Lecture 3

Conditional Independence
CS 886
Sept 21, 2010
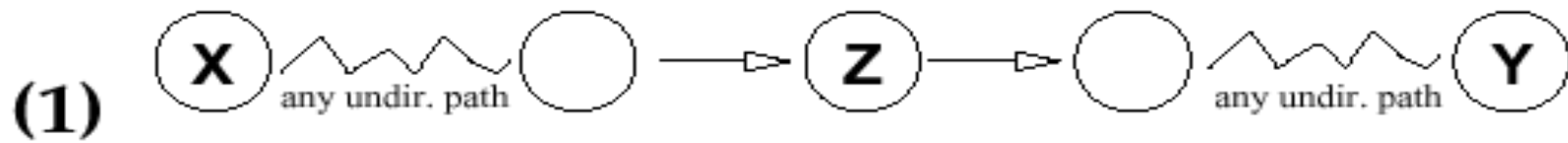
Testing Independence

- Given BN, how do we determine if two variables X, Y are independent (given evidence E)?
  - we use a (simple) graphical property

- **D-separation**: A set of variables **E** *d-separates* X and Y if it *blocks every undirected path* in the BN between X and Y.

- X and Y are conditionally independent given evidence **E** if **E** d-separates X and Y
  - thus BN gives us an easy way to tell if two variables are independent (set E = $\varnothing$) or cond. independent
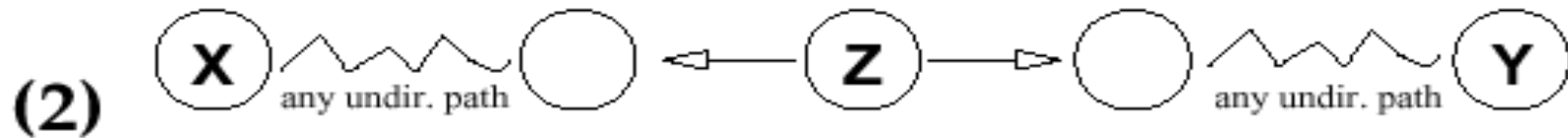
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Blocking in D-Separation

- Let P be an undirected path from X to Y in a BN. Let **E** be an evidence set. We say **E** *blocks path P* iff there is some node Z on the path such that:

  - **Case 1**: one arc on P *goes into* Z and one *goes out* of Z, and Z∈**E**; or

  - **Case 2**: both arcs on P leave Z, and Z∈**E**; or

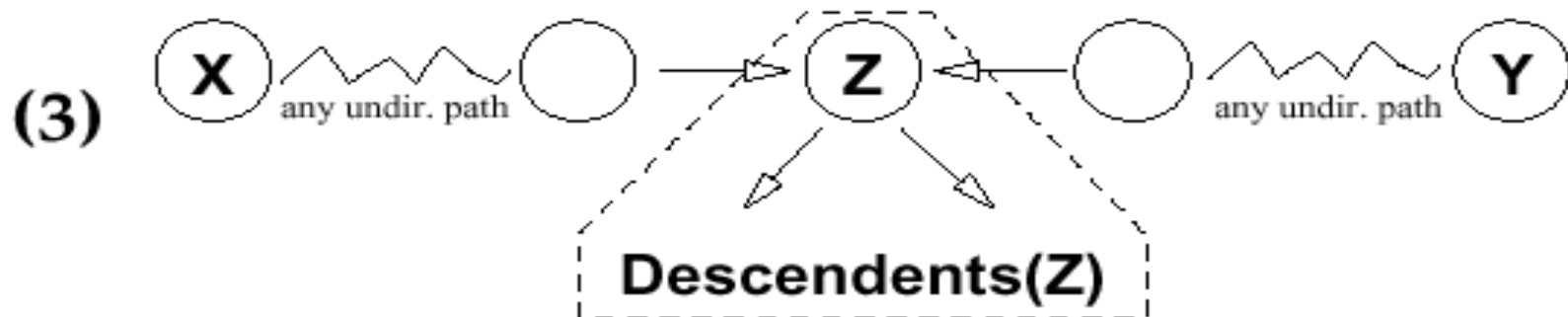 - **Case 3**: both arcs on P enter Z and *neither Z, nor any of its descendents*, are in **E**.

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Blocking: Graphical View

(1) If Z in evidence, the path between X and Y blocked

(2) If Z in evidence, the path between X and Y blocked

(3) Descendents(Z)

If Z is **not** in evidence and **no** descendent of Z is in evidence, then the path between X and Y is blocked
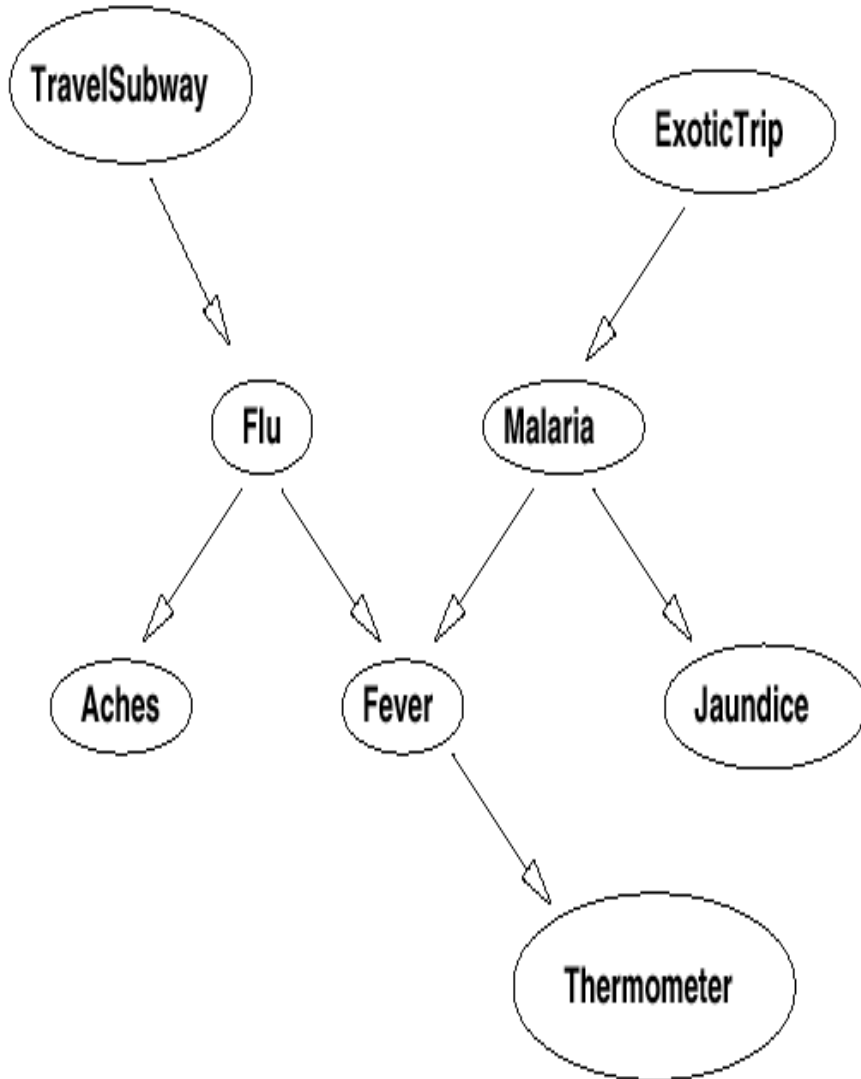
D-Separation: Intuitions

1. Subway and Thermometer?

2. Aches and Fever?

3. Aches and Thermometer?

4. Flu and Malaria?

5. Subway and ExoticTrip?

D-Separation: Intuitions

- Subway and Therm are dependent; but are independent given Flu (since Flu blocks the only path)

- Aches and Fever are dependent; but are independent given Flu (since Flu blocks the only path). Similarly for Aches and Therm (dependent, but indep. given Flu).

- Flu and Mal are indep. (given no evidence): Fever blocks the path, since it is *not in evidence*, nor is its descendant Therm. Flu,Mal are dependent given Fever (or given Therm): nothing blocks path now.

- Subway,ExoticTrip are indep.; they are dependent given Therm; they are indep. given Therm and Malaria. This for exactly the same reasons for Flu/Mal above.

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