CS885 Reinforcement Learning
Lecture 9: May 30, 2018

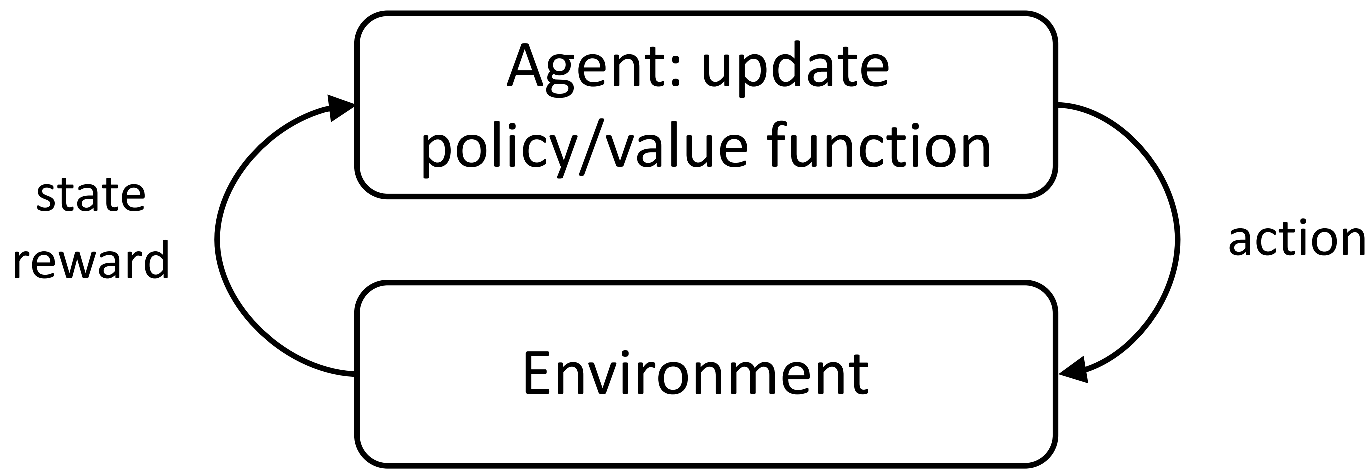
Model-based RL
[SutBar] Chap 8

Outline

- Model-based RL
- Dyna
- Monte-Carlo Tree Search

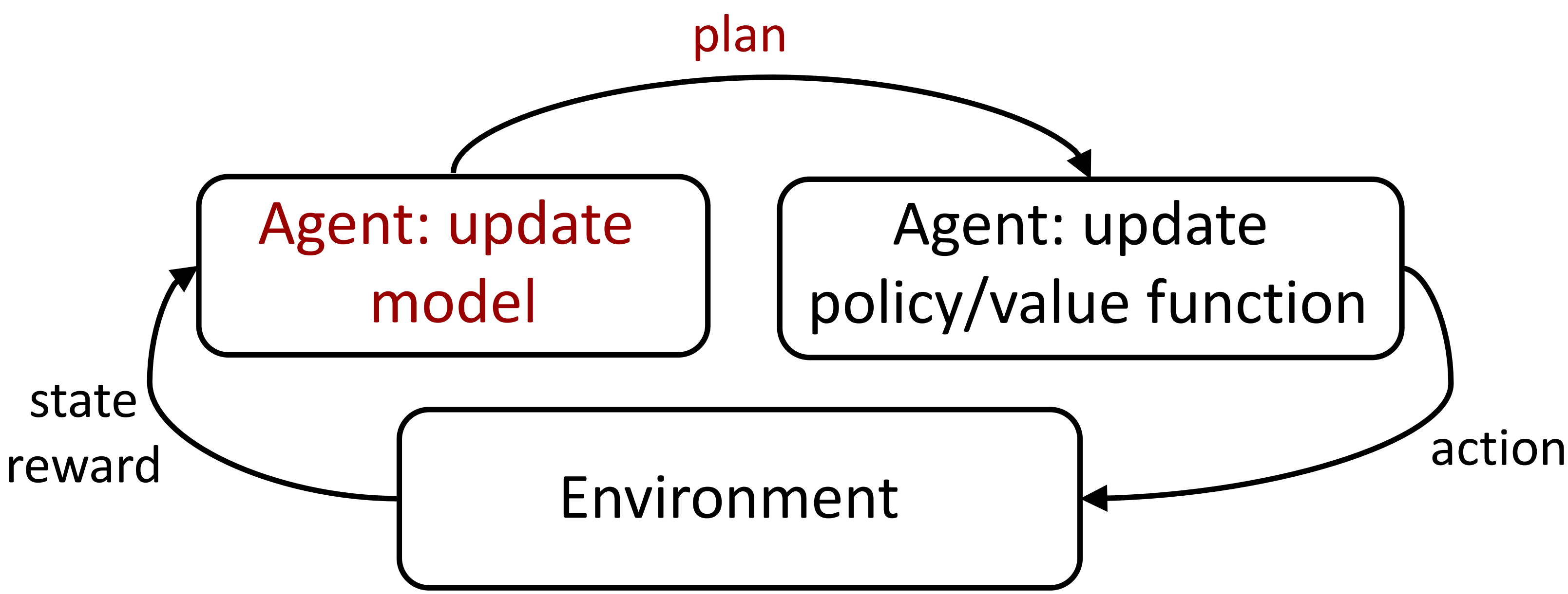
Model-free RL

- No explicit transition or reward models
  - Q-learning: **value-based method**
  - Policy gradient: **policy-based method**
  - Actor critic: **policy and value based method**



Model-based RL

- **Learn explicit transition and/or reward model**
  - **Plan based on the model**
  - Benefit: Increased sample efficiency
  - Drawback: Increased complexity



Maze Example

	1	2	3	4
3	r	r	r	+1
2	u		u	-1
1	u	l	l	l

$\gamma = 1$

Reward is -0.04 for non-terminal states

We need to learn all the transition probabilities!

(1,1)→ (1,2)→ **(1,3)**→ (1,2)→ **(1,3)**→ (2,3)→ (3,3)→ $(4,3)_{+1}$
(1,1)→ (1,2)→ **(1,3)**→ (2,3)→ (3,3)→ (3,2)→ (3,3)→ $(4,3)_{+1}$
(1,1)→ (2,1)→ (3,1)→ (3,2)→ $(4,2)_{-1}$

P((2,3)|(1,3),r) =2/3
P((1,2)|(1,3),r) =1/3

Use this information in

$$V^*(s) = \max_a R(s,a) + \gamma \sum_{s'} \Pr(s'|s,a)\, V^*(s')$$

Model-based RL

- Idea: at each step
  - Execute action
  - Observe resulting state and reward
  - Update transition and/or reward model
  - Update policy and/or value function

Model-based RL (with Value Iteration)

ModelBasedRL(s)

Repeat

Select and execute a

Observe s' and r

Update counts: $n(s,a) \leftarrow n(s,a) + 1$,

$n(s,a,s') \leftarrow n(s,a,s') + 1$

Update transition: $\Pr(s'|s,a) \leftarrow \frac{n(s,a,s')}{n(s,a)}\ \forall s'$

Update reward: $R(s,a) \leftarrow \frac{r + (n(s,a)-1)R(s,a)}{n(s,a)}$

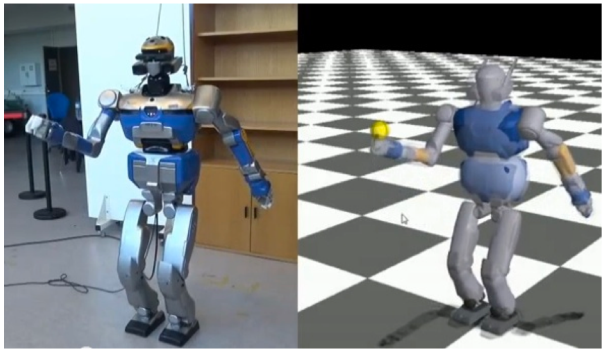
Solve: $V^*(s) = \max_a R(s,a) + \gamma \sum_{s'} \Pr(s'|s,a)\, V^*(s')\ \forall s$

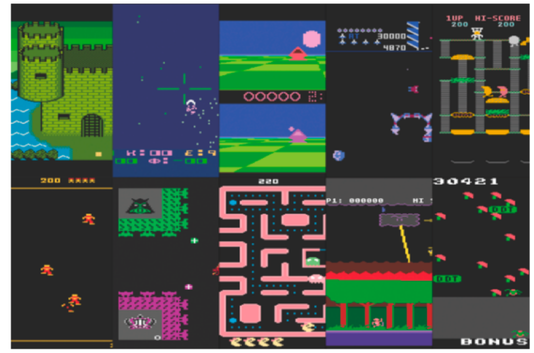
$s \leftarrow s'$

Until convergence of V^*

Return V^*

Complex models





- Use function approx. for transition and reward models
  - Linear model: $pdf(s'|s,a) = N(s'|w^T \begin{bmatrix} s \\ a \end{bmatrix}, \sigma^2 I)$
  - Non-linear models:
    - Stochastic (e.g. Gaussian process):
      $$pdf(s'|s,a) = GP(s|w^T \begin{bmatrix} s \\ a \end{bmatrix}, \sigma^2 I)$$
    - Deterministic (e.g., neural network): $s' = T(s,a) = NN(s,a)$

Partial Planning

- In complex models, fully optimizing the policy or value function at each time step is intractable

- Consider partial planning
  - A few steps of Q-learning
  - A few steps of policy gradient

Model-based RL (with Q-learning)

ModelBasedRL(s)

Repeat

Select and execute a, observe s' and r

Update transition: $w_T \leftarrow w_T - \alpha_T(T(s,a) - s')\nabla_{w_T} T(s,a)$

Update reward: $w_R \leftarrow w_R - \alpha_R(R(s,a) - r)\nabla_{w_R} R(s,a)$

Repeat a few times:

sample $\hat{s}, \hat{a}$ arbitrarily

$\delta \leftarrow R(\hat{s},\hat{a}) + \gamma \max_{\hat{a}'} Q(T(\hat{s},\hat{a}), \hat{a}') - Q(\hat{s},\hat{a})$

Update Q: $w_Q \leftarrow w_Q - \alpha_Q \delta \nabla_{w_Q} Q(\hat{s},\hat{a})$

$s \leftarrow s'$

Until convergence of Q

Return Q

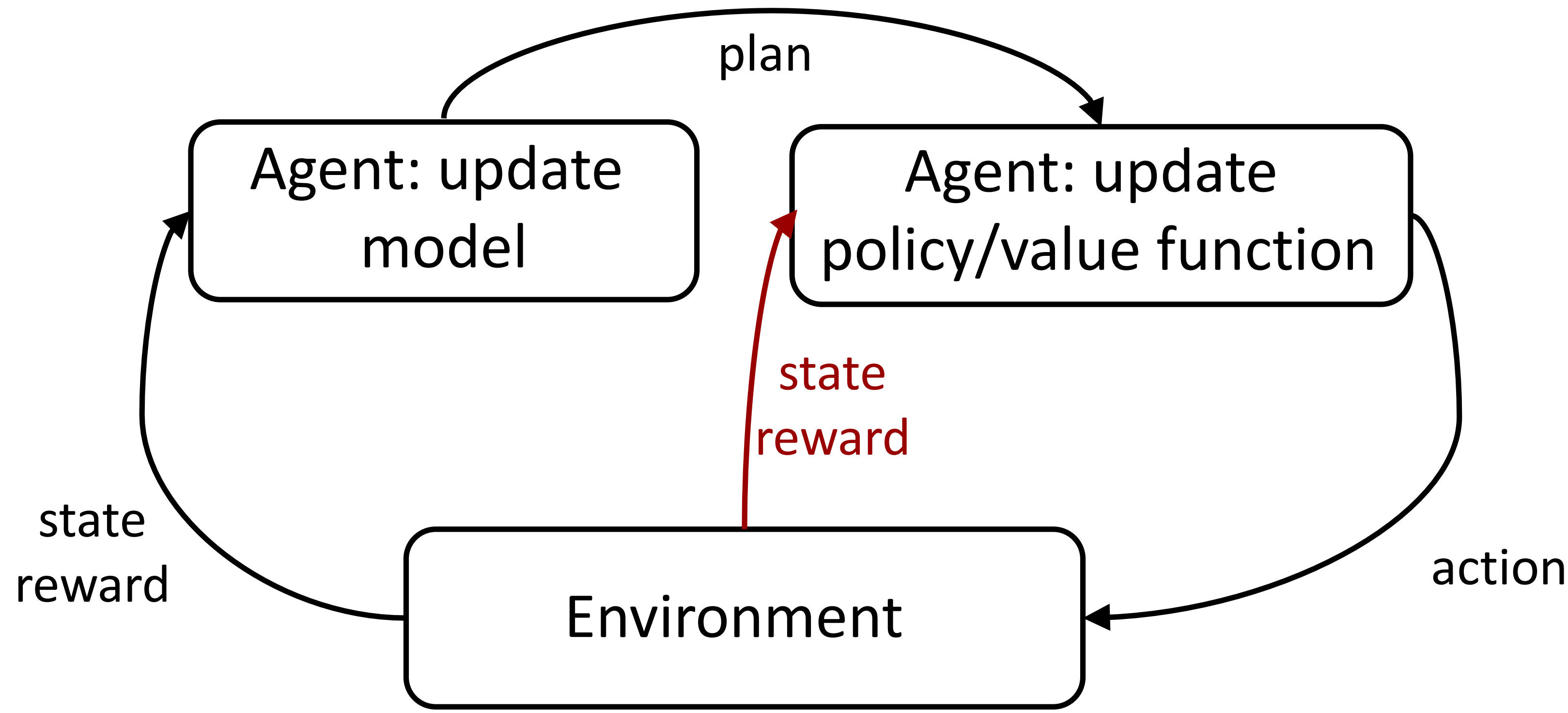
Partial Planning vs Replay Buffer

- Previous algorithm is very similar to Model-free Q-learning with a replay buffer
- Instead of updating Q-function based on samples from replay buffer, generate samples from model

- Replay buffer:
  - Simple, real samples, no generalization to other sate-action pairs
- Partial planning with a model
  - Complex, simulated samples, generalization to other state-action pairs (can help or hurt)

Dyna

- **Learn explicit transition and/or reward model**
  - Plan based on the model
- **Learn directly from real experience**

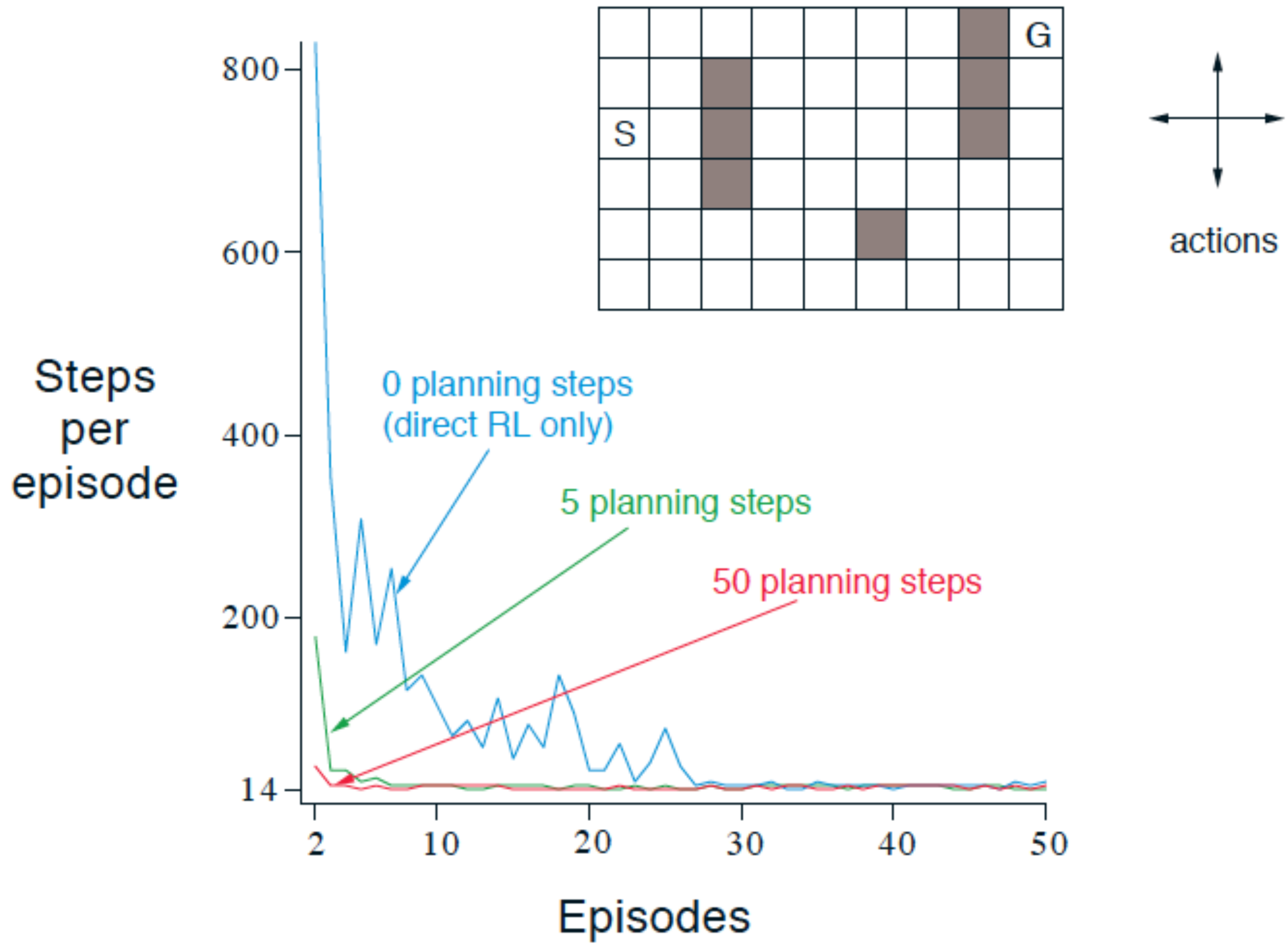


Dyna-Q

Dyna-Q(s)
Repeat
Select and execute a, observe s' and r
Update transition: $w_T \leftarrow w_T - \alpha_T(T(s,a) - s')\nabla_{w_T} T(s,a)$
Update reward: $w_R \leftarrow w_R - \alpha_R(R(s,a) - r)\nabla_{w_R} R(s,a)$
$\delta \leftarrow r + \gamma \max_{a'} Q(s',a') - Q(s,a)$
Update Q: $w_Q \leftarrow w_Q - \alpha_Q \delta \nabla_{w_Q} Q(s,a)$
Repeat a few times:
sample $\hat{s}, \hat{a}$ arbitrarily
$\delta \leftarrow R(\hat{s},\hat{a}) + \gamma \max_{\hat{a}'} Q(T(\hat{s},\hat{a}),\hat{a}') - Q(\hat{s},\hat{a})$
Update Q: $w_Q \leftarrow w_Q - \alpha_Q \delta \nabla_{w_Q} Q(\hat{s},\hat{a})$
$s \leftarrow s'$
Return Q

Dyna-Q

- Task: reach G from S

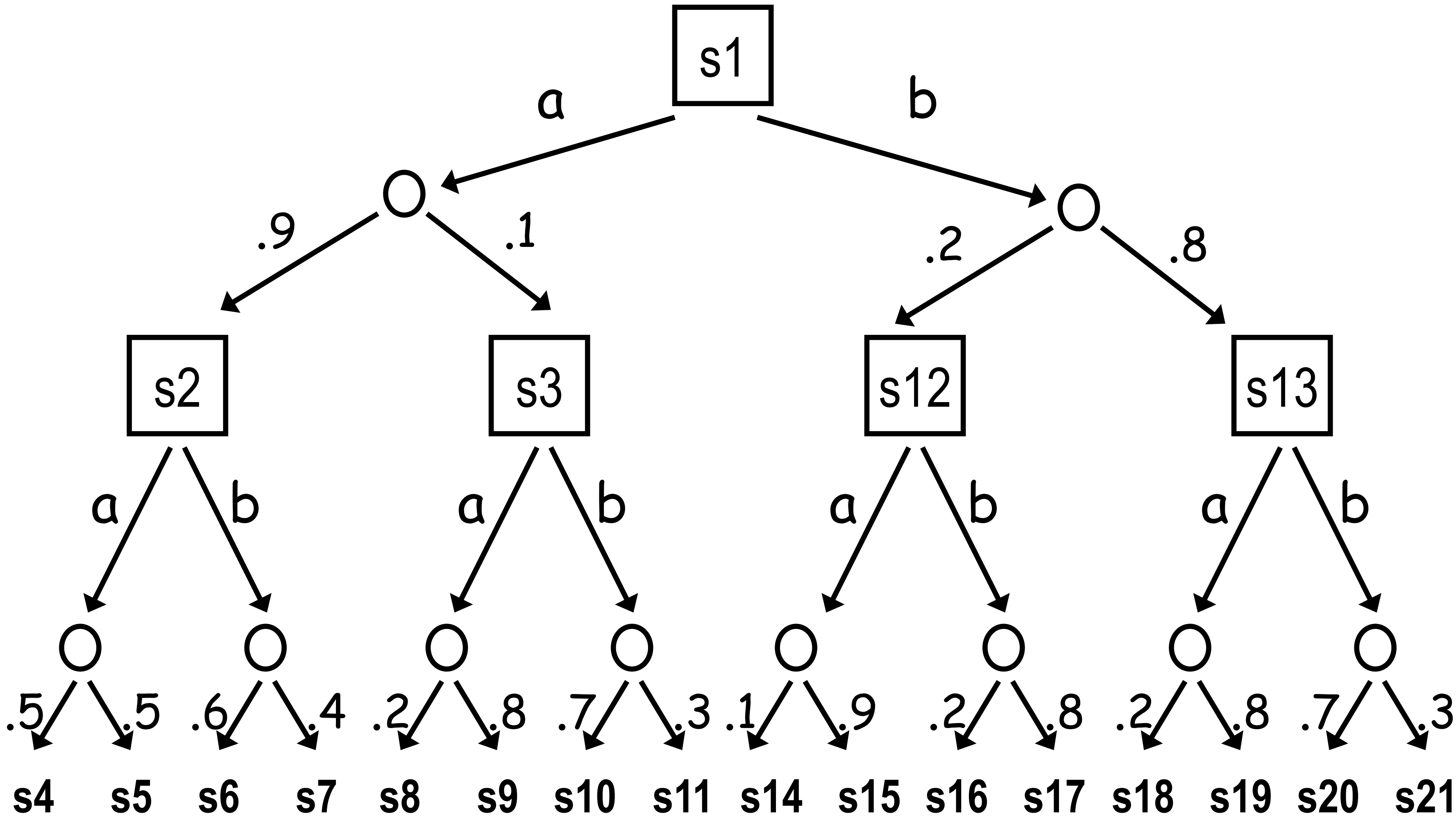


Planning from current state

- Instead of planning at arbitrary states, plan from the current state
  – This helps improve next action

- Monte Carlo Tree Search

Tree Search



s1
a
b
.9
.1
.2
.8
s2
s3
s12
s13
a
b
a
b
a
b
a
b
.5
.5
.6
.4
.2
.8
.7
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.1
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s16
s17
s18
s19
s20
s21

Tractable Tree Search

- Combine 3 ideas:
  - Leaf nodes: approximate leaf values with value of default policy π

$$Q^*(s,a) \approx Q^\pi(s,a) \approx \frac{1}{n(s,a)} \sum_{k=1}^{n} G_k$$

  - Chance nodes: approximate expectation by sampling from transition model

$$Q^*(s,a) \approx R(s,a) + \gamma \frac{1}{n(s,a)} \sum_{s' \sim \Pr(s'|s,a)} V(s')$$

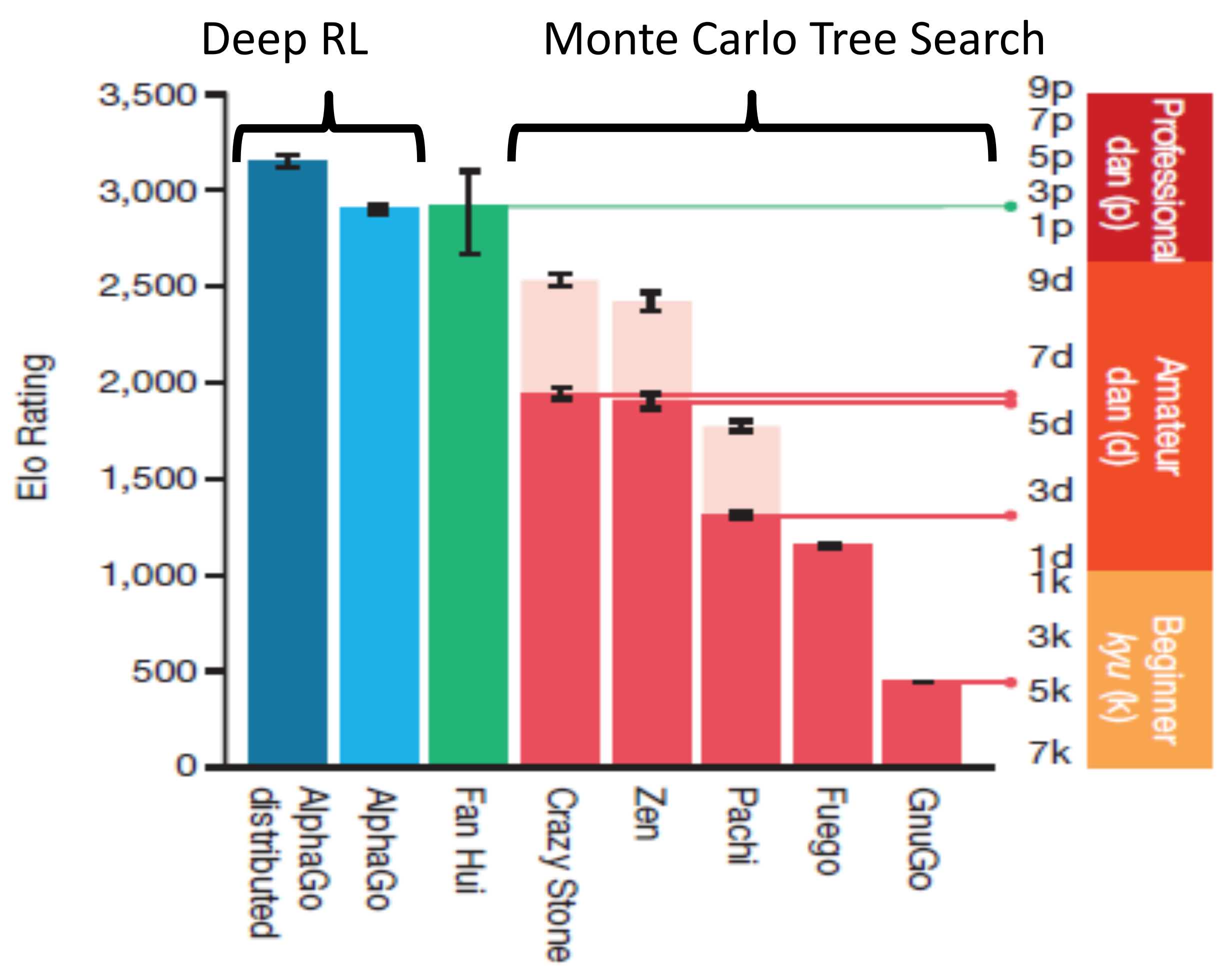
  - Decision nodes: expand only most promising actions

$$a^* = argmax_a Q(s,a) + c\sqrt{\frac{2 \ln n(s)}{n(s,a)}} \quad \text{and} \quad V^*(s) = Q(s,a^*)$$

- Resulting algorithm: Monte Carlo Tree Search

Computer Go

- Oct 2015:



Monte Carlo Tree Search (with upper confidence bound)

UCT(s_0)

create root $node_0$ with state $state(node_0) \leftarrow s_0$
while within computational budget do
  $node_l \leftarrow TreePolicy(node_0)$
  $value \leftarrow DefaultPolicy(state(node_l))$
  $Backup(node_l, value)$
return $action(BestChild(node_0, 0))$

TreePolicy($node$)

while $node$ is nonterminal do
  if $node$ is not fully expanded do
    return $Expand(node)$
  else
    $node \leftarrow BestChild(node, C)$
return $node$

Monte Carlo Tree Search (continued)

Expand($node$)
  choose $a \in$ untried actions of $A(state(node))$
  add a new child $node'$ to $node$
    with $state(node') \leftarrow T(state(node), a)$
  return $node'$

deterministic transition

BestChild($node$,c)

$$\text{return } \arg\max_{node' \in children(node)} V(node') + c\sqrt{\frac{(2 \ln n(node))}{n(node')}}$$

DefaultPolicy($node$)
  while $node$ is not terminal do
    sample $a \sim \pi(a|state(node))$
    $s' \leftarrow T(state(node), a)$
  return $R(s, a)$

Monte Carlo Tree Search (continued)

Single Player

Backup($node$,$value$)

while $node$ is not null do

$$V(node) \leftarrow \frac{n(node)V(node)+value}{n(node)+1}$$

$$n(node) \leftarrow n(node) + 1$$

$$node \leftarrow parent(node)$$

Two Players (adversarial)

BackupMinMax($node$,$value$)

while $node$ is not null do

$$V(node) \leftarrow \frac{n(node)V(node)+value}{n(node)+1}$$

$$n(node) \leftarrow n(node) + 1$$

$$value \leftarrow -value$$

$$node \leftarrow parent(node)$$

AlphaGo

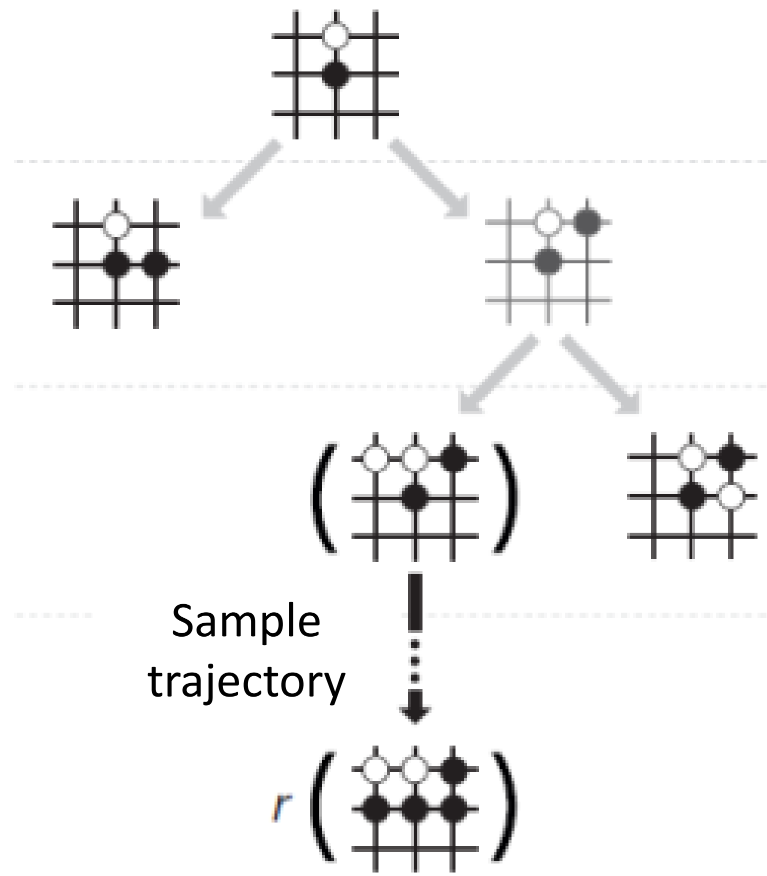
Four steps:

1. Supervised Learning of Policy Networks
2. Policy gradient with Policy Networks
3. Value gradient with Value Networks
4. Searching with Policy and Value Networks
   – Monte Carlo Tree Search variant

Search Tree

- At each edge store
  $Q(s,a), \pi(a|s), n(s,a)$

- Where $n(s,a)$ is the visit count of (s,a)



Simulation

- At each node, select edge a^* that maximizes

$$a^* = argmax_a \, Q(s,a) + u(s,a)$$

- where $u(s,a) \propto \frac{\pi(a|s)}{1+n(s,a)}$ is an exploration bonus

$$Q(s,a) = \frac{1}{n(s,a)} \sum_i 1_i(s,a) \, [\lambda V_{\boldsymbol{w}}(s) + (1-\lambda) G_i]$$

$$1_i(s,a) = \begin{cases} 1 & if \ (s,a) \ was \ visited \ at \ iteration \ i \\ 0 & otherwise \end{cases}$$