Lecture 3: Informed Search Techniques
CS486/686 Intro to Artificial Intelligence

2026-1-13

Pascal Poupart
David R. Cheriton School of Computer Science

UNIVERSITY OF WATERLOO

Outline

- Using knowledge
  - Heuristics
- Best-first search
  - Greedy best-first search
  - A* search
  - Other variations of A*

Recall from last lecture

- Uninformed search methods expand nodes based on "distance" from start node
  - Never look ahead to the goal, no domain specific info neded
- But, we often have some additional **knowledge** about the problem
  - E.g. in traveling around Romania we know the distances between cities so we can measure the overhead of going in the wrong direction

Informed Search

- Our knowledge is often about the merit of nodes
  - Value of being at a node

- Different notions of merit
  - If we are concerned about the cost of the solution, we might want a notion of how expensive it is to get from a state to a goal
  - If we are concerned with minimizing computation, we might want a notion of how easy it is to get from a state to a goal

  - We will focus on **cost of solution**

Informed search

- We need to develop a domain specific heuristic function, $h(n)$
- $h(n)$ guesses the cost of reaching the goal from node n
  - We often have some information about the problem that can be used in forming a heuristic function (i.e., heuristics are domain specific)

Informed search

- If $h(n_1) < h(n_2)$ then we guess that it is cheaper to reach the goal from n_1 than it is from n_2

- We require

$h(n) = 0$ when n is a goal node

$h(n) \geq \ 0$ for all other nodes

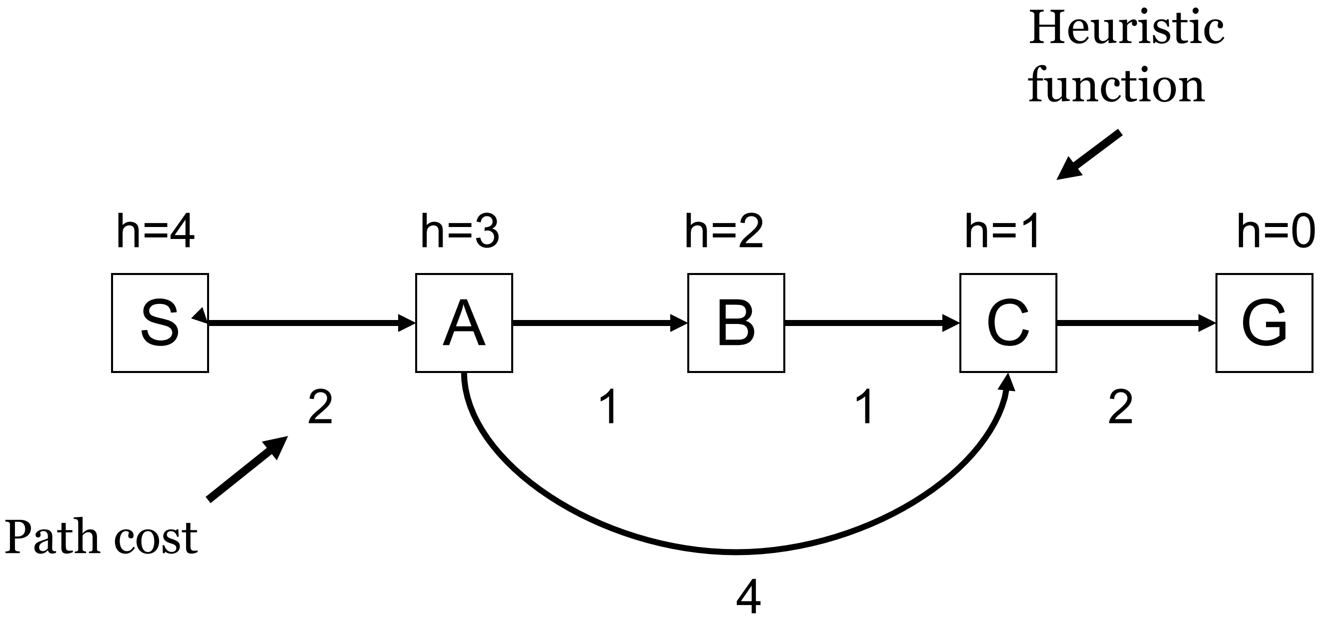
Greedy best-first search

- Use the heuristic function, $h(n)$, to rank the nodes in the fringe

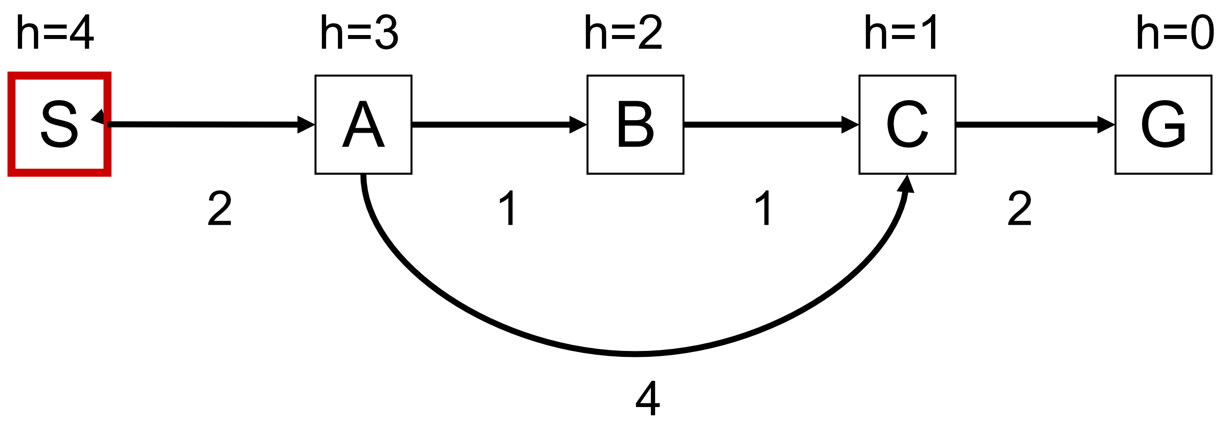
- Search strategy
  - Expand node with lowest h-value

- Greedily trying to find the least-cost solution

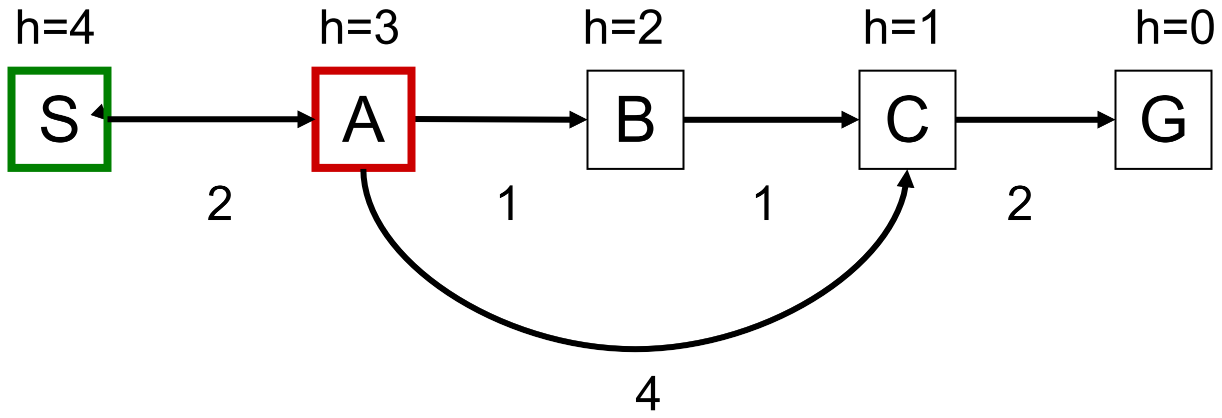
Greedy best-first search: Example



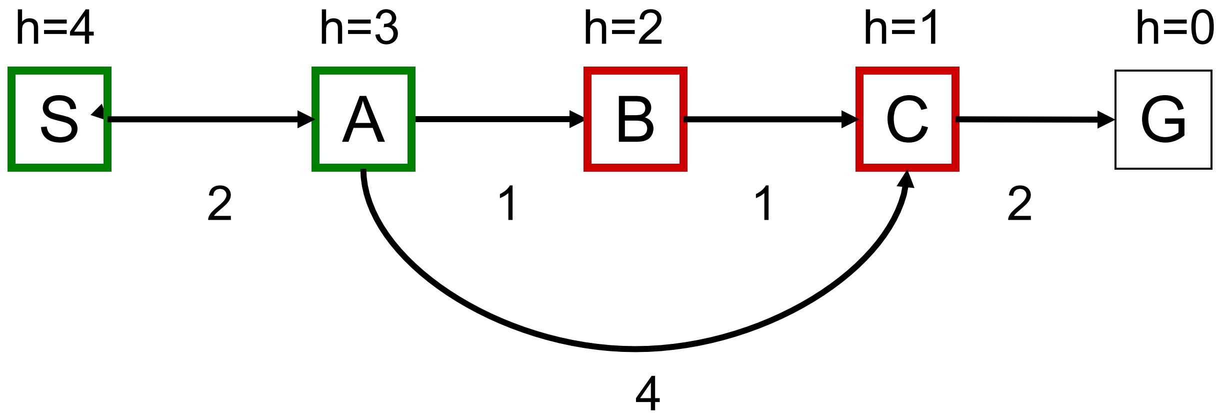
Example continued



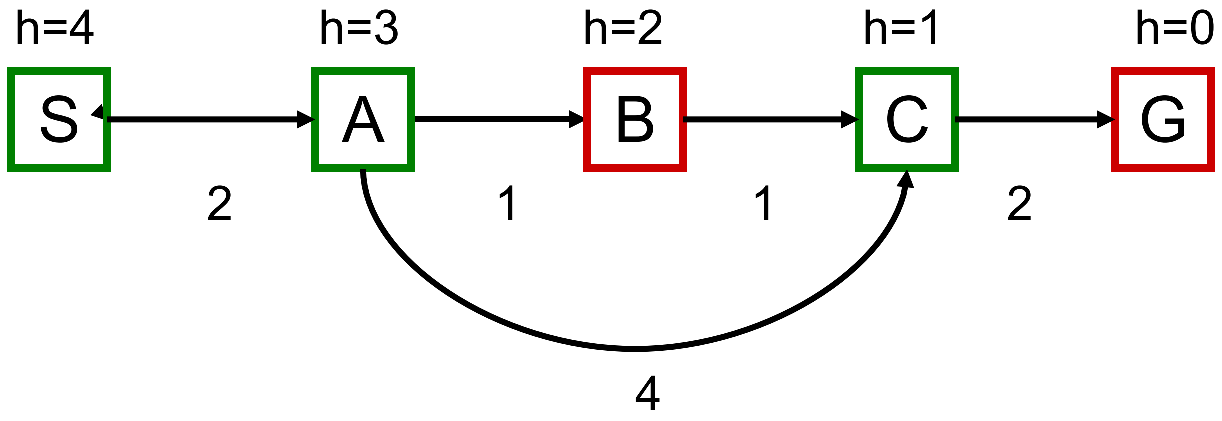
Example continued



Example continued

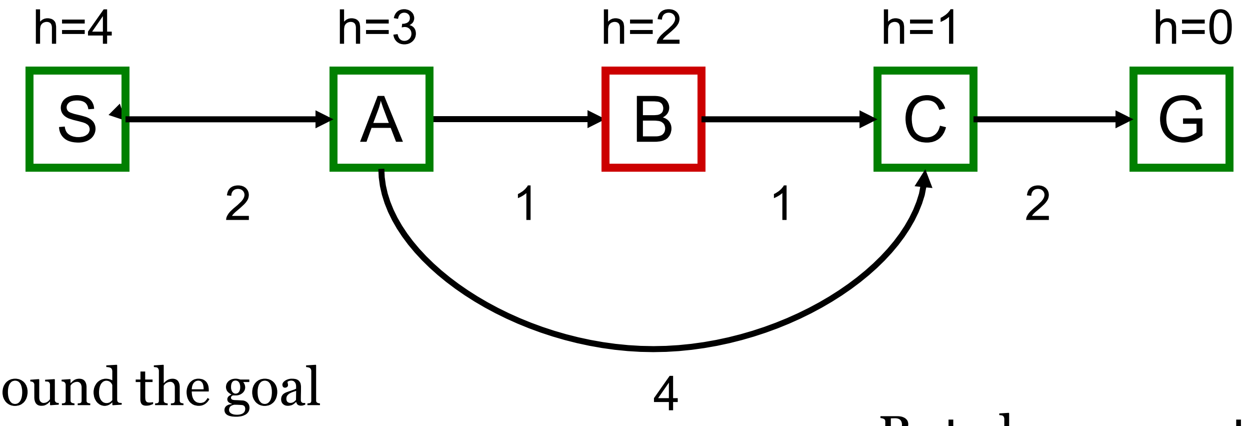


Example continued



Example continued

Greedy best-first is not optimal



Found the goal

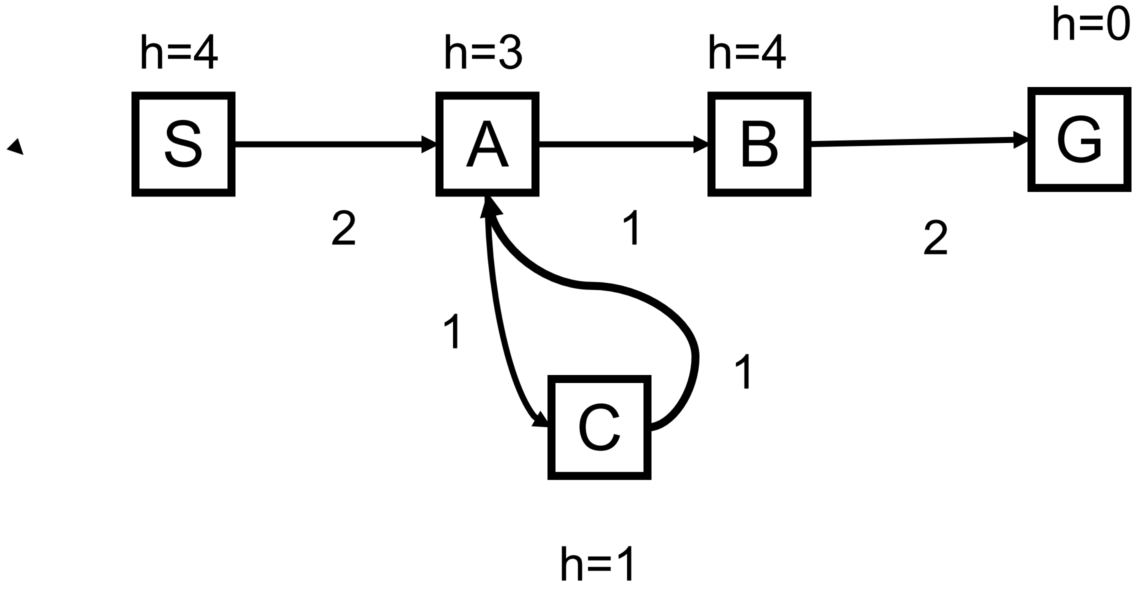
Path is S, A, C, G

Cost of the path is 2+4+2=8

But cheaper path is S, A, B, C, G

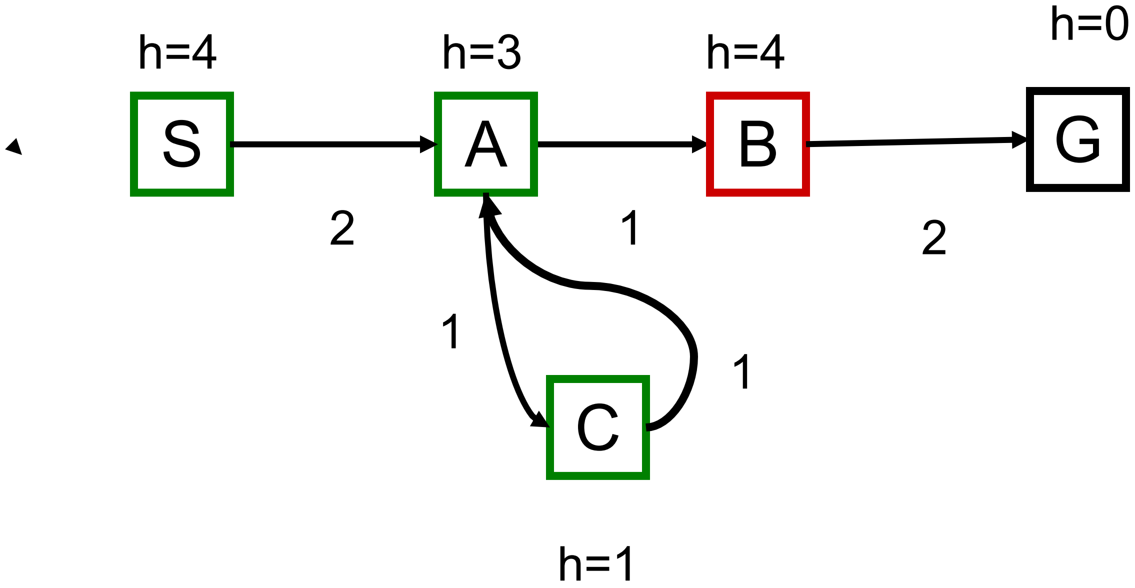
With cost **2+1+1+2=6**

Another Example



Another Example

Not complete



Greedy best-first can get stuck in loops

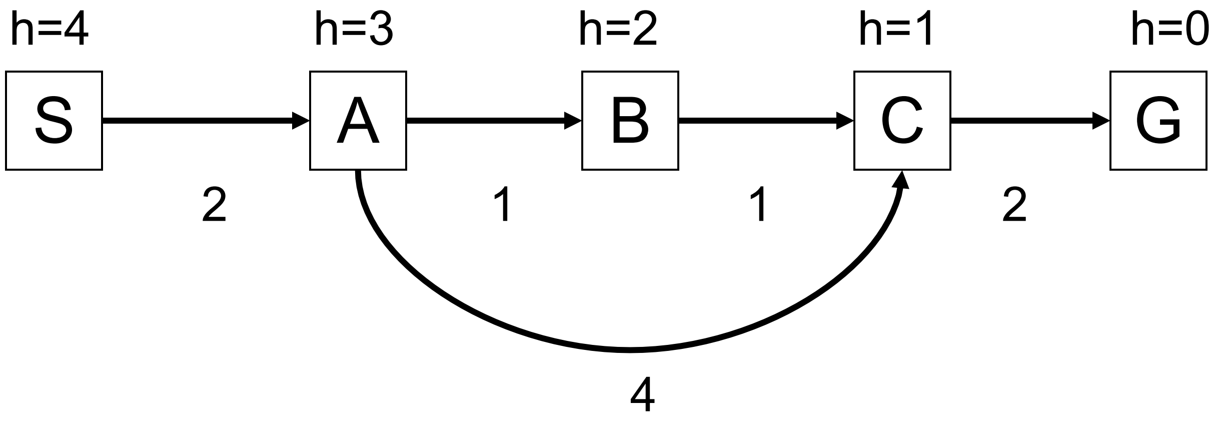
Properties of greedy search

- Not optimal!
- Not complete!
  - If we check for repeated states then we are ok
- Exponential space in worst case since need to keep all nodes in memory
- Exponential worst case time $O(b^m)$ where m is the maximum depth of the tree
  - If we choose a good heuristic then we can do much better

A* Search

- Greedy best-first search is too greedy
  - It does not take into account the cost of the path so far!
- Define $\boldsymbol{f(n) = g(n) + h(n)}$
  $g(n)$ is the cost of the path to node n
  $h(n)$ is the heuristic estimate of cost of reaching goal from node n
- A* search
  - Expand node in fringe (queue) with lowest f value

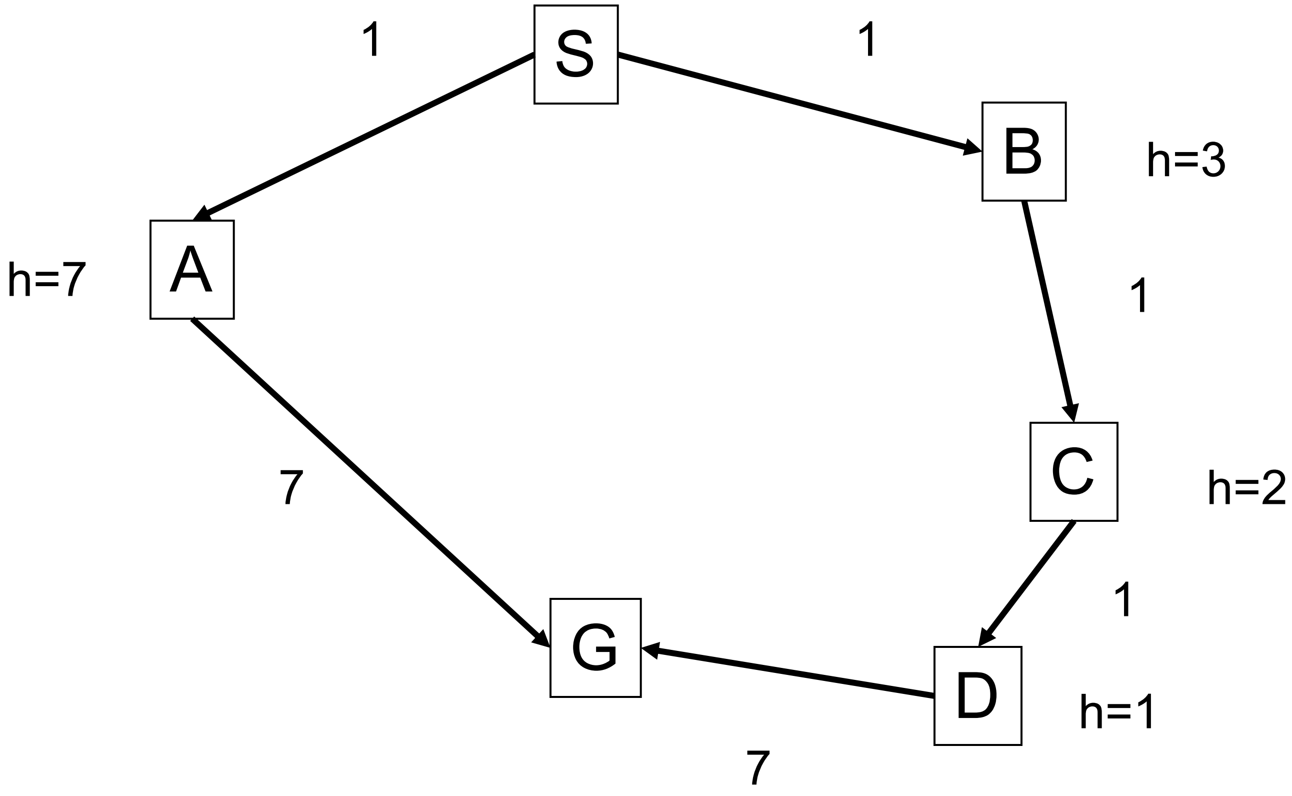
A* Example



1. Expand S
2. Expand A
3. Choose between B (f(B)=3+2=5) and C (f(C)=6+1=7)) expand B
4. Expand C
5. Expand G – recognize it is the goal

When should A* terminate?

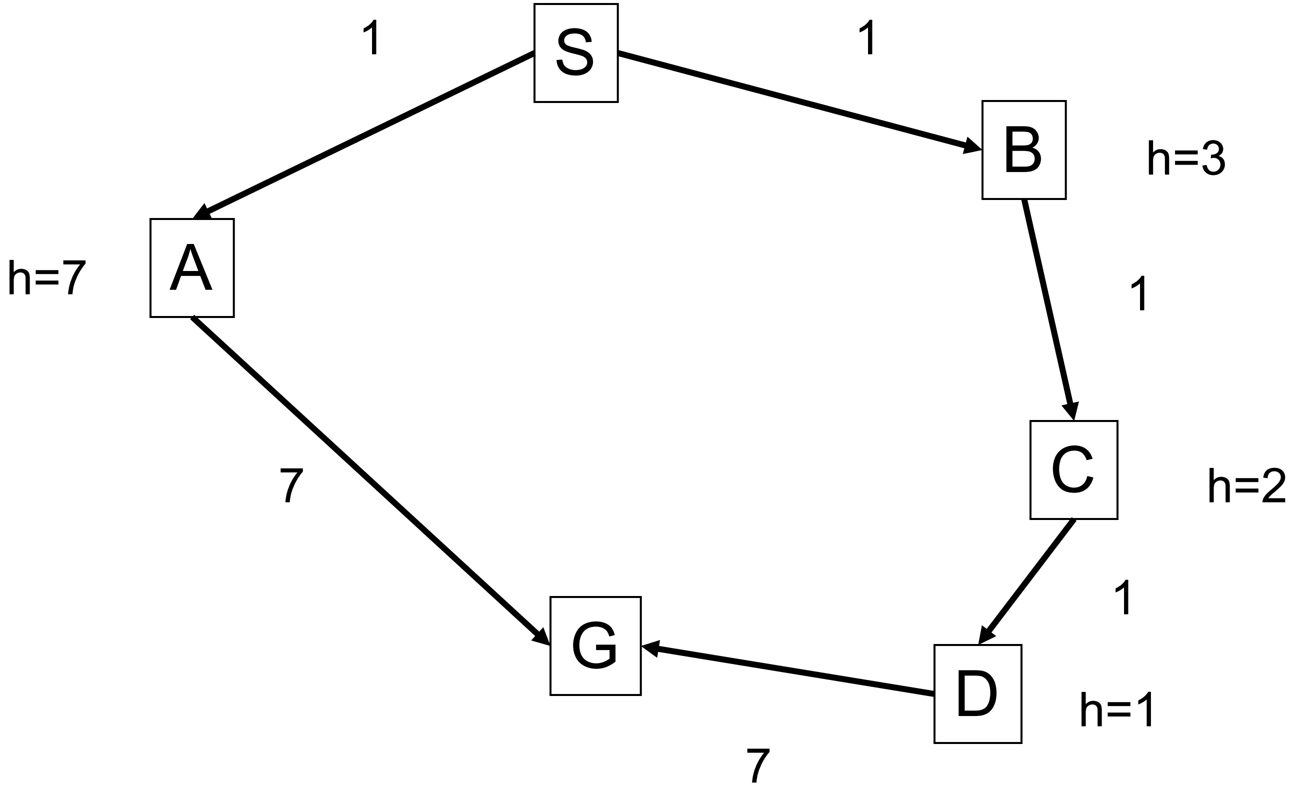
- As soon as we find a goal state?



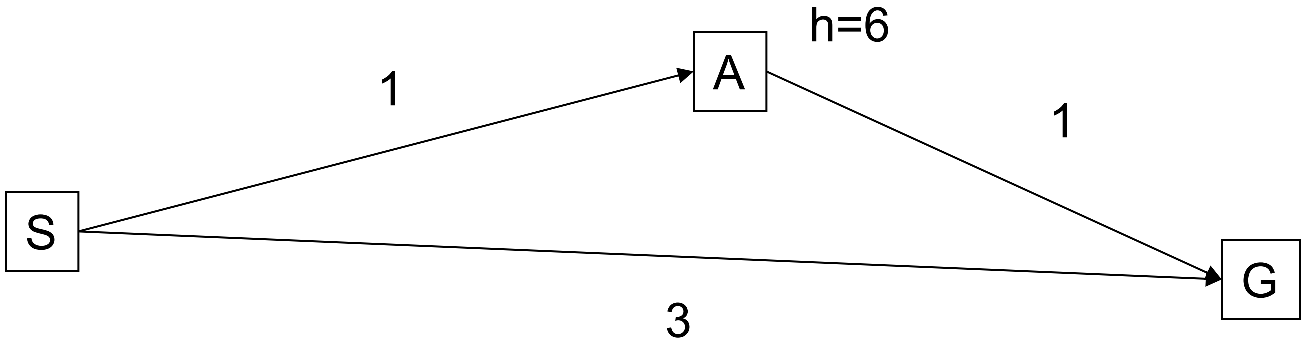
When should A* terminate?

- As soon as we find a goal state?

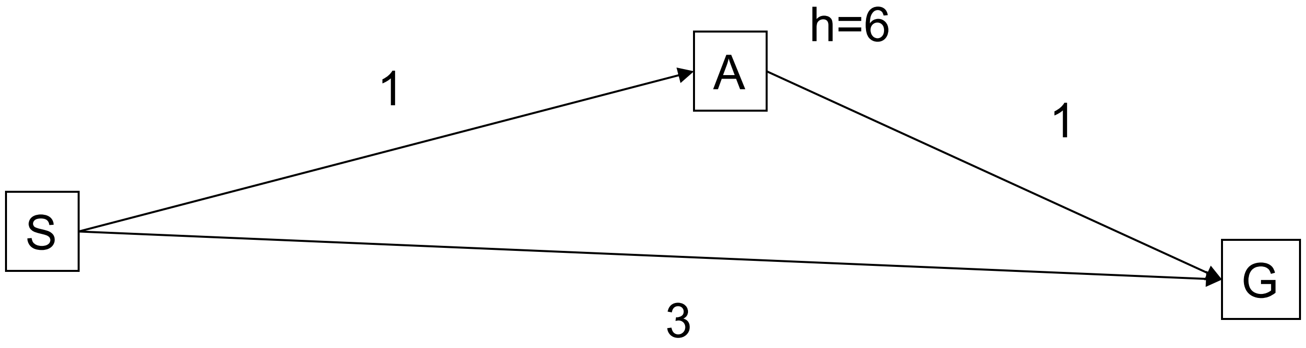
A* terminates only when goal state is popped from the queue



Is A* Optimal?



Is A* Optimal?



No. This example shows why not.

Admissible heuristics

- Let $h^*(n)$ denote the true minimal cost to the goal from node n

- A heuristic, h, is admissible if

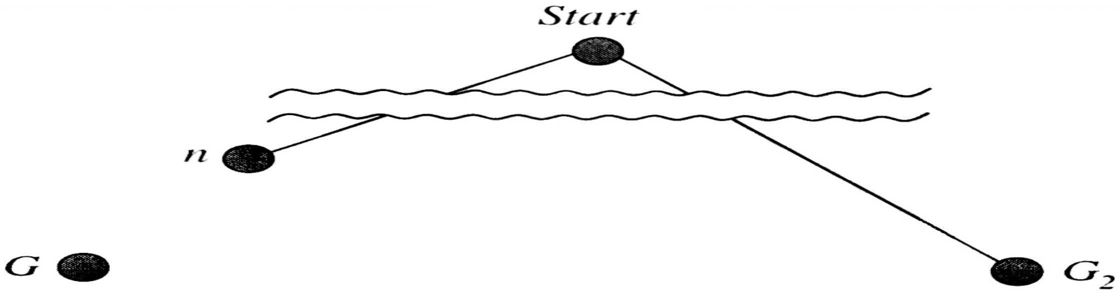
  $h(n) \leq h^*(n)$ for all n

- Admissible heuristics never overestimate the cost to the goal
  - Optimistic

Optimality of A*

If the heuristic is admissible then A* with tree-search is optimal

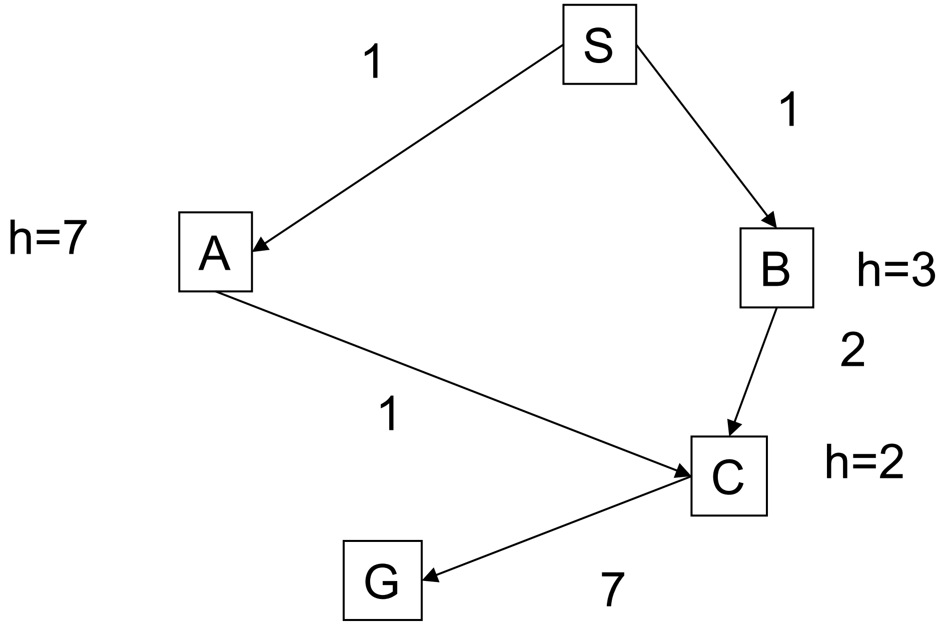
Let G be an optimal goal state, and $f(G) = f^* = g(G)$.
Let G_2 be a suboptimal goal state, i.e., $f(G_2) = g(G_2) > f^*$.
Assume for contradiction that A* selects G_2 from queue. (A* terminates with suboptimal solution)
Let n be a node that is currently a leaf node on an optimal path to G.



Since h is admissible, $f^* \geq f(n)$.
If n is not chosen for expansion over G_2, we must have $f(n) \geq f(G_2)$
So $f^* \geq f(G_2)$. Because $h(G_2) = 0$, we have $f^* \geq g(G_2)$, contradiction.

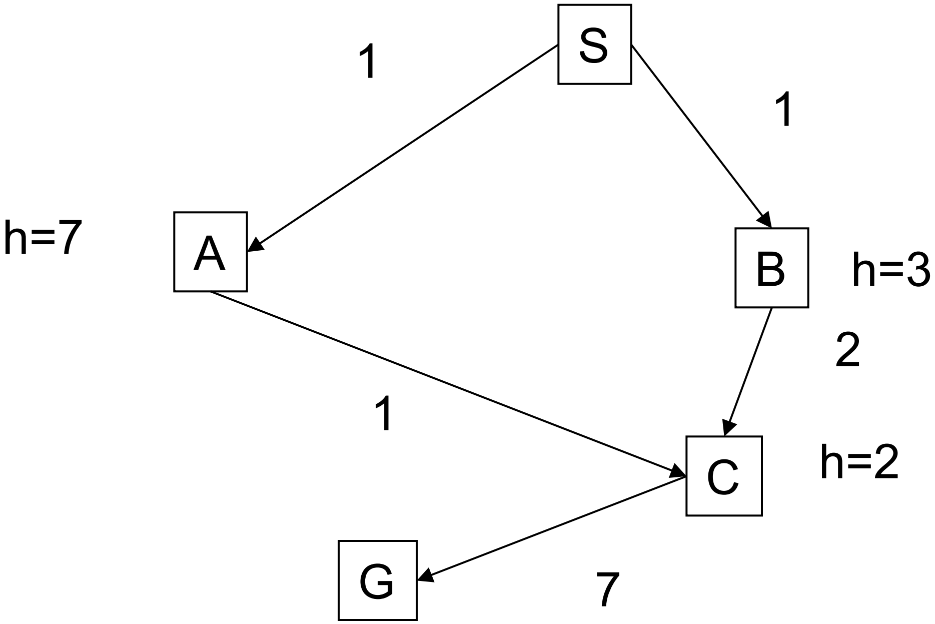
A* and revisiting states

What if we revisit a state that was already expanded?



A* and revisiting states

What if we revisit a state that was already expanded?



If we allow states to be expanded again, we might get a better solution!

Optimality of A*

- To search graphs, we need something stronger than admissibility
  - **Consistency (monotonicity):** $h(n) \leq cost(n, n') + h(n') \ \ \forall n, n'$
  - Almost any admissible heuristic function will also be consistent

- A* graph-search with a consistent heuristic is optimal

Properties of A*

- Complete (assuming finite branching factor and positive costs)
  - Along any path, f will eventually increase and the algorithm will eventually try all paths. Hence a solution will be found if there exists one.
- Exponential time complexity in worst case
  - A good heuristic will help a lot here
  - $O(bm)$ if the heuristic is perfect
- Exponential space complexity

Memory-bounded heuristic search

- A* keeps most generated nodes in memory
  - On many problems A* will run out of memory

- Iterative deepening A* (IDA*)
  - Like IDS, but change f-cost rather than depth at each iteration

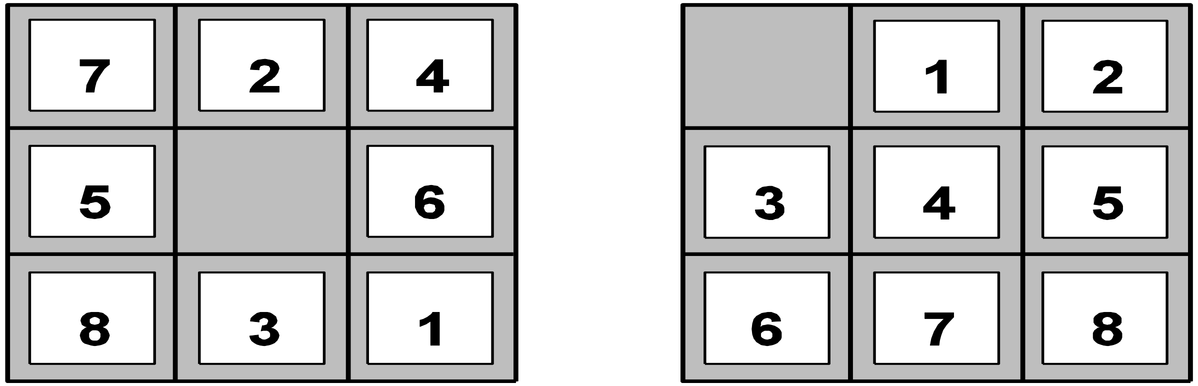
- SMA* (Simplified Memory-Bounded A*)
  - Uses all available memory
  - Proceeds like A* but when it runs out of memory it drops the worst leaf node (one with highest f-value)
  - If all leaf nodes have the same f-value then it drops oldest and expands the newest
  - Optimal and complete if depth of shallowest goal node is less than memory size

Heuristic Functions

- A good heuristic function can make all the difference!

- How do we get heuristics?
  - One approach is to think of an easier problem and let $h(n)$ be the cost of reaching the goal in the easier problem

8-puzzle



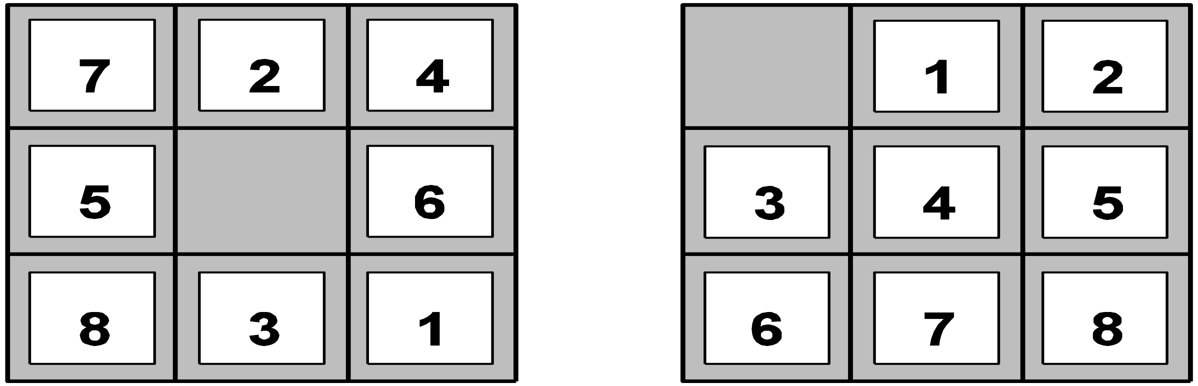
7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

8-puzzle



7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

Relax the game:

1. Can move tile from position A to position B if A is next to B (ignore whether or not position is blank)
2. Can move tile from position A to position B if B is blank (ignore adjacency)
3. Can move tile from position A to position B

8-puzzle continued

- 3) leads to misplaced tile heuristic
  - To solve this problem need to move each tile into its final position
  - Number of moves = number of misplaced tiles
  - Admissible
- 1) leads to manhattan distance heuristic
  - To solve the puzzle need to slide each tile into its final position
  - Admissible

8-puzzle continued

- h_3 = misplaced tiles
- h_1 = manhattan distance
- Note h_1 dominates h_3

  $h_3(n) \leq h_1(n)$ for all n

  Which heuristic is best?

Designing heuristics

- Relaxing the problem (as just illustrated)
- Precomputing solution costs of subproblems and storing them in a pattern database
- Learning from experience with the problem class

Conclusion

- What you should now know
  - Thoroughly understand A* and IDA*
  - Be able to trace simple examples of A* and IDA* execution
  - Understand admissibility and consistency of heuristics
  - Proof of completeness, optimality
  - Criticize greedy best-first search