Lecture 21: Distributed Algorithms

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Overview

- Distributed Computing: The Models
- Consensus with Byzantine Failures
- Conclusion
- Acknowledgements

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  - Data Management and Transmission
  - Synchronization
  - Consensus
  - many more

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  - Failure and recovery of processors or channels
- Many models
  - *Memory & Communication*: shared memory, message-passing
  - *Timing*: synchronous (rounds), asynchronous, partially synchronous (bounds on message delay, processor speeds, clock rates)
  - *Failures*: processor (stop, Byzantine), communication (message loss/altered), system state corruption

Synchronous Model

- processors are vertices of directed graph
  - *Memory*: each processor has its own memory
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- Σ is the message alphabet, plus special symbol $\perp$
- For each vertex $i \in [n]$, a processor consists of:
  - S_i = non-empty set of states
  - σ_i = a start state
  - $\mu_i : S_i \times out_i \to \Sigma \cup \{\perp\}$ Message function
  - $\tau_i : S_i \times (\Sigma \cup \{\perp\})^{in_i} \to S_i$ Transition function

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- Complexity Measure: *number of rounds* (*total data communicated*) needed to solve problem
  - processors have *unlimited internal resources* (i.e., can compute anything)
  - For today, will assume each processor deterministic

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- To show this, simply look at execution and check that all processors will always be at identical states.

Leader Election: Algorithm

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    - if it is bigger, pass it on
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- Can reduce communication to $O(n \log n)$ by successively doubling (see reference)

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- Model: synchronous model, arbitrary number of message failures.
- **Input**: Each processor has one bit. 1 (attack) or 0 (don't attack)
- **Output**: *same decision bit* b satisfying *strong validity*.
  - if all processors start with bit b, then b is only allowed decision [1]
  - if all start with 1 and *all messages successfully delivered*, then 1 is the only allowed decision.

[1] Weak validity: the agreed upon output should be the initial value of some non-faulty processor

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- Two types of failures:
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- **Output**: all *non-faulty processors* should *terminate* and have
  1. *Agreement*: same decision bit b
  2. *Strong Validity*: if all *non-faulty processors* start with bit a, then b must be equal to a.

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- Complexity measures: *number of rounds* & *communication* (# messages exchanged in bit-size).

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  Each node now will keep track of what each node has told another and so on...
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- At each round, each vertex broadcasts its knowledge
- After a number of rounds, everyone must make a decision
- Does this work?
- How many rounds do we need?
- How many Byzantine failures can it tolerate?

Byzantine Consensus - Bad Example

- 3 vertices $\{v_1, v_2, v_3\}$, 1 faulty vertex
- Scenario 1: v_1, v_2 good with value 1, v_3 faulty with value 0
  1. Round 1: all vertices truthful
  2. Round 2: v_3 lies to v_1, saying that v_2 said 0, all other communications truthful
  3. Validity $\Rightarrow v_1, v_2$ must decide 1

Byzantine Consensus - Bad Example

- 3 vertices $\{v_1, v_2, v_3\}$, 1 faulty vertex
- Scenario 2: v_2, v_3 good with value 0, v_1 faulty with value 1
    1. Round 1: all vertices truthful
    2. Round 2: v_1 lies to v_3, saying that v_2 said 1, all other communications truthful
    3. Validity $\Rightarrow v_2, v_3$ must decide 0

Byzantine Consensus - Bad Example

- 3 vertices $\{v_1, v_2, v_3\}$, 1 faulty vertex
- Scenario 3: v_1, v_3 good with values $1, 0$ (resp.), v_2 faulty with value 0
  1. Round 1: v_2 tells v_1 its value is 1, tells v_3 its value is 0
  2. Round 2: all truthful

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- Scenario 1: v_1, v_2 good with value 1, v_3 faulty with value 0
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- Scenario 3: v_1, v_3 good with values $1, 0$ (resp.), v_2 faulty with value 0
  1. Round 1: v_2 tells v_1 its value is 1, tells v_3 its value is 0
  2. Round 2: all truthful
- Scenarios 1 and 3 identical to v_1, so it must return 1 (validity)
- Scenarios 2 and 3 identical to v_3, so it must return 0 (validity)
- Contradicts *agreement* in Scenario 3!

Byzantine Consensus - Algorithm

- Assumption:[2] $n > 3f$ (number of bad vertices $<$ third total vertices)

[2] It turns out that $n \leq 3f \Rightarrow$ *no algorithm* can reach consensus!

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- Data structure: *Exponential Information Gathering* (EIG) tree $T_{n,f}$
  - Depth: $f + 1$ (so $f + 2$ node levels)
  - Each tree node at level $k + 1$ labeled by string $i_1 i_2 \cdots i_k$ ($i_a \neq i_b$)

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  - Each tree node at level $k + 1$ labeled by string $i_1 i_2 \cdots i_k$ $(i_a \neq i_b)$
  - Node $i_1 i_2 \cdots i_k$ will store value v if the following happens: i_k told you that i_{k-1} told i_k that i_{k-2} told i_{k-1} ... that i_1 told i_2 that its initial value was v

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   - Each vertex decorates values of its $(r+1)^{th}$ level with values from messages
3. After $f+1$ rounds, redecorate tree bottom-up, taking strict majority of children (if there is no strict majority set value of tree node to $v_\perp$)

EIG Algorithm - Example

- $n = 4$, $f = 1$
- p_3 is faulty, initial values are $p_1 = p_2 = 1$, $p_3 = p_4 = 0$
- round 1: p_3 lies to p_2 and p_4
- round 2: p_3 lies to p_2 about p_1 and lies to p_1 about p_2

EIG Algorithm - Analysis

Lemma (Consistency of Non-Faulty Messages)

If i, j, k are non-faulty, then $T_i(x) = T_j(x)$ whenever label x ends with k.

(This is value of the tree before relabeling)

EIG Algorithm - Analysis

Lemma (Consistency of Upwards Relabeling)

If label x ends with non-faulty process, then for any two non-faulty processors i, j the new values of $T_i(x)$ and $T_j(x)$ are the same.

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- Inductive step: $|x| = t < f$ (x not a leaf)
  - By induction, if ℓ is a non-faulty element the new value of $T_i(x \circ \ell)$ is the same for any non-faulty $i \in [n]$.

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$$= n - t > 3f - f = 2f$$

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  - At most f are faulty. By taking majority, we get that new values $T_i(x) = T_j(x)$

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3. *Agreement*: all nodes must agree on same value
   - By first lemma, all values in the leaves x are consistent across processors so long as x ends on a non-faulty process
   - By second lemma, majority will cause all values in nodes from level r ending in non-faulty nodes to be *the same* across processors
   - Induction and $n > 3f$ ensures that labels in level 1 will look the same on non-faulty nodes $\Rightarrow$ agreement

Conclusion

- Today we learned about distributed computation
- It is cool
- Widely used in practice
  - Cryptocurrencies - all of them need to solve Byzantine Agreement!

    Happening at UW: Sergey Gorbunov (Algorand & Axelar)
  - Other peer-to-peer systems
  - Multi-core programming

    Happening at UW: Trevor Brown
  - Biology (social insect colony algorithms)
  - many more...
- Learned an (inefficient) algorithm for Byzantine Agreement (check out the more efficient one in [Attiya and Welch 2004])

Acknowledgement

- Lecture based largely on:
  - Nancy Lynch's 6.852 Fall 2015 course - lectures 1 and 6
  - Lecture 1

    `https://learning-modules.mit.edu/service/materials/groups/103042/files/271154f5-ea0f-41a0-9ed9-6f83a5222d8b/link?errorRedirect=%2Fmaterials%2Findex.html&download=true`
  - Lecture 6

    `https://learning-modules.mit.edu/service/materials/groups/103042/files/95f71f5e-7791-4a1a-aeb5-e3d97afb167f/link?errorRedirect=%2Fmaterials%2Findex.html&download=true`

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