Lecture 24: Distributed Algorithms

Rafael Oliveira

University of Waterloo
Cheriton School of Computer Science

rafael.oliveira.teaching@gmail.com

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Overview

- Administrivia
- Distributed Computing: The Models
- Consensus with Byzantine Failures
- Conclusion
- Acknowledgements

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Today is the **last day** to provide us (and the school) with your evaluation and feedback on the course!

- This would really help me figuring out what worked and what didn't for the course
- And let the school (and santa) know if I was a good boy this term!
- Teaching this course is also a learning experience for me :)

How can I learn more?

Consider taking more advanced courses next term!
See graduate course openings at:

- Current graduate course offerings for next term!

  `https://cs.uwaterloo.ca/current-graduate-students/courses/current-course-offerings/fall-2019-course-offerings/tentative-winter-2021-course-offerings`

- Classes by:
  1. Eric Blais (sublinear time algorithms)
  2. Shalev Ben-David (quantum query and communication)
  3. Gautam Kamath (intro to machine learning)
  4. Trevor Brown (multicore programming)
  5. Jeff Shallit (formal languages and parsing)
  6. Myself (intro to symbolic computation & advanced topics in algebra, complexity and optimization)!
- Or, try out some of the research opportunities at UW!

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- Many models
  - *Memory & Communication*: shared memory, message-passing
  - *Timing*: synchronous (rounds), asynchronous, partially synchronous (bounds on message delay, processor speeds, clock rates)
  - *Failures*: processor (stop, Byzantine), communication (message loss/altered), system state corruption

Synchronous Model

- Processes are vertices of directed graph
  - *Memory*: each processor has its own memory
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- Σ is the message alphabet, plus special symbol $\perp$
- For each vertex $i \in [n]$, a process consists of:
  - S_i = non-empty set of states
  - σ_i = a start state
  - $\mu_i : S_i \times out_i \to \Sigma \cup \{\perp\}$ — Message function
  - $\tau_i : S_i \times (\Sigma \cup \{\perp\})^{in_i} \to S_i$ — Transition function

Processes are deterministic algorithms

Vector of incoming messages

out_i ← set of outgoing edges

in_i ← incoming edges

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- Complexity Measure: *number of rounds* needed to solve problem
  - Processes have *unlimited internal resources* (i.e., can compute anything)
  - For today, will assume each process deterministic
  - total data communicated

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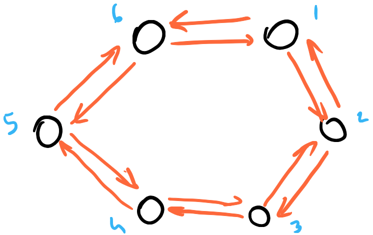

$n = 6$

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- **Theorem:** all processes identical (same set of states and transition functions) and deterministic then it is *impossible* to elect a leader!
- To show this, simply look at execution and check that all processes will always be at identical states.

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- Can reduce communication to $O(n \log n)$ by successively doubling (see reference)

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- Model: synchronous model, arbitrary number of message failures.
- **Input**: Each process has one bit of input. 1 (attack) or 0 (don't attack)
- **Output**: all should have *same decision bit* b satisfying *weak validity*.[1]
  - if all processes start with bit 0, then 0 is only allowed decision
  - if all start with 1 and *all messages successfully delivered*, then 1 is the only allowed decision.

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- Two types of failures:
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- Complexity measures: *number of rounds* & *communication* (# messages exchanged in bit-size).

Byzantine Consensus - Complete Graph

- Assume all vertices can talk to any other vertex ("broadcast" setting)

Broadcast here means they can reach in one hop any other vertex.

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n = size of network
f = # faulty processes

Example: $n = 3$, $f = 1$

p_1, p_2 honest p_3 faulty

inputs: $x_1 = 1$ $x_2 = 0$ $x_3 = 0$

p_3 sends 1 to p_1 ⇒ 1 0 1 attack

p_3 sends 0 to p_2 ⇒ 1 0 0 don't attack

violated agreement

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- At each round, each vertex broadcasts its knowledge
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- Does this work?
- How many rounds do we need?
- How many Byzantine failures can it tolerate?

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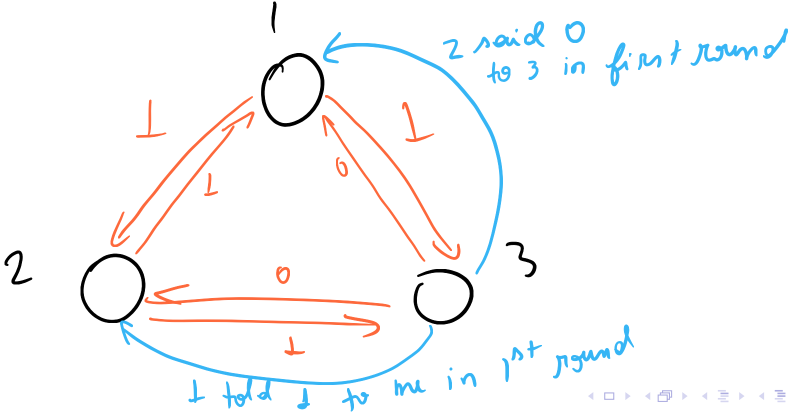

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- Scenario 3: v_1, v_3 good with values $1, 0$ (resp.), v_2 faulty with value 0
  1. Round 1: v_2 tells v_1 its value is 1, tells v_3 its value is 0
  2. Round 2: all truthful

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- Scenarios 1 and 3 identical to v_1, so it must return 1 (validity)

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- Scenarios 1 and 3 identical to v_1, so it must return 1 (validity)
- Scenarios 2 and 3 identical to v_3, so it must return 0 (validity)
- Contradicts *agreement* in Scenario 3!

Byzantine Consensus - Algorithm

- Assumption:[2] $n > 3f$ (number of bad vertices $<$ third total vertices)

[2] It turns out that $n \leq 3f \Rightarrow$ *no algorithm* can reach consensus!

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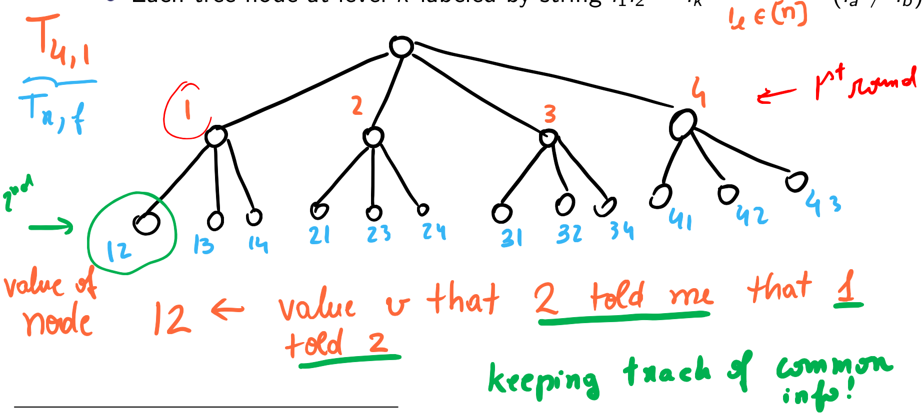
- How to perfectly gossip?
- Data structure: *Exponential Information Gathering* (EIG) tree
  - Depth: $f + 1$ (so $f + 2$ node levels)
  - Each tree node at level k labeled by string $i_1 i_2 \cdots i_k$ ($i_a \neq i_b$)



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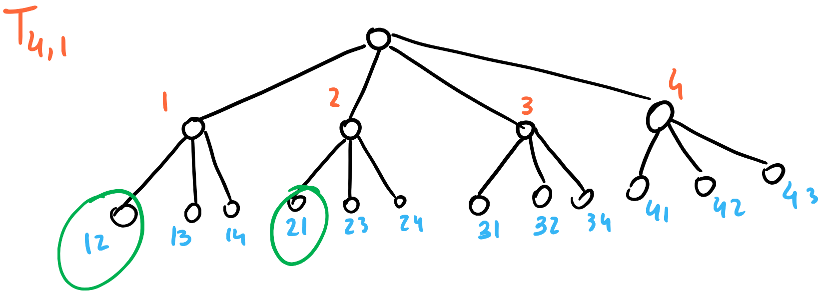
Byzantine Consensus - EIG Tree



$i_1 \; i_2 \cdots i_k$

$i_1 \to i_2 \to i_3 \to \cdots \to i_k \to me$

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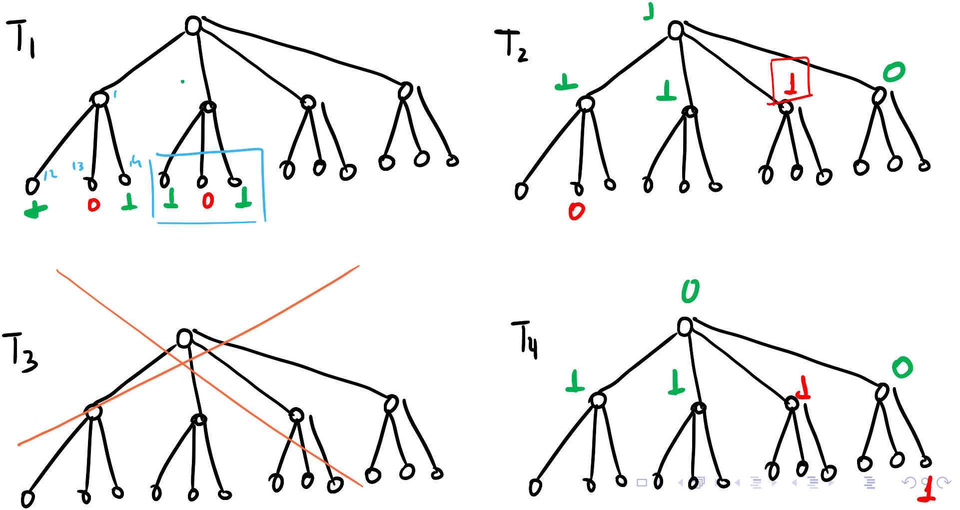
2. Relay messages for $f + 1$ rounds
   - At round r, each vertex sends the values of level r of its EIG tree
   - Each vertex decorates values of its $(r + 1)^{th}$ level with values from messages
3. After $f + 1$ rounds, redecorate tree bottom-up, taking strict majority of children (otherwise set value of tree node to $\perp$)

EIG Algorithm - Example

$n=4$
$f=1$

P_3 is faulty

$P_1 = P_2 = 1$
$P_3 = P_4 = 0$ } initially



EIG Algorithm - Analysis

Lemma (Consistency of Non-Faulty Messages)

If i, j, k are non-faulty, then $T_i(x) = T_j(x)$ whenever label x ends with k.

$x = 1\,2\,3\,4\,k$

$1 \to 2 \to 3 \to 4 \xrightarrow{v} k$

$T_i(x) = v$

$T_j(x) = v$

both of them are assigned after receiving (same) message from k

EIG Algorithm - Analysis

Lemma (Consistency of Upwards Relabeling)

If label x ends with non-faulty process, then for any two non-faulty processes i, j the new values of $T_i(x)$ and $T_j(x)$ are the same.

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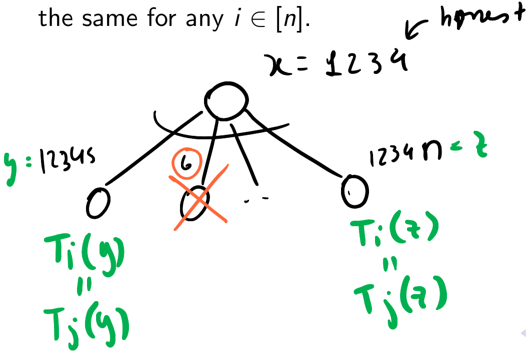

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  - So label x has same labeled children across trees

so long as $x\ell$ → is honest

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$x = 12 \cdots k$

$$= n - k > 3f - f = 2f$$

$x(k+1)$
$\vdots$
$x(n)$ $\Big\}$ $n-k$

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  - At most f are faulty. By taking majority, we get that new values $T_i(x) = T_j(x)$

$T_i(x\ell) = T_j(x\ell)$ for at least 1+ half the children

EIG Algorithm - Analysis

So far we have managed to prove:

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3. *Agreement*: all nodes must agree on same value
   - By first lemma, all values in the leaves x are consistent across processes so long as x ends on a non-faulty process
   - By second lemma, majority will cause all values in nodes from level r ending in non-faulty nodes to be *the same* across processes
   - Induction and $n > 3f$ ensures that labels in level 1 will look the same on non-faulty nodes $\Rightarrow$ agreement

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  - Multi-core programming

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  - many more...
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  - It is cool
- Learned an (inefficient) algorithm for Byzantine Agreement (check out the more efficient one in [Attiya and Welch 2004])

Acknowledgement

- Lecture based largely on:
  - Nancy Lynch's 6.852 Fall 2015 course - lectures 1 and 6
  - Lecture 1

    https://learning-modules.mit.edu/service/materials/groups/103042/files/271154f5-ea0f-41a0-9ed9-6f83a5222d8b/link?errorRedirect=%2Fmaterials%2Findex.html&download=true
  - Lecture 6

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