Lecture 3a: Intro to RL, Q-Learning
CS885 Reinforcement Learning

2025-01-14

Complementary readings: [SutBar] Sec. 5.1-5.3, 6.1-6.3, 6.5, [Sze] Sec. 3.1, 4.3, [SigBuf] Sec. 2.1-2.5, [RusNor] Sec. 21.1-21.3

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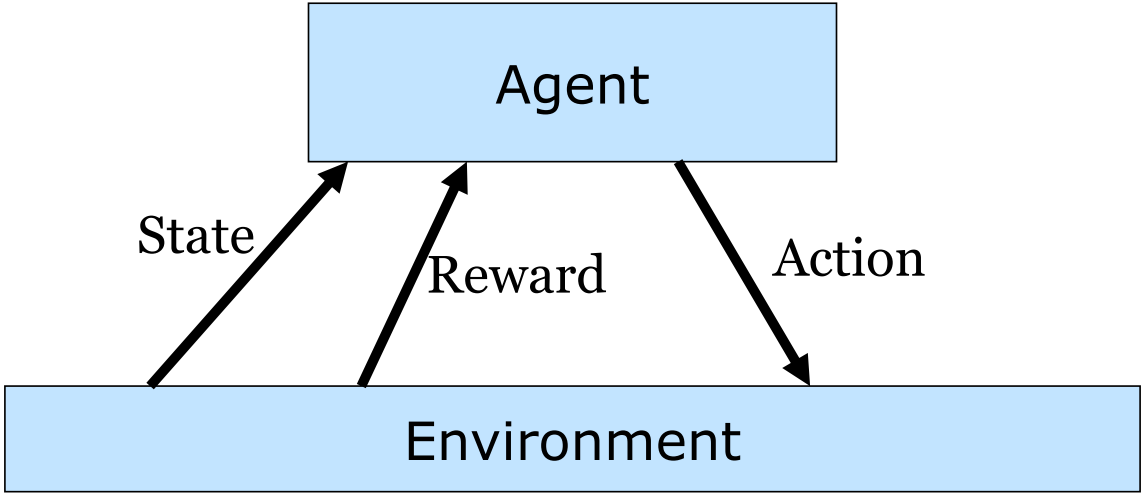
Outline

- Reinforcement Learning
    - Q-Learning
    - Exploration strategies

Recap: Reinforcement Learning Problem



Agent

State

Reward

Action

Environment

Goal: Learn to choose actions that maximize rewards

Reinforcement Learning

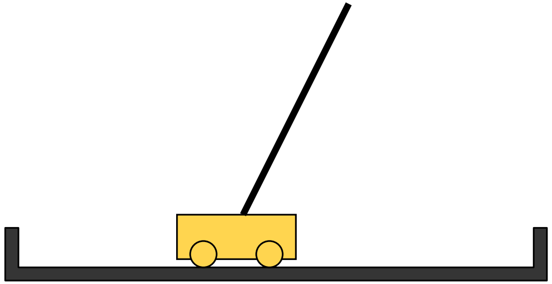
- Formal Definition
  - States: $s \in S$
  - Actions: $a \in A$
  - Rewards: $r \in \mathbb{R}$
  - ~~Transition model: $\Pr(s_t|s_{t-1}, a_{t-1})$~~
  - ~~Reward model: $\Pr(r_t|s_t, a_t)$~~
  - Discount factor: $0 \leq \gamma \leq 1$
  - Horizon (i.e., # of time steps): h

unknown

- Goal: find optimal policy $\pi^* = argmax_\pi \sum_{t=0}^{h} \gamma^t E_\pi[r_t]$

Example: Inverted Pendulum

- State: $x(t), x'(t), \theta(t), \theta'(t)$
- Action: Force F
- Reward: 1 for any step where pole balanced



Problem: Find $\pi: S \to A$ that maximizes rewards

Important Components in RL

RL agents may or may not estimate the following components:

- **Model**: $\Pr(s'|s,a)\,,\Pr(r|s,a)$
    - Environment dynamics and rewards
- **Policy**: $\pi(s)$
    - Agent action choices
- **Value function**: $V(s)$
    - Expected total rewards of the agent policy

Categorizing RL agents

Value based
- No policy (implicit)
- Value function

Policy based
- Policy
- No value function

Actor critic
- Policy
- Value function

Model based
- Transition and reward model

Model free
- No transition and no reward model (implicit)

Online RL
- Learn by interacting with environment

Offline RL
- No environment
- Learn only from saved data

Bellman's Equation

- Value Iteration:

$$V_n^*(s) \leftarrow \max_a E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) V_{n-1}^*(s')$$

- Bellman Equation (when $n \to \infty$):

$$V^*(s) = \max_a E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) V^*(s')$$

- State-action Bellman Equation:

$$Q^*(s,a) = E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) \max_{a'} Q^*(s',a')$$

where $V^*(s) = max_a Q^*(s,a)$, $\quad \pi^*(s) = argmax_a Q^*(s,a)$

Temporal Difference Control

- Approximate Q-function:

$$Q^*(s,a) = E[r|s,a] + \gamma \sum_{s'} \Pr(s'|s,a) \max_{a'} Q^*(s',a')$$

$$\approx r + \gamma \max_{a'} Q^*(s',a')$$ ← one sample approximation

- **Incremental update**

$$Q_n^*(s,a) \leftarrow Q_{n-1}^*(s,a) + \alpha_n \left(r + \gamma \max_{a'} Q_{n-1}^*(s',a') - Q_{n-1}^*(s,a) \right)$$

learning rate

Tabular Q-Learning

```
Qlearning()
    Initialize $s$ and $Q^*$ arbitrarily
    Repeat
        Select and execute $a$
        Observe $s$' and $r$
        Update counts: $n(s,a) \leftarrow n(s,a) + 1$
        Learning rate: $\alpha \leftarrow 1/n(s,a)$
        $Q^*(s,a) \leftarrow Q^*(s,a) + \alpha\left(r + \gamma \max_{a'} Q^*(s',a') - Q^*(s,a)\right)$
        $s \leftarrow s'$
    Until convergence of $Q^*$
Return $Q^*$
```

Q-learning Exercise

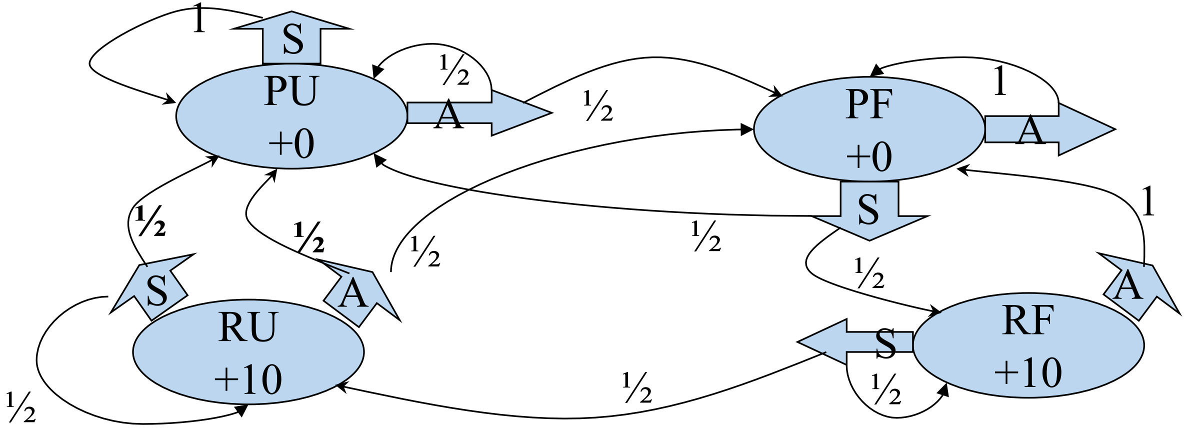
Current estimates:

$Q(RF,S) = 25$

$Q(RF,A) = 20$

$Q(RU,S) = 20$

$Q(RU,A) = 15$



Discount: $\gamma = 0.9$ Learning rate: $\alpha = 0.5$

Update $Q(RF,S)$ after executing S in RF and transitioning to RU:

$$Q(RF,S) \leftarrow Q(RF,S) + \alpha\left(r + \gamma \max\{Q(RU,S), Q(RU,A)\} - Q(RF,S)\right)$$

$$\leftarrow 25 \quad + 0.5(10 + 0.9 \max\{20 , 15\} - 25)$$

$$\leftarrow 26.5$$

Convergence

- Q-learning converges to optimal Q-values if
  - Every state is visited infinitely often (due to exploration)
  - The action selection becomes greedy as time approaches infinity
  - The learning rate α is decreased fast enough, but not too fast (sufficient conditions for α):

    (1) $\sum_t \alpha_t \to \infty$ (2) $\sum_t (\alpha_t)^2 < \infty$

- NB: $\alpha_t(s,a) = 1/n_t(s,a)$ satisfies the above conditions

Common Exploration Methods

- ϵ-greedy:
  - With probability ϵ, execute random action
  - Otherwise execute best action $a^* = argmax_a\, Q(s, a)$

- Boltzmann exploration
  - Increasing temperature T increases stochasticity

$$\Pr(a) = \frac{e^{\frac{Q(s,a)}{T}}}{\sum_a e^{\frac{Q(s,a)}{T}}}$$