (SP) edge-betweenness
- Current-flow (CF) edge-betweenness
Network epidemic modelling and control strategies

- SP and CF may not work well
- Global “bottlenecks” do not block local transmission
- Less effective in the presence of community outbreak
Quantifying edge importance locally

- We need a new edge-betweenness measure that detects local bottlenecks
Quantifying edge importance locally

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• Electrical current flow

\[
\begin{align*}
\min_{f \in \mathbb{R}^{|E|}} \|f\|^2_2 & \quad \text{s.t.} \quad B^T f + 1_s = 1_t \quad (P') \\
\min_{x \in \mathbb{R}^{|V|}} x^T L x - x^T (1_s - 1_t) & \quad (D')
\end{align*}
\]

- \( f \in \mathbb{R}^{|E|} \): incidence matrix
- \( 1_s, 1_t \): indicator vector of \( s \in V \)
- \( x \in \mathbb{R}^{|V|} \): Laplacian matrix
Quantifying edge importance locally

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\[
\min \|f\|_2^2 \quad \text{s.t.} \quad B^Tf + 1_s = 1_t \quad (P') \quad \min \quad x^TLx - x^T(1_s - 1_t) \quad (D')
\]

\( f \in \mathbb{R}^{|E|} \quad \text{incidence matrix} \quad \text{indicator vector of } s \in V \quad x \in \mathbb{R}^{|V|} \quad \text{Laplacian matrix} \)

*Global focus: All possible pairs \((s, t) \in V \times V\) are taken into account*
Quantifying edge importance locally

- We need a new edge-betweenness measure that detects local bottlenecks

- \textit{p}-norm flow diffusion (for brevity, \(p = 2\) in this presentation)

  \[
  \min \|f\|^2_2 \quad \text{s.t.} \quad B^T f + 1_s \leq T \quad (P) \quad \min \quad x^T Lx - x^T (1_s - T) \quad (D)
  \]

- Electrical current flow

  \[
  \min \|f\|^2_2 \quad \text{s.t.} \quad B^T f + 1_s = 1_t \quad (P') \quad \min \quad x^T Lx - x^T (1_s - 1_t) \quad (D')
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\(f \in \mathbb{R}^{\lvert E \rvert}\) \quad \text{incidence matrix} \quad \text{indicator vector of} \ s \in V \quad x \in \mathbb{R}^{\lvert V \rvert}\) \quad \text{Laplacian matrix}
Quantifying edge importance locally

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- Electrical current flow

$$\min x^T L x - x^T (1_s - T) \quad (D)$$

$$\min x^T L x - x^T (1_s - 1_t) \quad (D')$$

$t \in \mathbb{R}_{+}^{\|V\|}$ specifies node capacities

$f \in \mathbb{R}^{\|E\|}$

incidence matrix

indicator vector of $s \in V$

$x \in \mathbb{R}^{\|V\|}$

Laplacian matrix
Quantifying edge importance locally

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\]

- We set \( T(v) = \deg(v)/(2\lambda |E|) \), where \( \lambda \in (0,1] \) controls locality.
Quantifying edge importance locally

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- $p$-norm flow diffusion (for brevity, $p = 2$ in this presentation)

$$\min \|f\|_2^2 \text{ s.t. } B^Tf + 1_s \leq T \quad (P) \quad \min_{x \geq 0} x^T L x - x^T (1_s - T) \quad (D)$$

- We set $T(v) = \deg(v)/(2\lambda |E|)$, where $\lambda \in (0,1]$ controls locality

- Denote $f^\lambda_s$ the optimal flow arising from source node $s$ with locality $\lambda$

- Local-flow (LF) betweenness of an edge $e \in E$ is

$$lb(e; \lambda) := \frac{1}{|V|} \sum_{s \in V} |f^\lambda_s(e)|$$
Quantifying edge importance locally

- Local-flow (LF) betweenness
  - Colors and edge widths are chosen to reflect relative magnitude
Quantifying edge importance locally

SP

Remove top 20% edges

Global

CF

Remove top 20% edges

Global

LF

Local
Facebook-county network

- 3100 counties
- Two counties are connected with an edge if there exists strong social interaction
- Social interaction tends to happen mostly among nearby counties
Facebook-county network - simulated epidemic dynamics

- Network SEIR model
- Targeting top 25% edges

NI: No Intervention
UI: Uniform Intervention
EG: Eigenvector centrality
HD: Degree centrality
SP: Shortest-Path betweenness
CF: Current-Flow betweenness
LF: Local-Flow betweenness
Facebook-county network - simulated epidemic dynamics

- **NI:** No Intervention
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- **HD:** Degree centrality
- **SP:** Shortest-Path betweenness
- **CF:** Current-Flow betweenness
- **EG:** Eigenvector centrality
- **LF:** Local-Flow betweenness

Epidemic peak

Outbreak size
Why is LF most effective?

- **Distribution of top 25% edges** reflected by county-level colors:
  - **red** means most incident edges are reduced (in edge weights)
  - **dark blue** means few incident edges are reduced (in edge weights)
Why is LF most effective?

- Counties in red form a tightly-knit local cluster with few out-links
- 100% out-links are among top 5% edges identified by LF
- <20% out-links are identified by SP or CF
Are the results robust?

- Estimated reproduction number for Covid-19 is $R_0 = 2.5$
- We tried \textbf{varying reproduction numbers} $R_0 \in \{1.5, 2.5, 3.5, 4.5\}$

- 3 very \textbf{different initializations} from where epidemic starts
  - randomly chosen 1% counties spread across the network
  - a tightly-knit cluster of counties
  - single cities: Chicago, New York, Los Angeles

- \textbf{Delayed interventions} applied in the middle of the epidemic (not from the start)

* All these different settings produce consistent results
Thank you!
Computation time

- Computing LF for all edges requires $O(\lambda |V||E|), \lambda \in (0,1]$
Why is LF most effective?

- **Distribution of small-size clusters** (consisting of $\leq 100$ counties) by conductance
Facebook-county network - simulated epidemic dynamics

**15-day delay**

**30-day delay**

**45-day delay**

**60-day delay**
More datasets

- **Wi-Fi hotspots Montreal** network, $|V| = 103K$, $|E| = 631K$
- **Portland, Oregon** network, $|V| = 1.6M$, $|E| = 31M$
- **Sub-sampled Portland, Oregon** network, $|V| = 10K$, $|E| = 199K$
- Agent-based SEIR network model