

# Lecture 8: Causal Inference

## CS486/686 Intro to Artificial Intelligence

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# Outline

- Models
  - Causal Bayesian Networks
  - Structural Causal Models
- Causal inference
  - Interventions
  - Counterfactuals

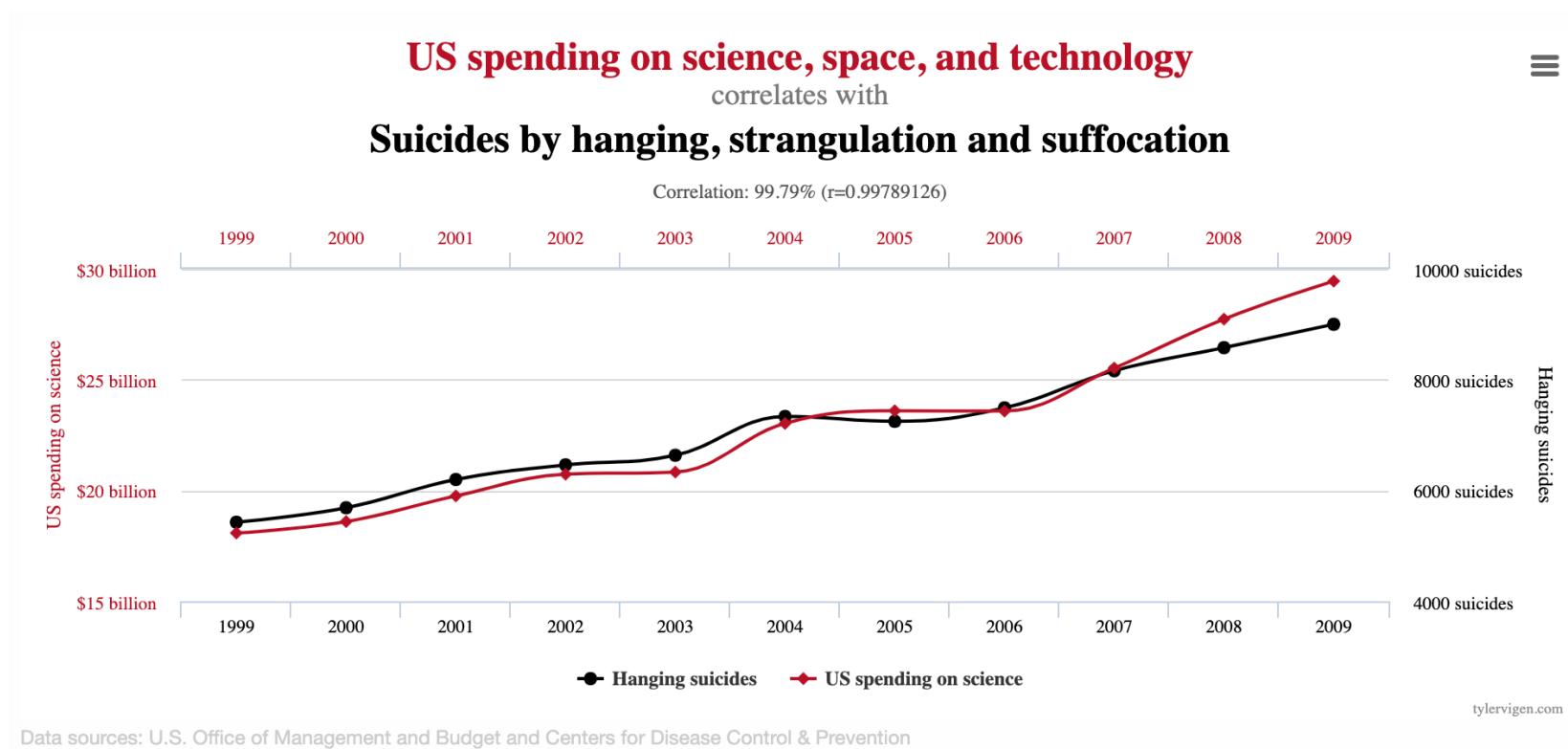
# Causality

- **Causality** is the study of **how things influence one other, how causes lead to effects.**
- **Causal dependence:**  $X$  causes  $Y$  iff changes to  $X$  induce changes to  $Y$ 
  - Example: Diseases cause symptoms, but symptoms do not cause diseases

# Causal and Non-Causal Correlations

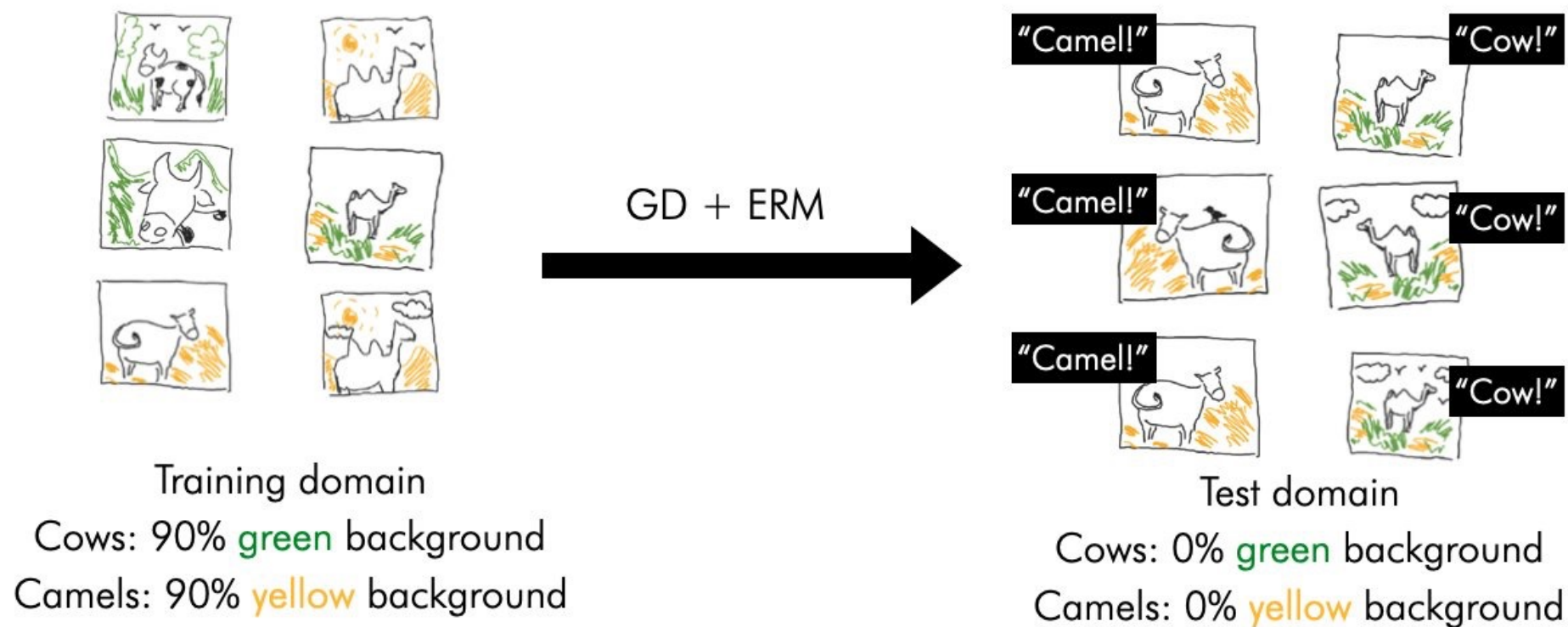
- A **joint distribution**  $P(X, Y)$  **captures correlations** between  $X$  and  $Y$ , but does not indicate whether a causal relation exists between  $X$  and  $Y$  nor the direction of the causal relation when it exists.
- A **conditional distribution**  $P(Y|X)$  **does not necessarily indicate  $X$  causes  $Y$** 
  - Recall Bayes' rule:  $P(Y|X) = \frac{\Pr(X|Y)P(Y)}{P(X)}$
  - Since we can transform  $P(X|Y)$  into  $P(Y|X)$ , conditional distributions do not always indicate causal dependences, otherwise  $Y$  would cause  $X$  and  $X$  would cause  $Y$

# Spurious Correlations



From <https://www.tylervigen.com/spurious-correlations>

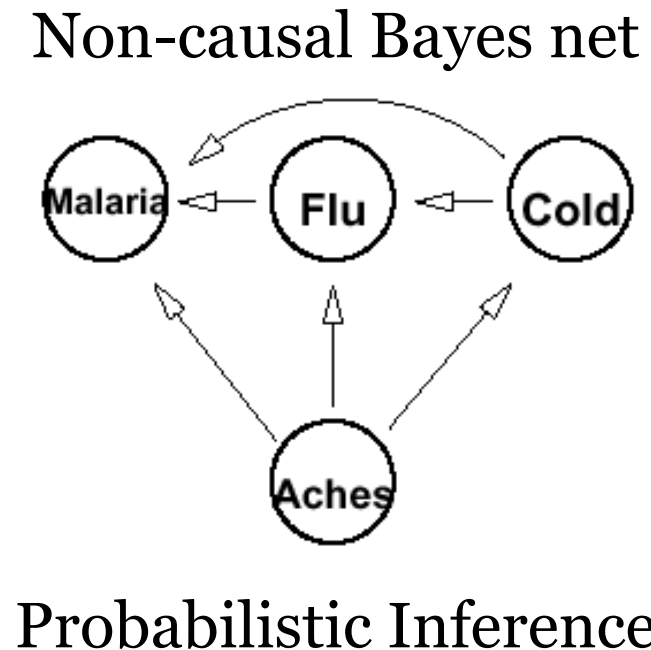
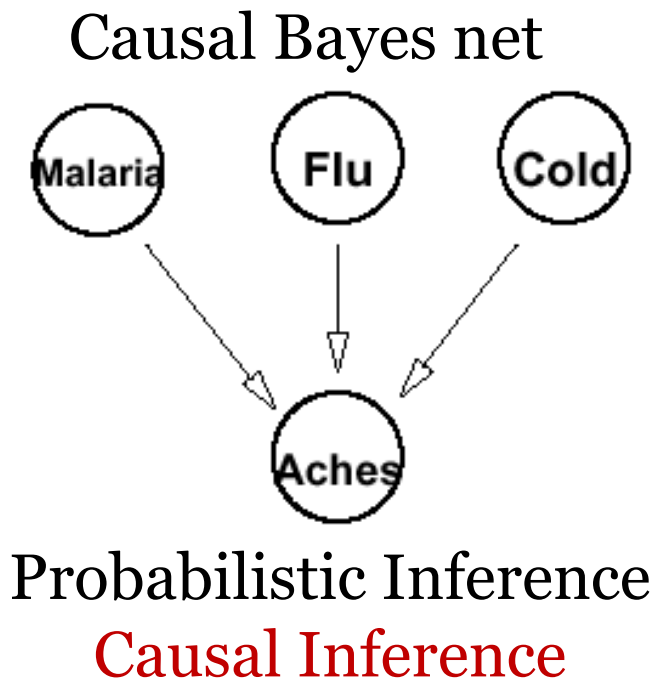
# Spurious Correlations



Standard example (Beery et al., '18 Arjovsky et al., '19)

# Causal Bayesian Network

Definition: Bayesian network where all edges indicate direct causal effects.



# Causal Inference

**Intervention:** What is the effect of an action?

E.g., What is the effect of a treatment?

**Causal networks can easily support intervention queries, but non-causal networks do not.**



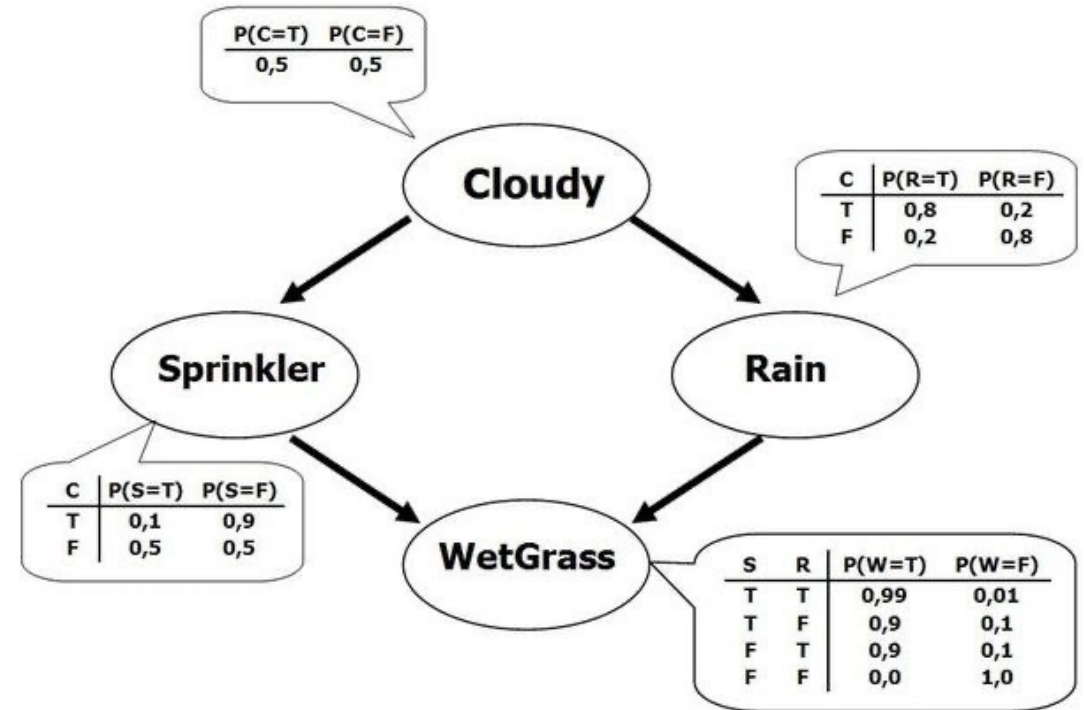
# Observation versus Intervention

**Observation:** What is the likelihood that the grass is wet when the sprinkler is observed to be on?

$$P(WG|S = \text{true})?$$

**Intervention:** How does turning on the sprinkler affect the grass?

$$P(WG|\text{do}(S = \text{true}))?$$



# Do Operator

## Observational query:

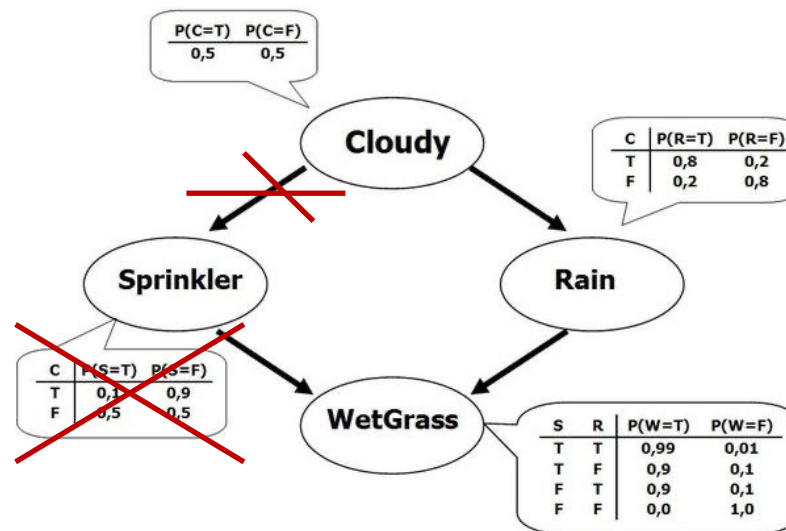
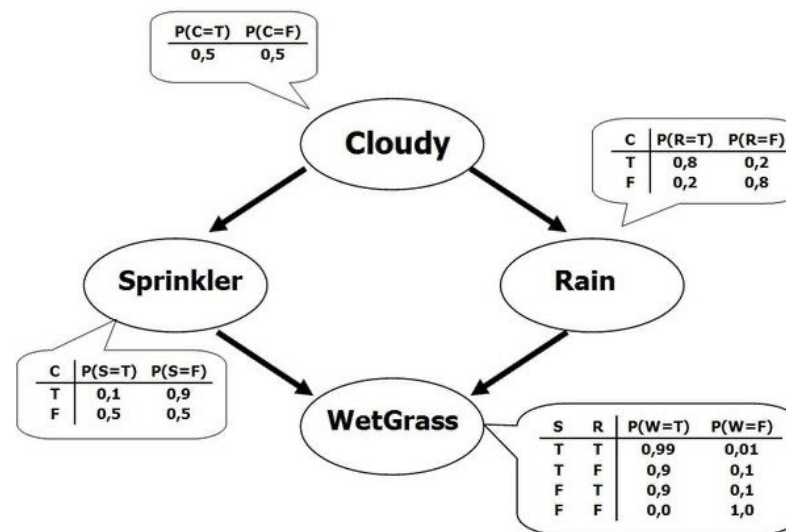
$$P(WG|S = \text{true})?$$

- Factors:
- Evidence:
- Eliminate:

## Intervention query:

$$P(WG|\text{do}(S = \text{true}))?$$

- Factors:
- Evidence:
- Eliminate:



# Inference with Do Operator

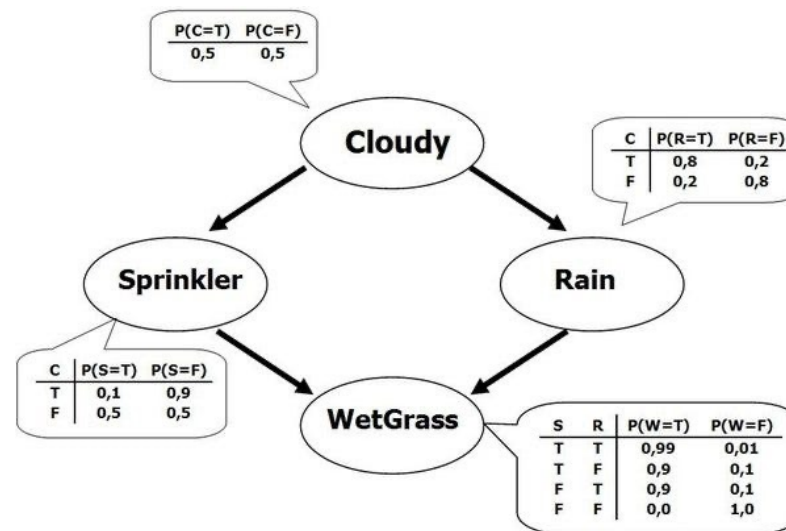
$$P(X|do(Y = y), Z = z)$$

In a causal graph:

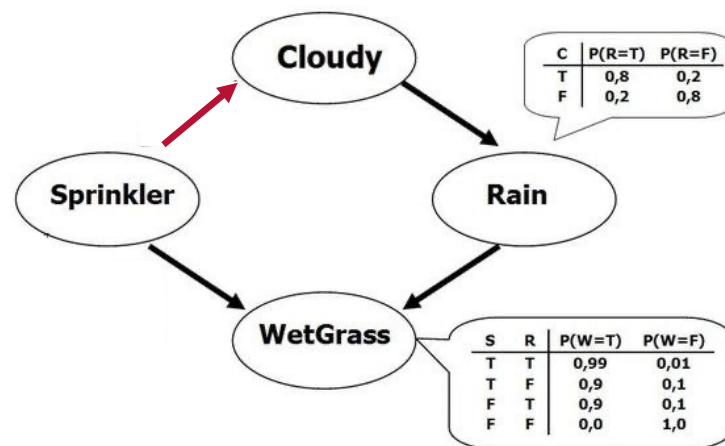
- 1) Remove edges pointing to  $Y$  and  $P(Y|parents(Y))$
- 2) Perform variable elimination on remaining graph:
  - a) Restrict factors to evidence:  $Y = y$  and  $Z = z$
  - b) Eliminate variables
  - c) Multiply remaining factors and normalize

# Non-Causal Graph

Original causal graph:



Equivalent non-causal graph:



# Counterfactual Analysis

**Intervention:** What is the effect of an action?

E.g., What is the effect of a treatment?

**Counterfactual analysis** (or counterfactual thinking): explores outcomes that did not actually occur, but which could have occurred under different conditions. It's a kind of what if? analysis and is a useful way for testing cause-and-effect relationships.

E.g., Would the patient have died if he was not treated?

E.g., Would a goal be scored had the player not tripped?

# Counterfactual Analysis

How can we answer counterfactual questions with a causal Bayes net?

Treatment  $\rightarrow$  Dead

**Fact:** patient was treated and then died

**Counterfactual question:** Had the patient not been treated, would the patient have survived?

Can't answer this question directly since we can't revive the patient to try no treatment...

# Structural Causal Models (SCMs)

**Idea:** separate causal relations from noise

**Structural Causal Model** contains:

- **X:** endogenous variables (domain variables)
- **U:** exogenous variables (noise)
- Only **deterministic** relations given by equations
  - $X_i = f(\text{parents}(X_i), U_i)$

# Equivalence

- Structural Causal Model (SCM)  $\rightarrow$  Causal Bayes Net
  - Trivial since SCMs are special Causal Bayes nets with deterministic relations
- Causal Bayes Net  $\rightarrow$  Structural Causal Model (SCM)
  - For every endogenous variable  $X$ , it is possible to create an exogenous variable  $U$  with distribution  $P(U)$  and deterministic function  $f(\text{parents}(X), U) = X$  such that  $P(X = x | \text{parents}(X)) = \sum_U P(U) \delta(f(\text{parents}(X), U) = x)$   
where  $\delta$  is the Kronecker delta function:  $\delta(a) = \begin{cases} 1 & a = \text{true} \\ 0 & a = \text{false} \end{cases}$

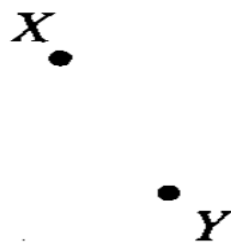


# Example

Let  $P(X, Y)$  be uniformly distributed i.e.,  $P(X = x, Y = y) = 0.25 \quad \forall x, y$

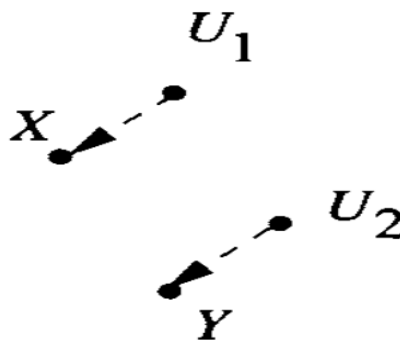
$X$ : treatment

$Y$ : dead



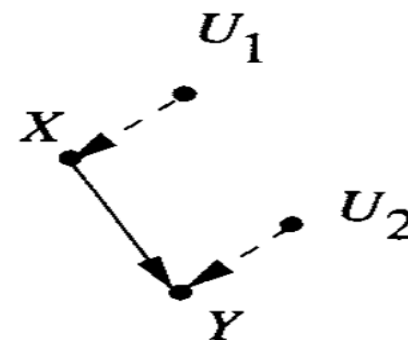
(a)

$$P(X) = 0.5$$
$$P(Y) = 0.5$$



(b)

$$P(U_1) = 0.5$$
$$P(U_2) = 0.5$$
$$X = U_1$$
$$Y = U_2$$



(c)

$$P(U_1) = 0.5$$
$$P(U_2) = 0.5$$
$$X = U_1$$
$$Y = XU_2 + (1 - X)(1 - U_2)$$

# Example

Model B	$u_2 = 0$		$u_2 = 1$		Marginal	
	$x = 1$	$x = 0$	$x = 1$	$x = 0$	$x = 1$	$x = 0$
$y = 1$ (death)	0	0	0.25	0.25	0.25	0.25
$y = 0$ (recovery)	0.25	0.25	0	0	0.25	0.25

Model C	$u_2 = 0$		$u_2 = 1$		Marginal	
	$x = 1$	$x = 0$	$x = 1$	$x = 0$	$x = 1$	$x = 0$
$y = 1$ (death)	0	0.25	0.25	0	0.25	0.25
$y = 0$ (recovery)	0.25	0	0	0.25	0.25	0.25

# Counterfactual Analysis

These three steps can be generalized to any causal model  $M$  as follows. Given evidence  $e$ , to compute the probability of  $Y = y$  under the hypothetical condition  $X = x$  (where  $X$  is a subset of variables), apply the following three steps to  $M$ .

**Step 1 (abduction):** Update the probability  $P(u)$  to obtain  $P(u \mid e)$ .

**Step 2 (action):** Replace the equations corresponding to variables in set  $X$  by the equations  $X = x$ .

**Step 3 (prediction):** Use the modified model to compute the probability of  $Y = y$ .

# Example

**Model b:**

**Model c:**

# DoWhy Library (Microsoft)

- <https://github.com/py-why/dowhy>

Case Studies using DoWhy: [Hotel booking cancellations](#) | [Effect of customer loyalty programs](#) | [Optimizing article headlines](#) | [Effect of home visits on infant health \(IHDP\)](#) | [Causes of customer churn/attrition](#)

