Proposition: If $\Pi_k^P \subseteq \Sigma_k^P$ then $PH = \Sigma_k^P$.
Similarly, if $\Sigma_k^P \subseteq \Pi_k^P$ then $PH = \Pi_k^P$.

Theorem (Karp-Lipton): $NP \subseteq P_{/poly} \Rightarrow PH = \Sigma_2^P$.

Proof: By above proposition, enough to prove that
$NP \subseteq P_{/poly} \Rightarrow \Pi_2^P \subseteq \Sigma_2^P$.
In particular, suffices to show $\Pi_2 SAT \in \Sigma_2^P$

$$\Pi_2 SAT := \{ \underbrace{\langle \phi \rangle}_{\text{inputs } \{0,1\}^{2n}} \text{ 3CNF} \mid \forall u \in \{0,1\}^n \; \exists v \in \{0,1\}^n \text{ s.t. } \phi(u,v) = 1 \}$$

$NP \subseteq P_{/poly} \Rightarrow \exists\, c \in \mathbb{N}$ and a circuit family $C := \{C_n\}_{n \geq 0}$ in $SIZE(n^c)$ s.t. for every 3CNF ϕ and $u \in \{0,1\}^n$ $C_n(\phi, u) = 1$ iff $\exists\, v \in \{0,1\}^n$ s.t. $\phi(u,v) = 1$.

From the family C, we can construct multi-output circuit family $\Gamma := \{\Gamma_n\}_{n \geq 0}$ in $SIZE(n^{2c})$ s.t. $\Gamma_n(\phi, u)$ outputs a satisfying assignment v in case one exists.

Note that $NP \subseteq P_{/poly}$ simply guarantees the existence of Γ in $SIZE(n^{2c})$. Hence, we can use the $\exists$ quantifier

Note that $NP \subseteq P_{/poly}$ simply guarantees the existence of Γ in $SIZE(n^{2c})$. However we can use the $\exists$ quantifier in Σ_2^P to "guess" Γ as follows:

Let $\gamma \in \mathbb{N}$ be s.t. $s_p(n) \leq \gamma \cdot n^{2c}$ $\forall\, n \in \mathbb{N}$.

Then, consider the following language in Σ_2^P:

$$L := \left\{ \underset{2n \text{ inputs}}{\langle \phi \rangle \overset{\text{3CNF}}{\text{on}}} \;\middle|\; \exists\, w \in \{0,1\}^{\gamma n^{2c}} \; \forall u \in \{0,1\}^n \; \begin{array}{r} w \text{ encodes a circuit } \tilde{C} \text{ and} \\ \phi(u, \tilde{C}(\phi, u)) = 1 \end{array} \right\}$$

Note that $\phi \in \Pi_2 SAT \Rightarrow \phi \in L$, as we can take $w = \Gamma_n$ and $\Gamma_n(\phi, u)$ always outputs a satisfying assignment to $\phi(u, -)$.

Now, if $\phi \notin \Pi_2 SAT$ then $\exists\, u \in \{0,1\}^n$ s.t. no $v \in \{0,1\}^n$ satisfies $\phi(u, -) \Rightarrow \phi \notin L$.

$\therefore \Pi_2 SAT \leq_p L \Rightarrow \Pi_2^P = \Sigma_2^P$. $\square$