

Lecture 5a: Multi-Armed Bandits

CS885 Reinforcement Learning

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Complementary readings: [SutBar] Sec. 2.1-2.7, [Sze] Sec. 4.2.1-4.2.2

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Outline

- Exploration/exploitation tradeoff
- Regret
- Multi-armed bandits
 - ϵ -greedy strategies
 - Upper confidence bounds

Exploration/Exploitation Tradeoff

- Fundamental problem of RL due to the active nature of the learning process
- Consider one-state RL problems known as **bandits**

Stochastic Bandits

- Formal definition:
 - Single state: $S = \{s\}$
 - A : set of actions (also known as **arms**)
 - Space of rewards (often re-scaled to be $[0,1]$)
- **No transition function to be learned** since there is a single state
- We simply need to **learn the stochastic** reward function

Origin

- The term bandit comes from gambling where **slot machines can be thought as one-armed bandits.**
- Problem: which slot machine should we play at each turn when their payoffs are not necessarily the same and initially unknown?



Examples

- Design of experiments (Clinical Trials)
- Online ad placement
- Web page personalization
- Recommender systems
- Networks (packet routing)

Online Ad Placement

The screenshot shows a web browser window displaying the homepage of the Globe and Mail. At the top, there is a purple banner for IBM with the text "Can your business anticipate shifts in the marketplace?" and "Learn how to use Big Data and Analytics to get better business outcomes". Below this is the site's navigation bar, including the "THE GLOBE AND MAIL" logo, a search bar, and links for "Login", "Register", "Subscribe", and "Help". A secondary navigation bar lists various news categories like "Home", "News", "Opinion", "Business", etc. A yellow banner for "GLOBE UNLIMITED FLASH SALE" offers a 50% discount. The main content area features several news articles and a large yellow advertisement for Porter's plans. The advertisement includes the text "ASK your Toronto City Councillor TO VOTE YES on April 1 for Porter's plans" and a "Take Action" button.

Can your business anticipate shifts in the marketplace? Learn how to use Big Data and Analytics to get better business outcomes

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GLOBE UNLIMITED FLASH SALE SAVE 50% ON THE FIRST 6 MONTHS | OFFER ENDS MARCH 31ST ACCESS EVERYTHING GLOBEANDMAIL.COM HAS TO OFFER SEE MY OPTIONS

Six Ontarians charged in alleged \$200-million investment fraud

- WATCH Video: How to protect your bank account from fraud

122 'potential objects' spotted in ocean offer fresh jet lead

- WATCH Sailing the waters where Flight 370 went down

TORONTO Chow presses Ford to 'take down the circus tent' as candidates hammer each other in mayoral debate

ASK your Toronto City Councillor TO VOTE YES on April 1 for Porter's plans Take Action porterplans.com

Online Ad Optimization

- Problem: **which ad should be presented?**
- Answer: present ad with highest payoff

$$\text{payoff} = \text{clickThroughRate} \times \text{payment}$$

- Click through rate: probability that user clicks on ad
- Payment: \$\$ paid by advertiser
 - Amount determined by an auction

Simplified Problem

- Assume payment is 1 unit for all ads
- Need to estimate click through rate
- Formulate as a bandit problem:
 - Arms: the set of possible ads
 - Rewards: 0 (no click) or 1 (click)
- In what order should ads be presented to maximize revenue?
 - **How should we balance exploitation and exploration?**

Simple yet Difficult Problem

- Simple: description of the problem is short
- Difficult: **no known tractable optimal solution**

Simple Heuristics

- **Greedy strategy:** select the arm with the highest average so far
 - May get stuck due to lack of exploration
- **ϵ -greedy:** select an arm at random with probability ϵ and otherwise do a greedy selection
 - Convergence rate depends on choice of ϵ

Regret

- Let $R(a)$ be the unknown average reward of a
- Let $r^* = \max_a R(a)$ and $a^* = \operatorname{argmax}_a R(a)$

- Denote by $loss(a)$ the **expected regret** of a

$$loss(a) = r^* - R(a)$$

- Denote by $Loss_n$ the **expected cumulative regret** for n time steps

$$Loss_n = \sum_{t=1}^n loss(a_t)$$

Theoretical Guarantees

- When ϵ is constant, then
 - For large enough t : $\Pr(a_t \neq a^*) \approx \epsilon$
 - Expected cumulative regret: $Loss_n \approx \sum_{t=1}^n \epsilon = O(n)$
 - Linear regret
- When $\epsilon_t \propto 1/t$
 - For large enough t : $\Pr(a_t \neq a^*) \approx \epsilon_t = O\left(\frac{1}{t}\right)$
 - Expected cumulative regret: $Loss_n \approx \sum_{t=1}^n \frac{1}{t} = O(\log n)$
 - Logarithmic regret

Empirical Mean

- Problem: how far is the empirical mean $\tilde{R}(a)$ from the true mean $R(a)$?
- If we knew that $|R(a) - \tilde{R}(a)| \leq bound$
 - Then we would know that $R(a) < \tilde{R}(a) + bound$
 - And we could select the arm with best $\tilde{R}(a) + bound$
- Overtime, additional data will allow us to refine $\tilde{R}(a)$ and compute a tighter *bound*.

Positivism in the Face of Uncertainty

- Suppose that we have an oracle that returns an **upper bound** $UB_n(a)$ on $R(a)$ for each arm based on n trials of arm a .
- Suppose the upper bound returned by this oracle converges to $R(a)$ in the limit:
 - i.e., $\lim_{n \rightarrow \infty} UB_n(a) = R(a)$
- **Optimistic algorithm**
 - At each step, **select** $\operatorname{argmax}_a UB_n(a)$

Convergence

- **Theorem:** An optimistic strategy that always selects $\operatorname{argmax}_a UB_n(a)$ will converge to a^*
- Proof by contradiction:
 - Suppose that we converge to suboptimal arm a after infinitely many trials.
 - Then $R(a) = UB_\infty(a) \geq UB_\infty(a') = R(a') \forall a'$
 - But $R(a) \geq R(a') \forall a'$ contradicts our assumption that a is suboptimal.

Probabilistic Upper Bound

- Problem: We can't compute an upper bound with certainty since we are sampling
- However we can obtain measures f that are upper bounds most of the time
 - i.e., $\Pr(R(a) \leq f(a)) \geq 1 - \delta$
 - Example: Hoeffding's inequality

$$\Pr \left(R(a) \leq \tilde{R}(a) + \sqrt{\frac{\log\left(\frac{1}{\delta}\right)}{2n_a}} \right) \geq 1 - \delta$$

where n_a is the number of trials for arm a

Upper Confidence Bound (UCB)

- Set $\delta_n = 1/n^4$
in Hoeffding's bound
- Choose a with
highest Hoeffding bound

UCB(h)

$V \leftarrow 0, n \leftarrow 0, n_a \leftarrow 0 \quad \forall a$

Repeat until $n = h$

Execute $\operatorname{argmax}_a \tilde{R}(a) + \sqrt{\frac{2 \log n}{n_a}}$

Receive r

$V \leftarrow V + r$

$\tilde{R}(a) \leftarrow \frac{n_a \tilde{R}(a) + r}{n_a + 1}$

$n \leftarrow n + 1, n_a \leftarrow n_a + 1$

Return V

UCB Convergence

- **Theorem:** Although Hoeffding's bound is probabilistic, **UCB converges.**
- **Idea:** As n increases, the term $\sqrt{\frac{2 \log n}{n_a}}$ increases, ensuring that all arms are tried infinitely often
- Expected cumulative regret: $Loss_n = O(\log n)$
 - **Logarithmic regret**

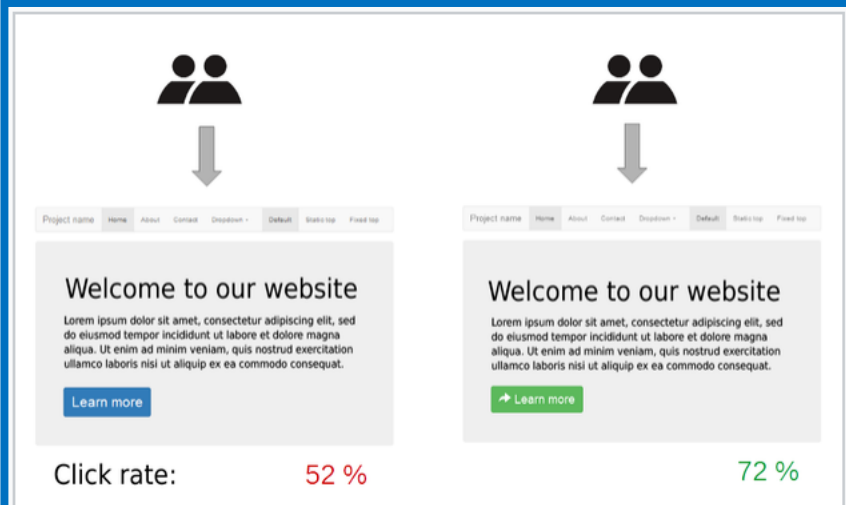
Extension of A/B Testing

- **A/B Testing:** randomized experiment with 2 variants
 - Select best variant after completion of experiment

Example: email marketing

- "Offer ends this Saturday! Use code A" (response rate: 5%)
- "Offer ends soon! Use code B" (response rate: 3%)

- **Multi-armed bandits:** form of **continual A/B testing**



Example of A/B testing on a website. By randomly serving visitors two versions of a website that differ only in the design of a single button element, the relative efficacy of the two designs can be measured.

Summary

- Stochastic bandits
 - Exploration/exploitation tradeoff
- ϵ -greedy and UCB
 - Theory: logarithmic expected cumulative regret
- In practice:
 - UCB often performs better than ϵ -greedy
 - Many variants of UCB improve performance