Lecture 24: Conclusion

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Overview

- Course Overview and Recap
  - What have we learned?
  - High-Level Principles
  - Interconnectedness
- Administrivia
- Where do we go from here?
  - Next steps
  - "Real" Life

What was this course about?

In your previous algorithms/optimization/data structures course, you learned some of the following:

- combinatorial techniques (divide-and-conquer, greedy algorithms, dynamic programming, local search, etc.)
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The techniques above emphasized **two computational models** (sequential & deterministic computation, query model).

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In this course we used the algorithmic lens to:

- explore several models of computation:
  1. deterministic sequential
  2. randomized sequential
  3. randomized parallel
  4. sublinear-time
  5. memory constrained (streaming)
  6. distributed
  7. online (competitive analysis)
  8. algebraic
  9. interactive

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- expand your algorithmic toolkit
  1. amortized analysis
  2. use of randomness
  3. concentration inequalities
  4. dealing with NP-complete problems (approximation algorithms)
  5. exploring the limits of approximation algorithms

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  7. Can we show that our algorithm is the best?
     That is, we could try to reduce it to another problem which is known to be optimum (perhaps under certain complexity assumptions)

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  - use of data structures in sequential algorithms
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- Learned how to use amortized analysis to provide better overall guarantees
  - vanilla amortization — count all costs
  - charging scheme — assign charges to operations
  - potential function — assign charges and potential to data structure

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  - **online algorithms**: want *fast updates*, need to *decide on the spot*. Want to do as best as we can compared to *best in hindsight* (algorithms that can see entire input beforehand)
  - *competitive analysis*
  - Examples: multiplicative weights update, paging, k-server

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  - approximation algorithms (randomized rounding)

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- when the above does not happen, randomness to the rescue

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- Also saw how we can use interactive proofs (PCPs) to construct reductions which preserve a gap between YES and NO instances
- Interaction not only good for hardness of approximation - saw how to use interaction to give *zero knowledge proofs*

  How to convince someone that you know something without revealing any knowledge on how you do it.

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- More generally: "any polynomial you can compute is a determinant"

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- Many models
  - *Memory & Communication*: shared memory, message-passing
  - *Timing*: synchronous (rounds), asynchronous, partially synchronous (bounds on message delay, processor speeds, clock rates)
  - *Failures*: processor (stop, Byzantine), communication (message loss/altered), system state corruption

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- Algorithms used to prove lower bounds (recent trend - highly recommended!)
- Algorithms in forms of reductions, used to prove that even easy problems cannot be improved! (fine-grained complexity)

Rate this course!

Please log in to

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- This would really help me figuring out what worked and what didn't for the course
- And let the school know if I was a good boy this term!
- Teaching this course is also a learning experience for me :)

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How can I learn more?

Consider taking more advanced courses next term!
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- Current graduate course offerings for next term!

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  `https://cs.uwaterloo.ca/current-undergraduate-students/research-opportunities/undergraduate-research-fellowship-urf`

But is this theory stuff useful?

- Certainly so - and lately the gap between theory and practice has been quite short
- intense use of theoretical cryptography and distributed computing in cryptocurrencies
- cryptography highly used in e-commerce
- several algorithms used in computational biology
- Markov chains used in page rank, simulations of physical systems
- many more applications

Questions

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