Lecture 3b: Deep Q-networks
CS885 Reinforcement Learning

2025-01-14

Complementary readings: [GBC] Chap. 6, 7, 8, [SutBar] Sec. 9.4, 9.7, [Sze] Sec. 4.3.2

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Outline

- RL with function approximation
  - Linear approximation
  - Neural network approximation
- Algorithms:
  - Gradient Q-learning
  - Deep Q-Network (DQN)

Quick Recap

- Markov decision processes: value iteration

$$V(s) \leftarrow \max_a R(s,a) + \gamma \sum_{s'} \Pr(s'|s,a) V(s')$$

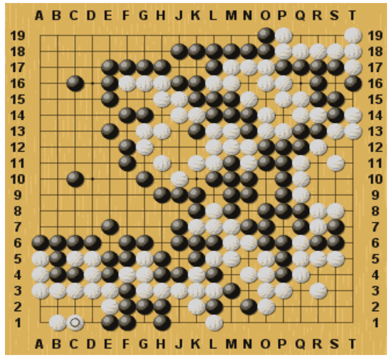
- Reinforcement learning: Q-learning

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left[r + \gamma \max_{a'} Q(s',a') - Q(s,a) \right]$$

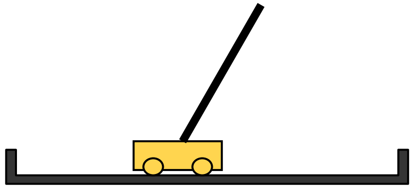
- Complexity depends on number of states and actions

Large State Spaces

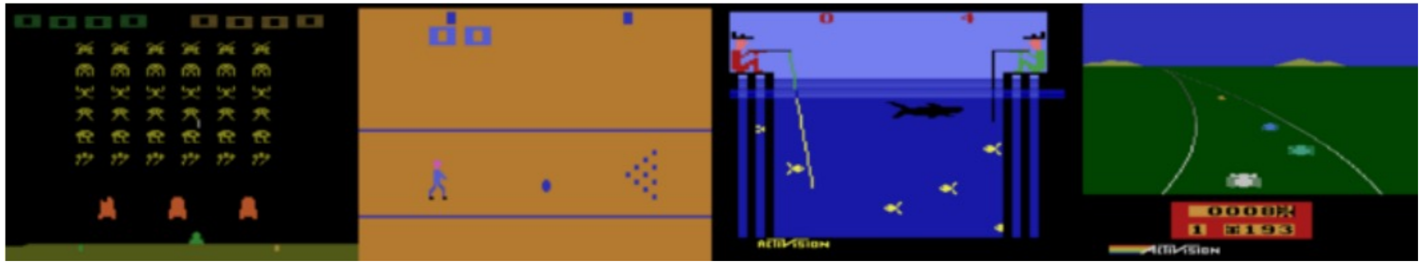
- Computer Go: 3^{361} states



- Inverted pendulum: (x, x', θ, θ')
  - 4-dimensional continuous state space



- Atari: 210 x 160 x 3 dimensions (pixel values)



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Functions to be Approximated

- Policy: $\pi(s) \rightarrow a$

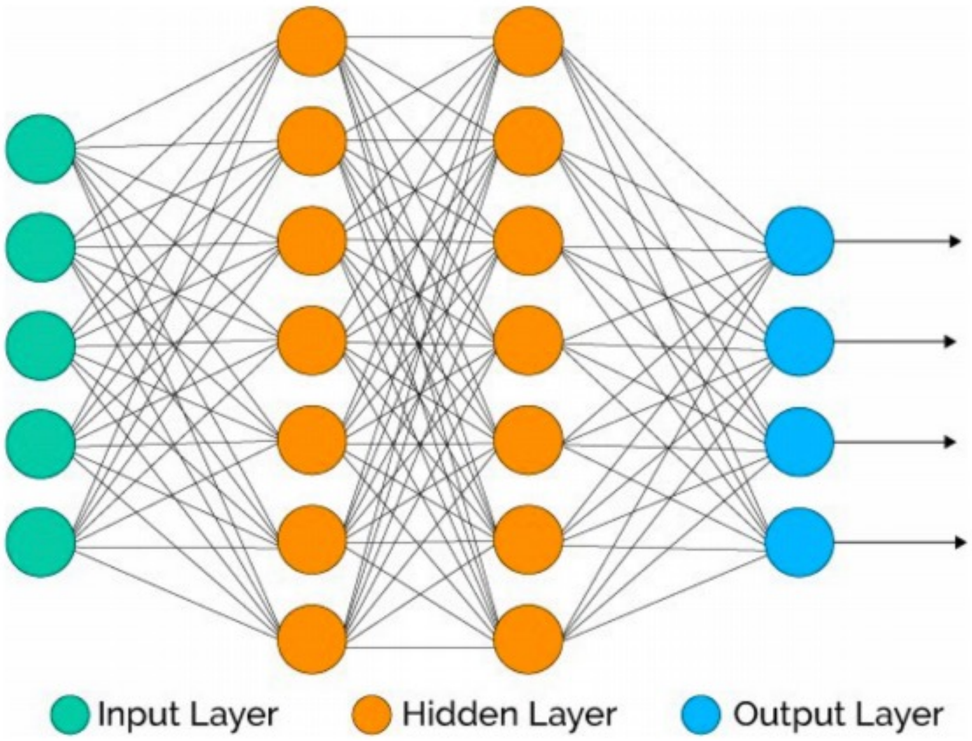
- Q-function: $Q(s,a) \in \Re$

- Value function: $V(s) \in \Re$

Traditional Neural Network

- Network of units (computational neurons) linked by weighted edges

- Each unit computes: $z = h(\boldsymbol{w}^T \boldsymbol{x} + b)$
  - Inputs: $\boldsymbol{x}$
  - Outputs: z
  - Weights (parameters): $\boldsymbol{w}$
  - Bias: b
  - Activation function (usually non-linear): h



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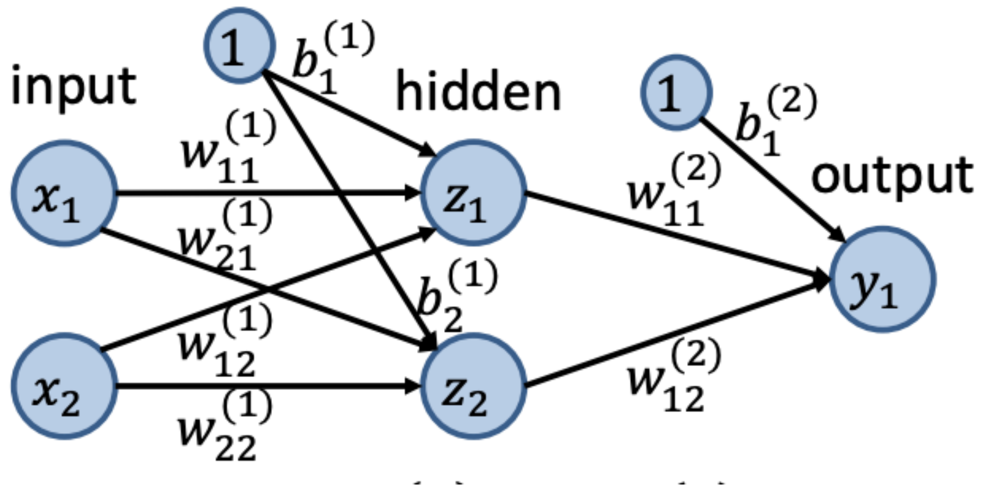
One Hidden Layer Architecture



- Feed-forward neural network
  - Hidden units: $z_j = h_1(\boldsymbol{w}_j^{(1)}\boldsymbol{x} + b_j^{(1)})$
  - Output units: $y_k = h_2(\boldsymbol{w}_k^{(2)}\boldsymbol{z} + b_k^{(2)})$
  - Overall: $y_k = h_2\left(\sum_j w_{kj}^{(2)} h_1\left(\sum_i w_{ji}^{(1)} x_i + b_j^{(1)}\right) + b_k^{(2)}\right)$

Common Activation Functions

- Sigmoid: $h(a) = \sigma(a) = \frac{1}{1+e^{-a}}$
- Softmax: $h(\boldsymbol{a})_i = \frac{e^{a_i}}{\sum_j e^{a_j}}$
- Tanh (hyperbolic tangent): $h(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$
- Gaussian: $h(a) = e^{-0.5\left(\frac{a-\mu}{\sigma}\right)^2}$
- ReLU (Rectified Linear Unit): $h(a) = \max(a, 0)$
- Identity: $h(a) = a$

Universal Function Approximator

Theorem: Neural networks with at least one hidden layer of sufficiently many sigmoid/tanh/Gaussian units can approximate any function arbitrarily closely.

Q-function Approximation

- Let $s = (x_1, x_2, \dots, x_n)^T$
- Linear: $Q(s,a) \approx \sum_i w_{ai} x_i$
- Non-linear (e.g., neural network): $Q(s,a) \approx g(\boldsymbol{x}; \boldsymbol{w})$

Gradient Q-learning

- Minimize squared error between Q-value estimate and target
  - Q-value estimate: $Q_{\boldsymbol{w}}(s,a)$
  - Target: $r + \gamma \max_{a'} Q_{\overline{\boldsymbol{w}}}(s',a')$
- Squared error: $Err(\boldsymbol{w}) = \frac{1}{2}[Q_{\boldsymbol{w}}(s,a) - r - \gamma \max_{a'} Q_{\overline{\boldsymbol{w}}}(s',a')]^2$

  $\overline{\boldsymbol{w}}$ fixed
- Gradient: $\frac{\partial Err}{\partial \boldsymbol{w}} = \left[Q_{\boldsymbol{w}}(s,a) - r - \gamma \max_{a'} Q_{\overline{\boldsymbol{w}}}(s',a')\right] \frac{\partial Q_{\boldsymbol{w}}(s,a)}{\partial \boldsymbol{w}}$

Gradient Q-learning

Initialize weights $\boldsymbol{w}$ at random in $[-1,1]$

Observe current state s

Loop

Select action a and execute it

Receive immediate reward r

Observe new state s'

Gradient: $\frac{\partial Err}{\partial \boldsymbol{w}} = \left[Q_{\boldsymbol{w}}(s,a) - r - \gamma \max_{a'} Q_{\boldsymbol{w}}(s',a')\right] \frac{\partial Q_{\boldsymbol{w}}(s,a)}{\partial \boldsymbol{w}}$

Update weights: $\boldsymbol{w} \leftarrow \boldsymbol{w} - \alpha \frac{\partial Err}{\partial \boldsymbol{w}}$

Update state: $s \leftarrow s$'

Recap: Convergence of Tabular Q-learning

- Tabular Q-Learning converges to optimal Q-function under the following conditions:

  $\sum_{t=0}^{\infty} \alpha_t = \infty$ and $\sum_{t=0}^{\infty} \alpha_t^2 < \infty$

- Let $\alpha(s,a) = 1/n(s,a)$
  - Where $n(s,a)$ is # of times that (s,a) is visited

- Q-learning: $Q(s,a) \leftarrow Q(s,a) + \alpha(s,a)[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$

Convergence of Linear Gradient Q-Learning

- Linear Q-Learning converges under the same conditions:

  $\sum_{t=0}^{\infty} \alpha_t = \infty$ and $\sum_{t=0}^{\infty} \alpha_t^2 < \infty$

- Let $\alpha_t = 1/t$
- Let $Q_{\boldsymbol{w}}(s,a) = \sum_i w_i x_i$

- Q-learning: $\boldsymbol{w} \leftarrow \boldsymbol{w} - \alpha_t \left[Q_{\boldsymbol{w}}(s,a) - r - \gamma \max_{a'} Q_{\boldsymbol{w}}(s',a') \right] \frac{\partial Q_{\boldsymbol{w}}(s,a)}{\partial \boldsymbol{w}}$

Divergence of Non-linear Gradient Q-learning

- Even when the following conditions hold

  $\sum_{t=0}^{\infty} \alpha_t = \infty$ and $\sum_{t=0}^{\infty} \alpha_t^2 < \infty$

  non-linear Q-learning may diverge

- Intuition:
  - Adjusting $\boldsymbol{w}$ to increase Q at (s, a) might introduce errors at nearby state-action pairs.

Mitigating divergence

- Two tricks are often used in practice:
  1. Experience replay
  2. Use two networks:
     - Q-network
     - Target network

Experience Replay

- Idea: store previous experiences (s, a, s', r) into a buffer and sample a mini-batch of previous experiences at each step to learn by Q-learning

- Advantages
  - Break correlations between successive updates (more stable learning)
  - Less interactions with environment needed to converge (better data efficiency)

Target Network

- Idea: Use a separate target network that is updated only periodically

repeat for each (s, a, s', r) in mini-batch:

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \alpha_t \Big[\underbrace{Q_{\boldsymbol{w}}(s,a)}_{\text{update}} - r - \gamma \max_{a'} \underbrace{Q_{\overline{\boldsymbol{w}}}(s',a')}_{\text{target}}\Big] \frac{\partial Q_{\boldsymbol{w}}(s,a)}{\partial \boldsymbol{w}}$$

$$\overline{\boldsymbol{w}} \leftarrow \boldsymbol{w}$$

- Advantage: mitigate divergence

Target Network

- Similar to value iteration:

repeat for all s

$$\underbrace{V(s)}_{\text{update}} \leftarrow \max_a R(s) + \gamma \sum_{s'} \Pr(s'|s,a) \underbrace{\bar{V}(s')}_{\text{target}} \quad \forall s$$

$$\bar{V} \leftarrow V$$

repeat for each (s, a, s', r) in mini-batch:

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \alpha_t \left[\underbrace{Q_{\boldsymbol{w}}(s,a)}_{\text{update}} - r - \gamma \max_{a'} \underbrace{Q_{\bar{\boldsymbol{w}}}(s',a')}_{\text{target}} \right] \frac{\partial Q_{\boldsymbol{w}}(s,a)}{\partial \boldsymbol{w}}$$

$$\bar{\boldsymbol{w}} \leftarrow \boldsymbol{w}$$

Deep Q-network (DQN)

- Deep Mind

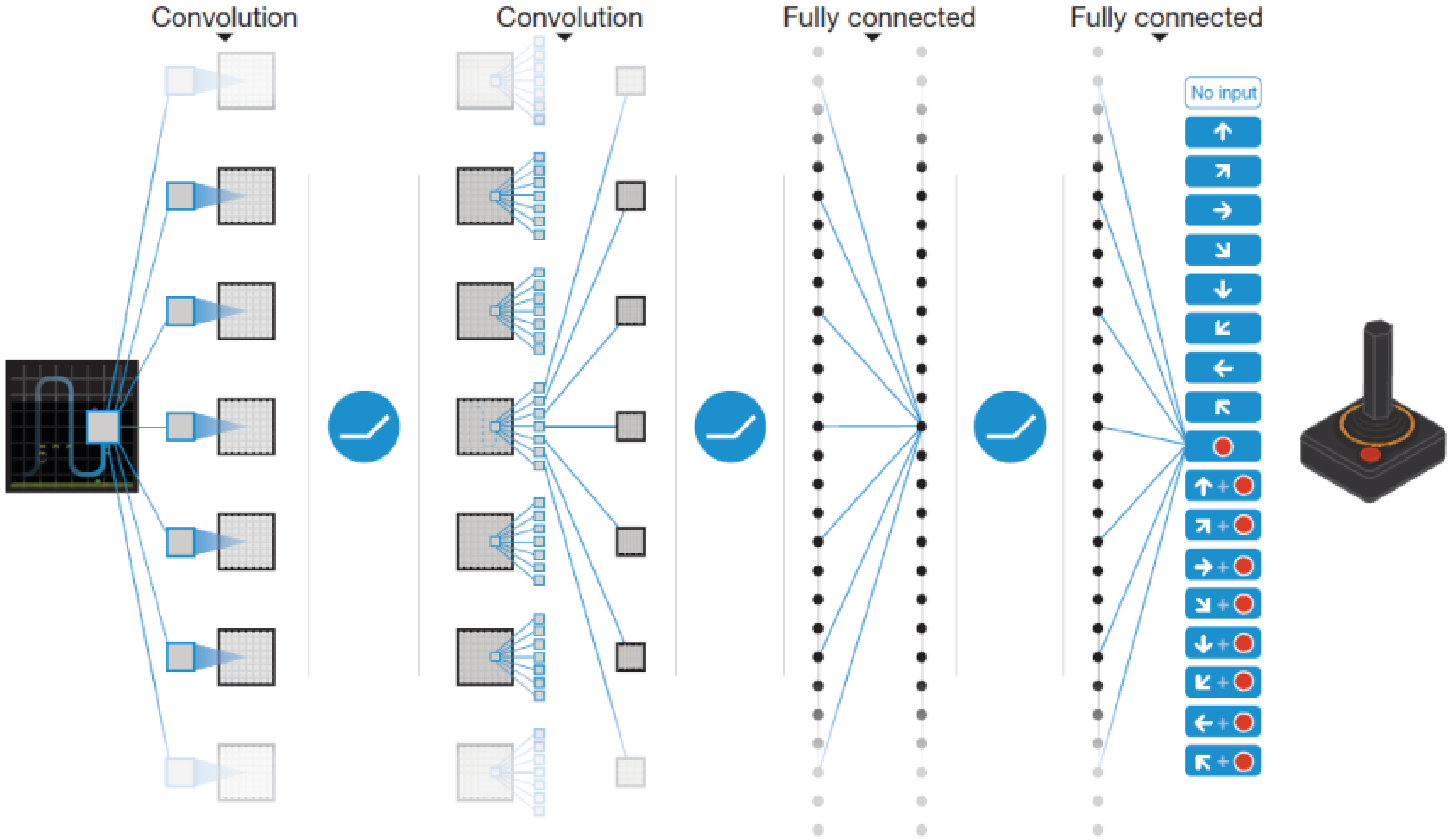
- Deep Q-network: Gradient Q-learning with
  - Deep neural networks
  - Experience replay
  - Target network

- Breakthrough: human-level play in many Atari video games

Deep Q-network (DQN)

Initialize weights $\boldsymbol{w}$ and $\overline{\boldsymbol{w}}$ at random in $[-1,1]$
Observe current state s
Loop
    Select action a and execute it
    Receive immediate reward r
    Observe new state s'
    Add (s, a, s', r) to experience buffer
    Sample mini-batch of experiences from buffer
    For each experience $(\hat{s}, \hat{a}, \hat{s}', \hat{r})$ in mini-batch
        Gradient: $\frac{\partial Err}{\partial \boldsymbol{w}} = \left[Q_{\boldsymbol{w}}(\hat{s}, \hat{a}) - \hat{r} - \gamma \max_{\hat{a}'} \mathrm{Q}_{\overline{\mathbf{w}}}(\hat{s}', \hat{a}') \right] \frac{\partial Q_{\boldsymbol{w}}(\hat{s},\hat{a})}{\partial \boldsymbol{w}}$
        Update weights: $\boldsymbol{w} \leftarrow \boldsymbol{w} - \alpha \frac{\partial Err}{\partial \boldsymbol{w}}$
    Update state: $s \leftarrow s$'
    Every c steps, update target: $\overline{\boldsymbol{w}} \leftarrow \boldsymbol{w}$

Deep Q-Network for Atari



DQN versus Linear Approximation