HMM Example

Consider a simplified activity recognition problem with two classes

- $c_1 \equiv walking$
- $c_2 \equiv sitting$

and two measurements (e.g., from an accelerometer)

- $v_1 \equiv high$
- $v_2 \equiv low$

Suppose we have 100 sequences of varying lengths that are summarized by the following statistics

- $\#c_1^{start} = 90$ $\quad \#c_2^{start} = 10$
- $\#(c_1, c_1) = 80{,}000$ $\quad \#(c_2, c_1) = 20{,}000$ $\quad \#(c_1, c_2) = 10{,}000$ $\quad \#(c_2, c_2) = 90{,}000$
- $\#(v_1, c_1) = 75{,}000$ $\quad \#(v_2, c_1) = 25{,}000$ $\quad \#(v_1, c_2) = 30{,}000$ $\quad \#(v_2, c_2) = 70{,}000$

Maximum likelihood allows us to compute the following estimates for the parameters

- Initial distribution: $\Pr(y_1) = \begin{cases} 0.9 & y_1 = c_1 \\ 0.1 & y_1 = c_2 \end{cases}$
- Transition distribution: $\Pr(y_{t+1}|y_t) = \begin{cases} 0.8 & y_{t+1} = c_1, y_t = c_1 \\ 0.2 & y_{t+1} = c_2, y_t = c_1 \\ 0.1 & y_{t+1} = c_1, y_t = c_2 \\ 0.9 & y_{t+1} = c_2, y_t = c_2 \end{cases}$
- Emission distribution: $\Pr(x_t|y_t) = \begin{cases} 0.75 & x_t = v_1, y_t = c_1 \\ 0.25 & x_t = v_2, y_t = c_1 \\ 0.30 & x_t = v_1, y_t = c_2 \\ 0.70 & x_t = v_2, y_t = c_2 \end{cases}$

Suppose that we observe the following sequence of measurements: v_1, v_1, v_2, v_2.

We can estimate the probability of each class at each time step by belief monitoring

1. $\Pr(y_1|v_1) \propto \Pr(v_1|y_1)\Pr(y_1) = \begin{cases} 0.675 \\ 0.030 \end{cases} \propto \begin{cases} 0.9574 & y_1 = c_1 \\ 0.0426 & y_1 = c_2 \end{cases}$
2. $\Pr(y_2|v_1 v_1) \propto \Pr(v_1|y_2)\sum_{y_1}\Pr(y_2|y_1)\Pr(y_1|v_1) = \begin{cases} 0.5777 \\ 0.0689 \end{cases} \propto \begin{cases} 0.8934 & y_2 = c_1 \\ 0.1066 & y_2 = c_2 \end{cases}$
3. $\Pr(y_3|v_1 v_1 v_2) \propto \Pr(v_2|y_3)\sum_{y_2}\Pr(y_3|y_2)\Pr(y_2|v_1 v_1) = \begin{cases} 0.1813 \\ 0.1922 \end{cases} \propto \begin{cases} 0.4854 & y_3 = c_1 \\ 0.5146 & y_3 = c_2 \end{cases}$
4. $\Pr(y_4|v_1 v_1 v_2 v_2) \propto \Pr(v_2|y_4)\sum_{y_3}\Pr(y_4|y_3)\Pr(y_3|v_1 v_1 v_2) = \begin{cases} 0.1099 \\ 0.3921 \end{cases} \propto \begin{cases} 0.2190 & y_4 = c_1 \\ 0.7810 & y_4 = c_2 \end{cases}$

We can also estimate the most likely sequence of classes by the Viterbi algorithm

1. $f(y_1, y_2) = \Pr(y_2|y_1)\Pr(v_1|y_1)\Pr(y_1) = \begin{cases} 0.540 & y_1 = c_1, y_2 = c_1 \\ 0.003 & y_1 = c_2, y_2 = c_1 \\ 0.135 & y_1 = c_1, y_2 = c_2 \\ 0.027 & y_1 = c_2, y_2 = c_2 \end{cases}$

   $\max_{y_1} \Pr(y_1, y_2|v_1) \propto \max_{y_1} f(y_1, y_2) = \begin{cases} 0.540 & y_2 = c_1 \\ 0.135 & y_2 = c_2 \end{cases}$

   Hence if $y_2 = c_1$ then $y_1 = c_1$ and if $y_2 = c_2$ then $y_1 = c_1$

2. $f(y_2, y_3) = \Pr(y_3|y_2)\Pr(v_1|y_2)\max_{y_1} \Pr(y_1, y_2|v_1) = \begin{cases} 0.3240 & y_2 = c_1, y_3 = c_1 \\ 0.0041 & y_2 = c_2, y_3 = c_1 \\ 0.0810 & y_2 = c_1, y_3 = c_2 \\ 0.0365 & y_2 = c_2, y_3 = c_2 \end{cases}$

   $\max_{y_1, y_2} \Pr(y_1, y_2, y_3|v_1, v_1) \propto \max_{y_2} f(y_2, y_3) = \begin{cases} 0.3240 & y_3 = c_1 \\ 0.0810 & y_3 = c_2 \end{cases}$

   Hence if $y_3 = c_1$ then $y_2 = c_1$ and if $y_3 = c_2$ then $y_2 = c_1$

3. $f(y_3, y_4) = \Pr(y_4|y_3)\Pr(v_2|y_3)\max_{y_1, y_2} \Pr(y_1, y_2, y_3|v_1, v_1) = \begin{cases} 0.0648 & y_3 = c_1, y_4 = c_1 \\ 0.0057 & y_3 = c_2, y_4 = c_1 \\ 0.0162 & y_3 = c_1, y_4 = c_2 \\ 0.0510 & y_3 = c_2, y_4 = c_2 \end{cases}$

   $\max_{y_1, y_2, y_3} \Pr(y_1, y_2, y_3, y_4|v_1, v_1, v_2) \propto \max_{y_3} f(y_3, y_4) = \begin{cases} 0.0648 & y_4 = c_1 \\ 0.0510 & y_4 = c_2 \end{cases}$

   Hence if $y_4 = c_1$ then $y_3 = c_1$ and if $y_4 = c_2$ then $y_3 = c_2$

4. $f(y_4) = \Pr(v_2|y_4) \max_{y_1, y_2, y_3} \Pr(y_1, y_2, y_3, y_4|v_1, v_1, v_2) = \begin{cases} 0.0162 & y_4 = c_1 \\ 0.0357 & y_4 = c_2 \end{cases}$

   $\max_{y_1, y_2, y_3, y_4} \Pr(y_1, y_2, y_3, y_4|v_1, v_1, v_2, v_2) \propto \max_{y_4} f(y_4) = 0.0357$

   Hence $y_4 = c_2$

   By backtracking through the rules obtained at each time step, we can conclude that the most likely sequence of classes is c_1, c_1, c_2, c_2