Reinforcement Learning for Integer Programming: Learning to Cut

CS885: Paper Presentation

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Introduction

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It solves them by using an RL agent to make heuristic choices, within a classic/hard-coded algorithm.

Integer Programming

Integer Programming (IP) is the optimization problem:

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- $A, \; b, \; c$ have rational components ($\in \mathbb{Q}$)
- The x satisfying the constraints are said to form the **feasible set**.
- Denote the optimum as x^*_{IP}, with minimum objective $c^\top x^*_{IP}$

Why IPs?

Many difficult combinatorial problems boil down to this form:

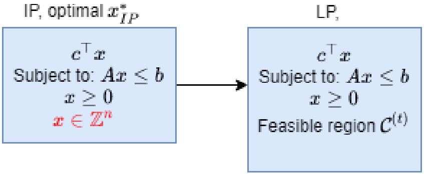
- Graph optimization (eg. traveling salesman problem)
- Scheduling
- Production Planning

The problem is (NP-)hard, and algorithms tend to use heuristics

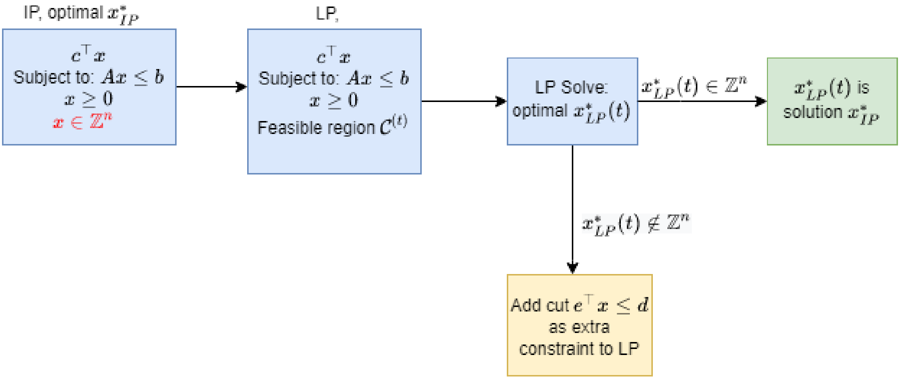
Cutting Plane Method

- One method used by commercial IP solvers
- Used on its own, or as a subroutine in a larger algorithm (called B&C - Branch and Cut)
- Idea is to convert to a linear program, and successively shrink the feasible set

Method: First, we drop integer constraints (results in easy to solve LP - Linear Program)

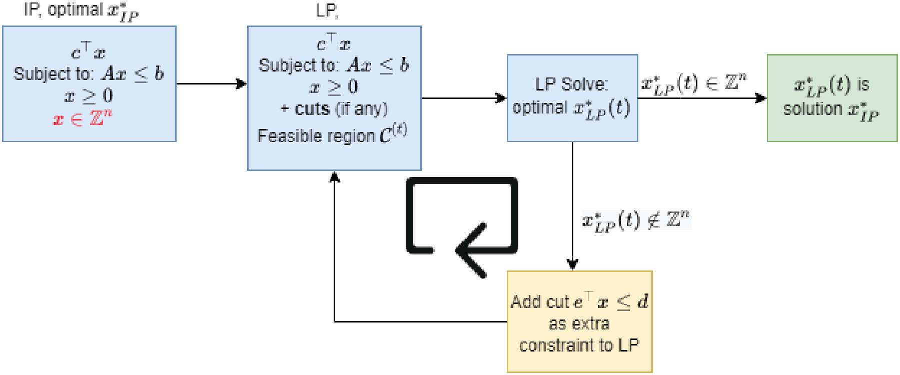


Next, solve the LP, and check if components are integers:



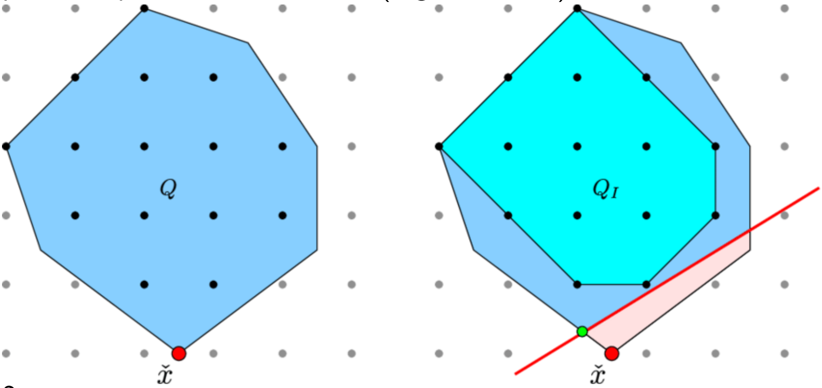
Cutting Plane Method 3

If they aren't, add an extra linear constraint (**called a cutting plane, or cut**) to the problem, and repeat:



Cutting Plane Visualization

A **cut** excludes the previous LP solution $x^*_{LP}(t)$, but keeps all integer points in previous feasible set. (Figure from 2)



2: Achterberg, Tobias. (2009). SCIP: Solving constraint integer programs. Mathematical Programming Computation. 1. 1-41.

Selection of Cuts

Different algorithms propose different cut choices:

- *Gomory's* method proposes a possible cut choice for every non-integer component.
- Cut choices formed using information extracted in solving the LP ("tableau matrix" $\tilde{A}$)
- **It proposes $l_t \leq n$ possible cuts to choose from.**

Typically, **heuristics** are used to select which (single) cut to apply

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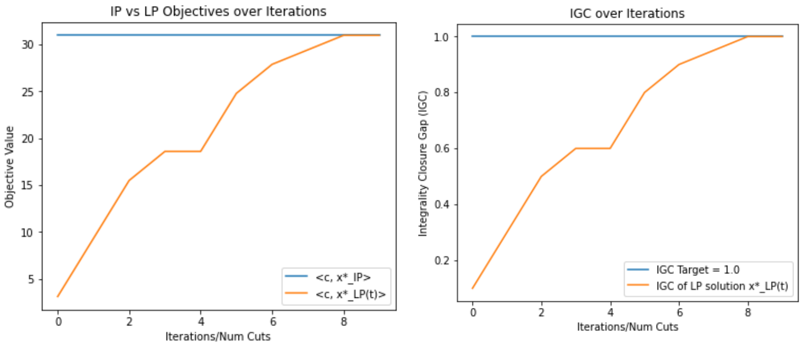
In paper: RL agent is trained to select cuts (given by Gomory's method).

Cut Quality Metric

Over the iterations, the (LP's) feasible set $\mathcal{C}^{(t)}$ reduces, and the objective $c^\top x^*_{LP}(t)$ increases to IP optimal $c^\top x^*_{IP}$ (left plot):



Integrality Gap Closure (IGC) rescales axis to $[0, 1]$ (right plot). Cut heuristics are evaluated based on IGC (closer to 1 is better).

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Setting Up MDP: States

The state is the LP problem in iteration t, specified by $s_t = \{\mathcal{C}^{(t)},\ c,\ x^*_{LP}(t),\ \mathcal{D}^{(t)}\}$:

- Feasible region $\mathcal{C}^{(t)} = \{a_i^\top x \leq b_i\}_{i=1}^N$ (defined by initial constraints + prev cuts)
- Objective parameter c (objective $= c^\top x$)
- Minimizing solution (to LP) $x^*_{LP}(t)$
- $\mathcal{D}^{(t)} = \{e_i^\top x \leq d_i\}_{i=1}^{I_t}$ is the set of candidate cuts given by Gomory's method

Setting up MDP: Actions

Discrete action space: the possible cutting planes given by Gomory's method, $\mathcal{D}^{(t)}$.

- Number of actions varies each t, but is $\leq n$
- Each action parameterized by real vectors $e_i, \ d_i$ (for plane $e_i^\top x \leq d_i$)

Setting up MDP: Reward

In each iteration, we promote as much immediate progress on LP objective as possible:

Reward in iteration t is: $r_t = c^\top x^*_{LP}(t+1) - c^\top x^*_{LP}(t)$ (which is ≥ 0)

Total expected return for a policy is $J(\pi) = \mathbb{E}_\pi[\sum_{t=0}^{T-1} \gamma^t r_t]$

(Where $\gamma \in (0, 1]$, and T is **finite** horizon (we stop after fixed number of iterations, or after finding x^*_{IP})).

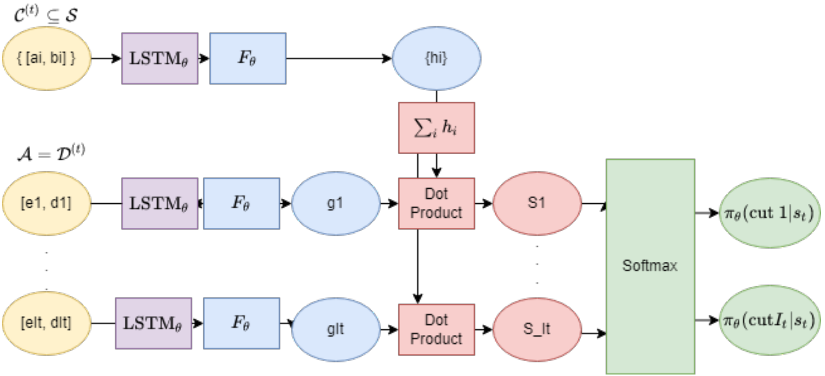
Setting up MDP: Transitions

Starting in state $s_t = \{\mathcal{C}^{(t)},\ c,\ x^*_{LP}(t),\ \mathcal{D}^{(t)}\}$, making action of selecting cut $e_i^\top x \leq d_i$

- Shrink feasible region as: $\mathcal{C}^{(t+1)} = \mathcal{C}^{(t)} \cup \{e_i^\top x \leq d_i\}$
- Solve the new LP (with simplex method) to obtain $x^*_{LP}(t+1)$
- Gomory's method on solution info gives new candidate cuts $\mathcal{D}^{(t+1)}$

Get to next state $s_{t+1} = \{\mathcal{C}^{(t+1)},\ c,\ x^*_{LP}(t+1),\ \mathcal{D}^{(t+1)}\}$ (**deterministic transition**)

Policy Network



- (Yellow): Inputs
- (Purple): LSTM encodes in fixed size (using last hidden state of LSTM)
- (Blue): Encode state and action info with learned function F_θ
- (Red): Attention computation to get scores "S"
- (Green): Softmax to get the **stochastic** policy $\pi_\theta(a_t|s_t)$: distribution over next cut

Learning Algorithm

We use **evolutionary strategies**: essentially a policy gradient method, updates policy parameters as $\theta \leftarrow \theta + \alpha \hat{g}_\theta$

Where $\hat{g}_\theta \approx \nabla_\theta J(\pi_\theta)$, but which estimates gradient without backprop:

- Sample N random "directions" $\epsilon_i \sim \mathcal{N}(0, I)$
- Step in the directions: $\theta' = \theta + \sigma \epsilon_i$
- Gradient estimate is

$$\hat{g}_\theta = \frac{1}{N} \sum_{i=1}^{N} J(\pi_{\theta'}) \frac{\epsilon_i}{\sigma}$$

Train using this over batch of IP instances (average gradient over batch).

Related Methods

Other methods consist of human designed heuristics for cut selection (in Gomory's method) eg:

- Random
- Max Violation (MV)
- Max Normalized Violation (MNV)
- Lexicographical Rule (LE)

RL agent needs to learn selection process, so is expensive to train, but can perform better.

No directly comparable ML methods for cut selection

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IP Problems

Train and test on multiple different classes of IP problems:

- Packing
- Production Planning
- Binary Packing
- Max-Cut

Each class gives problem with different structure and form.

Train and test on multiple sizes $n \times m$ (number of variables and constraints), ranging from $n \times m = 200$ to 5000.

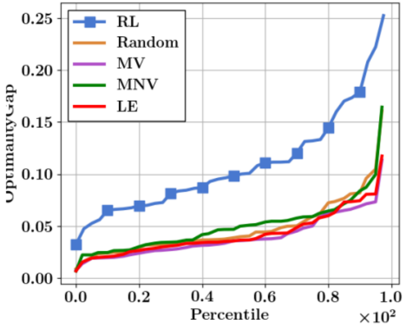
Experiment 1: Cut Efficiency

For small problems, monitor how many iterations (cuts) needed until reaching IP solution (table from paper):

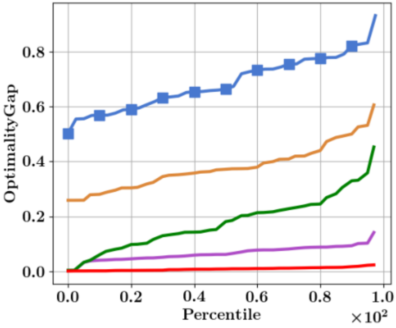
Table 1: Number of cuts it takes to reach optimality. We show mean $\pm$ std across all test instances.

Tasks	Packing	Planning	Binary	Max Cut
Size	10×5	13×20	10×20	10×22
RANDOM	48 ± 36	44 ± 37	81 ± 32	69 ± 34
MV	62 ± 40	48 ± 29	87 ± 27	64 ± 36
MNV	53 ± 39	60 ± 34	85 ± 29	47 ± 34
LE	34 ± 17	310 ± 60	89 ± 26	59 ± 35
RL	$\mathbf{14 \pm 11}$	$\mathbf{10 \pm 12}$	$\mathbf{22 \pm 27}$	$\mathbf{13 \pm 4}$

For small problems, monitor IGC attained in percentage of instances for RL heuristic vs human-designed baselines (plots from paper):



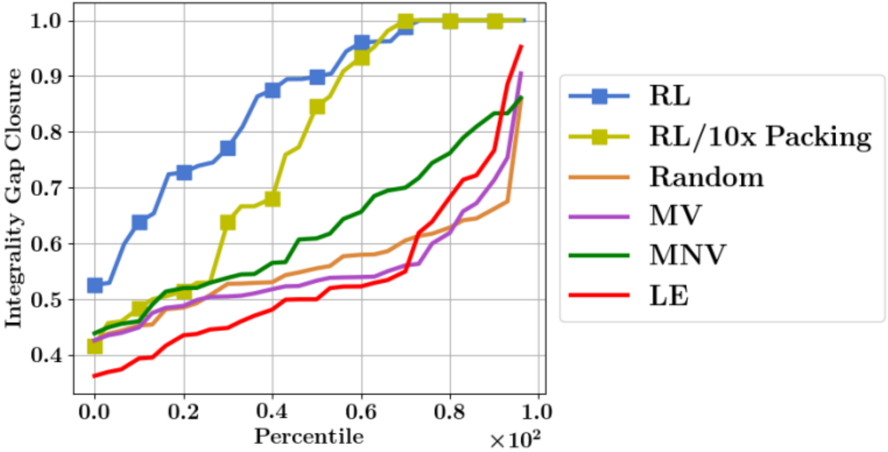
(a) Packing



(b) Planning

Experiment 3: Generalization

Evaluate difference in performance (IGC) for an agent trained on smaller problem instances AND different problem classes (plot from paper):



Experiment 4 and 5

- **Experiment 4 (Impact on efficiency of B&C)**: When using cutting plane method as subroutine in a larger B&C - branch and cut algorithm, the RL agent helps the downstream application more than the baselines.
- **Experiment 5 (Interpretability of Cuts)**: When applied to a simple, well-studied IP problem (knapsack problem), with known good cuts, the RL agent produces cuts resembling them.

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Conclusion

- This paper presented a novel application of RL to selecting cuts in IP problems.
- The RL agent can learn to select cuts better than human designed heuristics, with a capacity to generalize.
- Further work can improve efficiency and possibly integrate ML into commercial IP solvers.