Spectrahedral Representations of Hyperbolicity Cones

Rafael Oliveira

June 2022

1 Introduction

Given the algorithmic importance of semidefinite programming, the last two decades have seen increasing interest in characterizing which convex cones are spectrahedral (that is, feasible sets of semidefinite programming). One particular class of convex cones are hyperbolicity cones (defined by hyperbolic polynomials), which has connections to several areas of mathematics.

An outstanding open question – the generalized Lax conjecture – asks whether every hyperbolicity cone is spectrahedral. In [COSW04] and subsequent works, several special classes of hyperbolic polynomials have been studied, and the question of whether such classes of hyperbolicity cones arising from them are spectrahedral remains wide open.

2 Project Proposal

The purpose of this project is to study certain classes of hyperbolicity cones arising from the aforementioned works, with the focus on the following three questions:

1. **Qualitative:** provide methods to prove whether such cones are spectrahedral,
2. **Algorithmic:** construct spectrahedral representations for such cones,
3. **Complexity-theoretic:** prove bounds on the minimal spectrahedral representation of such cones.

The student will be working with me on the three questions above. No prior experience with hyperbolic polynomials and hyperbolic programming, or semidefinite programming is required.

The student will be expected to meet with me on a weekly basis to discuss the problems and relevant background readings.

The ideal student should have a solid command of (in order of importance):

- linear algebra (equivalent of MATH 245, or MATH 235),
- experience writing rigorous proofs,
- basic abstract algebra (such as the material from both PMATH 336 and PMATH 347),
- complex analysis,
- matroid theory.

References

[COSW04] Young-Bin Choe, James G Oxley, Alan D Sokal, and David G Wagner. Homogeneous multivariate polynomials with the half-plane property. *Advances in Applied Mathematics*, 32(1-2):88–187, 2004.