

# Lecture 24: Conclusion

Rafael Oliveira

University of Waterloo  
Cheriton School of Computer Science

[rafael.oliveira.teaching@gmail.com](mailto:rafael.oliveira.teaching@gmail.com)

July 15, 2024

# Overview

- Course Overview and Recap
  - What have we learned?
  - High-Level Principles
  - Interconnectedness
- Where do we go from here?
  - Next steps
  - “Real” Life

# What was this course about?

In your previous algorithms/optimization/data structures course, you learned some of the following:

- combinatorial techniques (divide-and-conquer, greedy algorithms, dynamic programming, local search, etc.)
- data structures (heaps, balanced trees, etc.)

# What was this course about?

In your previous algorithms/optimization/data structures course, you learned some of the following:

- combinatorial techniques (divide-and-conquer, greedy algorithms, dynamic programming, local search, etc.)
- data structures (heaps, balanced trees, etc.)

The techniques above emphasized **two computational models** (sequential & deterministic computation, query model).

# What was this course about?

In this course we used the algorithmic lens to:

- explore several models of computation:
  - 1 deterministic sequential
  - 2 randomized sequential
  - 3 randomized parallel
  - 4 sublinear-time
  - 5 memory constrained (streaming)
  - 6 distributed
  - 7 online (competitive analysis)
  - 8 algebraic
  - 9 interactive

# What was this course about?

In this course we used the algorithmic lens to:

- explore several models of computation:
  - 1 deterministic sequential
  - 2 randomized sequential
  - 3 randomized parallel
  - 4 sublinear-time
  - 5 memory constrained (streaming)
  - 6 distributed
  - 7 online (competitive analysis)
  - 8 algebraic
  - 9 interactive
- expand your algorithmic toolkit
  - 1 amortized analysis
  - 2 use of randomness
  - 3 concentration inequalities
  - 4 dealing with NP-complete problems (approximation algorithms)
  - 5 exploring the limits of approximation algorithms

# High-level principles in algorithms

- When encountering a problem, follow template:
  - ① what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?

# High-level principles in algorithms

- When encountering a problem, follow template:
  - ① what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?
  - ② What *guarantees* are *possible* in the model of computation? Can we hope to get an optimal answer to the problem?  
That is: are there any impossibility results which are known in the computational model?



# High-level principles in algorithms

- When encountering a problem, follow template:
  - ① what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?
  - ② What *guarantees* are *possible* in the model of computation? Can we hope to get an optimal answer to the problem?  
That is: are there any impossibility results which are known in the computational model?
  - ③ what is the simplest/most natural algorithm to solve it, and what is its complexity?

# High-level principles in algorithms

- When encountering a problem, follow template:
  - ① what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?
  - ② What *guarantees* are *possible* in the model of computation? Can we hope to get an optimal answer to the problem?  
That is: are there any impossibility results which are known in the computational model?
  - ③ what is the simplest/most natural algorithm to solve it, and what is its complexity?
  - ④ Can we do better?

# High-level principles in algorithms

- When encountering a problem, follow template:
  - ① what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?
  - ② What *guarantees* are *possible* in the model of computation? Can we hope to get an optimal answer to the problem?  
That is: are there any impossibility results which are known in the computational model?
  - ③ what is the simplest/most natural algorithm to solve it, and what is its complexity?
  - ④ Can we do better?
  - ⑤ Is the problem hard? If so, can we hope to *relax the guarantees*?

# High-level principles in algorithms

- When encountering a problem, follow template:
  - 1 what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?
  - 2 What *guarantees* are *possible* in the model of computation? Can we hope to get an optimal answer to the problem?  
That is: are there any impossibility results which are known in the computational model?
  - 3 what is the simplest/most natural algorithm to solve it, and what is its complexity?
  - 4 Can we do better?
  - 5 Is the problem hard? If so, can we hope to *relax the guarantees*?
  - 6 Can we do better?

# High-level principles in algorithms

- When encountering a problem, follow template:
  - 1 what *model of computation* is most suitable?  
That is: what are the most important *resources* and *constraints* that we need to respect?
  - 2 What *guarantees* are *possible* in the model of computation? Can we hope to get an optimal answer to the problem?  
That is: are there any impossibility results which are known in the computational model?
  - 3 what is the simplest/most natural algorithm to solve it, and what is its complexity?
  - 4 Can we do better?
  - 5 Is the problem hard? If so, can we hope to *relax the guarantees*?
  - 6 Can we do better?
  - 7 Can we show that our algorithm is the best?  
That is, we could try to reduce it to another problem which is known to be optimum (perhaps under certain complexity assumptions)

## Example: Amortized Analysis

- In data structures, oftentimes one cares about worst-case per query
  - internet's client-server model

## Example: Amortized Analysis

- In data structures, oftentimes one cares about worst-case per query
  - internet's client-server model
- sometimes, we don't care about worst-case per query, but worst-case overall
  - use of data structures in sequential algorithms
  - minimum spanning tree

## Example: Amortized Analysis

- In data structures, oftentimes one cares about worst-case per query
  - internet's client-server model
- sometimes, we don't care about worst-case per query, but worst-case overall
  - use of data structures in sequential algorithms
  - minimum spanning tree
- Learned how to use amortized analysis to provide better overall guarantees
  - vanilla amortization count all costs
  - charging scheme assign charges to operations
  - potential function assign charges and potential to data structure



## Similar input setting - different models

- When input comes to you “online” (as a stream of events)  
YOLO: you only look once  
we may have different computational models/goals

## Similar input setting - different models

- When input comes to you “online” (as a stream of events)  
YOLO: you only look once  
we may have different computational models/goals
- We learned:
  - **data streaming:** *memory* is our main constraint. Content with *approximation* of best answer
  - Examples: median, heavy hitters, distinct elements

## Similar input setting - different models

- When input comes to you “online” (as a stream of events)  
YOLO: you only look once  
we may have different computational models/goals
- We learned:
  - **data streaming:** *memory* is our main constraint. Content with *approximation* of best answer
  - Examples: median, heavy hitters, distinct elements
  - **online algorithms:** want *fast updates*, need to *decide on the spot*.  
Want to do as best as we can compared to *best in hindsight* (algorithms that can see entire input beforehand)
  - *competitive analysis*
  - Examples: multiplicative weights update, paging, *k*-server

## Randomness gives us power

- We learned how to use randomness in algorithms
- To use randomness, tradeoff between an algorithm which works *all the time* (deterministic) to one that works *most of the time*
- Usually faster (or the only option)

## Randomness gives us power

- We learned how to use randomness in algorithms
- To use randomness, tradeoff between an algorithm which works *all the time* (deterministic) to one that works *most of the time*
- Usually faster (or the only option)
- Quite useful in many settings:
  - when we don't know what to do
  - when problem has some adversarial input format

## Randomness gives us power

- We learned how to use randomness in algorithms
- To use randomness, tradeoff between an algorithm which works *all the time* (deterministic) to one that works *most of the time*
- Usually faster (or the only option)
- Quite useful in many settings:
  - when we don't know what to do
  - when problem has some adversarial input format
  - when we have to make decisions before seeing the whole input (streaming)

## Randomness gives us power

- We learned how to use randomness in algorithms
- To use randomness, tradeoff between an algorithm which works *all the time* (deterministic) to one that works *most of the time*
- Usually faster (or the only option)
- Quite useful in many settings:
  - when we don't know what to do
  - when problem has some adversarial input format
  - when we have to make decisions before seeing the whole input (streaming)
  - when problem can be encoded in algebraic format (polynomial identity testing)
  - when we need to construct objects which are abundant, but “hard” to construct (hash functions, graph sparsification, dimension reduction, expanders)

## Randomness gives us power

- We learned how to use randomness in algorithms
- To use randomness, tradeoff between an algorithm which works *all the time* (deterministic) to one that works *most of the time*
- Usually faster (or the only option)
- Quite useful in many settings:
  - when we don't know what to do
  - when problem has some adversarial input format
  - when we have to make decisions before seeing the whole input (streaming)
  - when problem can be encoded in algebraic format (polynomial identity testing)
  - when we need to construct objects which are abundant, but “hard” to construct (hash functions, graph sparsification, dimension reduction, expanders)
  - when we need to estimate number of objects which are hard to count (random walks)



## Randomness gives us power

- We learned how to use randomness in algorithms
- To use randomness, tradeoff between an algorithm which works *all the time* (deterministic) to one that works *most of the time*
- Usually faster (or the only option)
- Quite useful in many settings:
  - when we don't know what to do
  - when problem has some adversarial input format
  - when we have to make decisions before seeing the whole input (streaming)
  - when problem can be encoded in algebraic format (polynomial identity testing)
  - when we need to construct objects which are abundant, but “hard” to construct (hash functions, graph sparsification, dimension reduction, expanders)
  - when we need to estimate number of objects which are hard to count (random walks)
  - approximation algorithms (randomized rounding)

## Dealing with NP-hard problems

- when faced with an NP-hard (optimization) problems, still want to solve them (as best as we can)
- to that task, important to relax the guarantees
- instead of trying to get best solution, get a solution which is *guaranteed* to be close to best

## Dealing with NP-hard problems

- when faced with an NP-hard (optimization) problems, still want to solve them (as best as we can)
- to that task, important to relax the guarantees
- instead of trying to get best solution, get a solution which is *guaranteed* to be close to best
- formulate problems as *integer programs* or *quadratic programs* and relax them to problems we can solve: *linear programs* or *semidefinite programs*

## Dealing with NP-hard problems

- when faced with an NP-hard (optimization) problems, still want to solve them (as best as we can)
- to that task, important to relax the guarantees
- instead of trying to get best solution, get a solution which is *guaranteed* to be close to best
- formulate problems as *integer programs* or *quadratic programs* and relax them to problems we can solve: *linear programs* or *semidefinite programs*
- when we have enough structure, deterministic methods are good

## Dealing with NP-hard problems

- when faced with an NP-hard (optimization) problems, still want to solve them (as best as we can)
- to that task, important to relax the guarantees
- instead of trying to get best solution, get a solution which is *guaranteed* to be close to best
- formulate problems as *integer programs* or *quadratic programs* and relax them to problems we can solve: *linear programs* or *semidefinite programs*
- when we have enough structure, deterministic methods are good
- when integer programs have nice vertex solutions, easy to obtain deterministic rounding

## Dealing with NP-hard problems

- when faced with an NP-hard (optimization) problems, still want to solve them (as best as we can)
- to that task, important to relax the guarantees
- instead of trying to get best solution, get a solution which is *guaranteed* to be close to best
- formulate problems as *integer programs* or *quadratic programs* and relax them to problems we can solve: *linear programs* or *semidefinite programs*
- when we have enough structure, deterministic methods are good
- when integer programs have nice vertex solutions, easy to obtain deterministic rounding
- when the above does not happen, randomness to the rescue

## Interaction gives us power and limitations

- We learned about complexity of proofs, and how we can use interaction to give different characterization of complexity classes

## Interaction gives us power and limitations

- We learned about complexity of proofs, and how we can use interaction to give different characterization of complexity classes
- PCP theorem:  $NP$  can be characterized as problems that have proofs which can be verified by making **3** random queries to the proof!



# Interaction gives us power and limitations

- We learned about complexity of proofs, and how we can use interaction to give different characterization of complexity classes
- PCP theorem:  $NP$  can be characterized as problems that have proofs which can be verified by making 3 random queries to the proof!
- Also saw how we can use interactive proofs (PCPs) to construct reductions which preserve a gap between YES and NO instances

# Interaction gives us power and limitations

- We learned about complexity of proofs, and how we can use interaction to give different characterization of complexity classes
- PCP theorem:  $NP$  can be characterized as problems that have proofs which can be verified by making **3** random queries to the proof!
- Also saw how we can use interactive proofs (PCPs) to construct reductions which preserve a gap between YES and NO instances
- Interaction not only good for hardness of approximation - saw how to use interaction to give *zero knowledge proofs*

How to convince someone that you know something without revealing any knowledge on how you do it.

# Algebraic/Arithmetic Algorithms

- Also learned about algebraic/arithmetic models of computation
  - when your problem is algebraic in nature, this is the most natural model

# Algebraic/Arithmetic Algorithms

- Also learned about algebraic/arithmetic models of computation
  - when your problem is algebraic in nature, this is the most natural model
  - widely used for some of the most used algorithms in real life: matrix multiplication, discrete Fourier transform

# Algebraic/Arithmetic Algorithms

- Also learned about algebraic/arithmetic models of computation
  - when your problem is algebraic in nature, this is the most natural model
  - widely used for some of the most used algorithms in real life: matrix multiplication, discrete Fourier transform
- Used widely for design of parallel algorithms, since “linear algebra can be done in parallel”

# Algebraic/Arithmetic Algorithms

- Also learned about algebraic/arithmetic models of computation
  - when your problem is algebraic in nature, this is the most natural model
  - widely used for some of the most used algorithms in real life: matrix multiplication, discrete Fourier transform
- Used widely for design of parallel algorithms, since “linear algebra can be done in parallel”
- More generally: “any polynomial you can compute is a determinant”

# Distributed Computation

- Algorithms which run on a network, or multiprocessors within a computer which share memory

# Distributed Computation

- Algorithms which run on a network, or multiprocessors within a computer which share memory
- Problems they solve:
  - Resource Management
  - Data Management and Transmission
  - Synchronization
  - Consensus
  - many more



# Distributed Computation

- Algorithms which run on a network, or multiprocessors within a computer which share memory
- Problems they solve:
  - Resource Management
  - Data Management and Transmission
  - Synchronization
  - Consensus
  - many more
- Challenges in this setting:
  - Concurrent Activity
  - Uncertainty of order of events
  - Failure and recovery of processors or channels

# Distributed Computation

- Algorithms which run on a network, or multiprocessors within a computer which share memory
- Problems they solve:
  - Resource Management
  - Data Management and Transmission
  - Synchronization
  - Consensus
  - many more
- Challenges in this setting:
  - Concurrent Activity
  - Uncertainty of order of events
  - Failure and recovery of processors or channels
- Many models
  - *Memory & Communication*: shared memory, message-passing
  - *Timing*: synchronous (rounds), asynchronous, partially synchronous (bounds on message delay, processor speeds, clock rates)
  - *Failures*: processor (stop, Byzantine), communication (message loss/altered), system state corruption

# Interconnectedness in Algorithms and Complexity

- We saw many different settings for algorithms, important problems and techniques
- Is this just a bunch of tricks though?
- Certainly not

# Interconnectedness in Algorithms and Complexity

- We saw many different settings for algorithms, important problems and techniques
- Is this just a bunch of tricks though?
- Certainly not
- we saw how randomness is applied in many different settings, and common techniques such as hashing, fingerprinting, polynomial identity testing

# Interconnectedness in Algorithms and Complexity

- We saw many different settings for algorithms, important problems and techniques
- Is this just a bunch of tricks though?
- Certainly not
- we saw how randomness is applied in many different settings, and common techniques such as hashing, fingerprinting, polynomial identity testing
- interactive proofs, from proof complexity, used to prove that certain algorithms cannot be approximated up to a certain point

# Interconnectedness in Algorithms and Complexity

- We saw many different settings for algorithms, important problems and techniques
- Is this just a bunch of tricks though?
- Certainly not
- we saw how randomness is applied in many different settings, and common techniques such as hashing, fingerprinting, polynomial identity testing
- interactive proofs, from proof complexity, used to prove that certain algorithms cannot be approximated up to a certain point
- hashing and fingerprinting highly used in interactive proofs

# Interconnectedness in Algorithms and Complexity

- We saw many different settings for algorithms, important problems and techniques
- Is this just a bunch of tricks though?
- Certainly not
- we saw how randomness is applied in many different settings, and common techniques such as hashing, fingerprinting, polynomial identity testing
- interactive proofs, from proof complexity, used to prove that certain algorithms cannot be approximated up to a certain point
- hashing and fingerprinting highly used in interactive proofs
- Algorithms used to prove lower bounds (recent trend - highly recommended!)

# Interconnectedness in Algorithms and Complexity

- We saw many different settings for algorithms, important problems and techniques
- Is this just a bunch of tricks though?
- Certainly not
- we saw how randomness is applied in many different settings, and common techniques such as hashing, fingerprinting, polynomial identity testing
- interactive proofs, from proof complexity, used to prove that certain algorithms cannot be approximated up to a certain point
- hashing and fingerprinting highly used in interactive proofs
- Algorithms used to prove lower bounds (recent trend - highly recommended!)
- Algorithms in forms of reductions, used to prove that even easy problems cannot be improved! (fine-grained complexity)



- Course Overview and Recap
  - What have we learned?
  - High-Level Principles
  - Interconnectedness
  
- Where do we go from here?
  - Next steps
  - “Real” Life

# How can I learn more?

Consider taking more advanced courses next term!

See graduate course openings at:

- Current graduate course offerings for next term!

`https://cs.uwaterloo.ca/current-graduate-students/courses`

- Or, try out some of the research opportunities at UW!

`https://cs.uwaterloo.ca/computer-science/  
current-undergraduate-students/research-opportunities/  
undergraduate-research-assistantship-ura-program`

`https://cs.uwaterloo.ca/current-undergraduate-students/  
research-opportunities/undergraduate-research-fellowship-urf`

## But is this theory stuff useful?

- Certainly so - and lately the gap between theory and practice has been quite short
- intense use of theoretical cryptography and distributed computing in cryptocurrencies
- cryptography highly used in e-commerce
- several algorithms used in computational biology
- Markov chains used in page rank, simulations of physical systems
- many more applications

# Advice

- graduate school
- undergraduate research
- how to do research
- anything else?

# Questions

Questions?