Lecture 16: Reinforcement Learning CS486/686 Intro to Artificial Intelligence

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Outline

- Reinforcement Learning
  - Q-Learning
  - Exploration strategies

What is Reinforcement Learning?

- Reinforcement learning is also known as
  - Optimal control
  - Approximate dynamic programming
  - Neuro-dynamic programming

- Wikipedia: reinforcement learning is an area of machine learning inspired by behavioural psychology, concerned with how software **agents** ought to take **actions** in an **environment** so as to maximize some notion of cumulative **reward**.

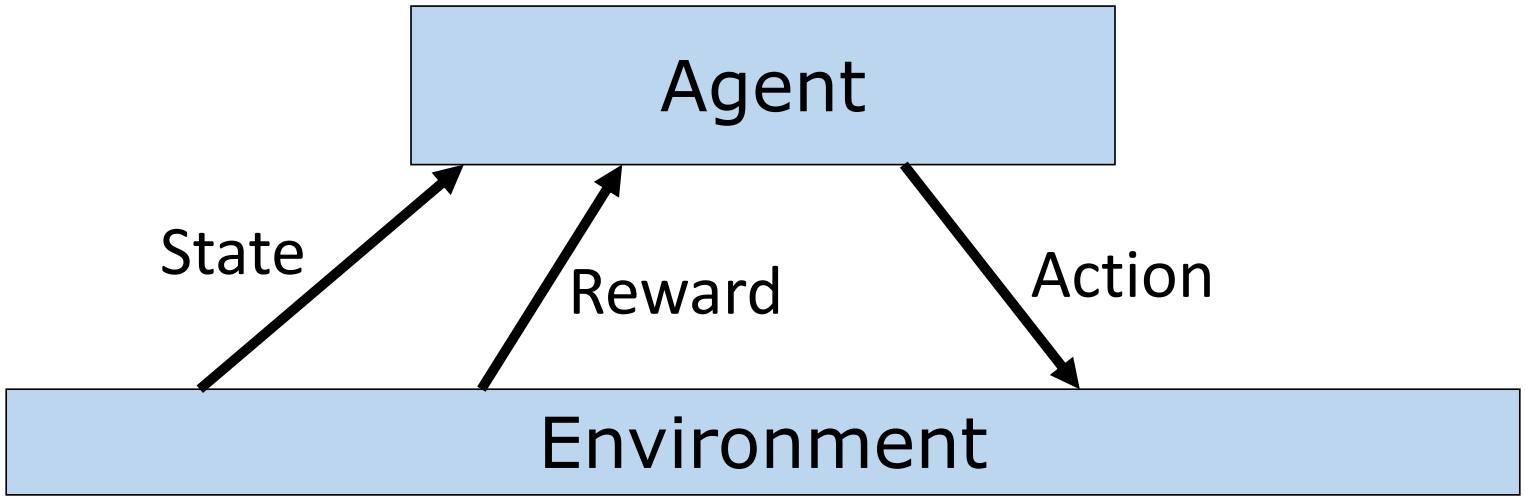
Animal Psychology

- Negative reinforcements
  - Pain and hunger
- Positive reinforcements
  - Pleasure and food

- Reinforcements used to train animals

- Let's do the same with computers

Reinforcement Problem



Goal: Learn to choose actions that maximize rewards

Reinforcement Learning

- Comprehensive, but challenging form of machine learning
  - Stochastic environment
  - Incomplete model
  - Interdependent sequence of decisions
  - No supervision
  - Partial and delayed feedback

- Long term goal: continual learning

Reinforcement Learning

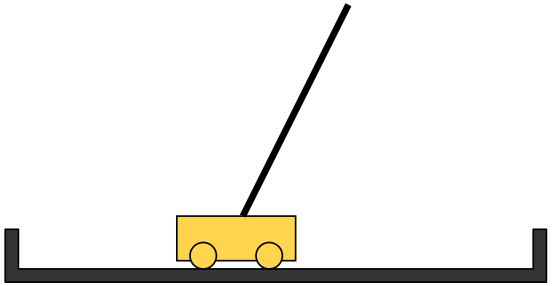
- Formal Definition
  - States: $s \in S$
  - Actions: $a \in A$
  - Rewards: $r \in \mathbb{R}$
  - ~~Transition model: $\Pr(s_t|s_{t-1}, a_{t-1})$~~
  - ~~Reward model: $\Pr(r_t|s_t, a_t)$~~
  - Discount factor: $0 \leq \gamma \leq 1$
  - Horizon (i.e., # of time steps): h

} unknown

- Goal: find optimal policy $\pi^* = argmax_\pi \sum_{t=0}^{h} \gamma^t E_\pi[r_t]$

Example: Inverted Pendulum

- State: $x(t), x'(t), \theta(t), \theta'(t)$
- Action: Force F
- Reward: 1 for any step where pole balanced



Problem: Find $\pi: S \rightarrow A$ that maximizes rewards

Sample Industrial Use Cases

Less Complex

More Complex

Contextual Bandits

Marketing
- ad placement,
- recommender systems

Loyalty programs
- personalized offers

Price management
- airline seat pricing
- cargo shipment pricing
- food pricing

Optimal design
- interface personalization

Bayesian Optimization

Hyperparameter optimization

Troubleshooting
- Customer assistance

Diagnostics
- Fault detection

Design of experiments
- Drug design
- Material design

Sequential decision Making

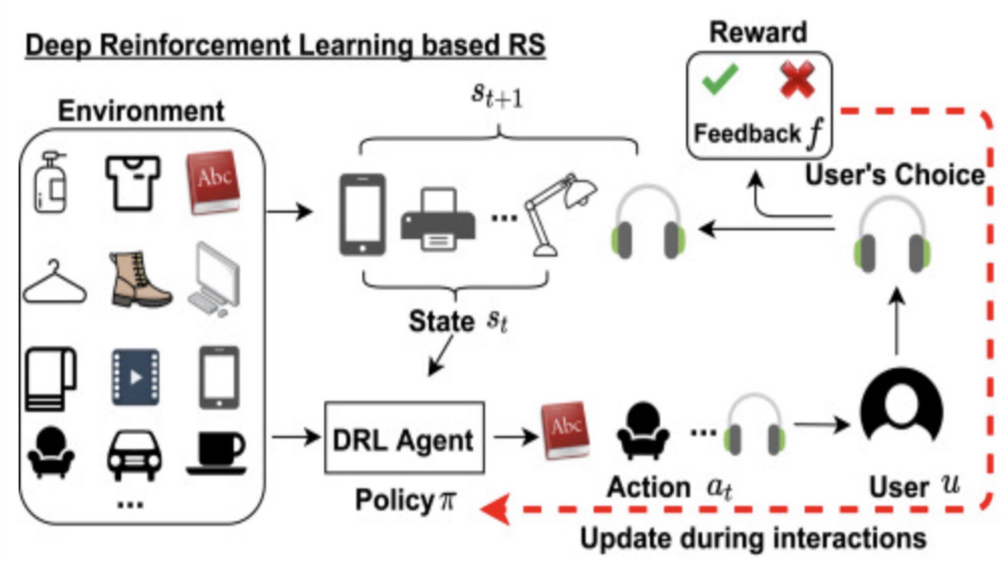
Automated trading
- Stocks, energy

Optimization
- Path planning
- Routing
- Energy consumption

Control
- Robotics
- Autonomous driving

Marketing (Recommender System)

- **Agent:** recommender system
- **Environment:** user
- **State:** context, past recommendations and feedback
- **Action:** recommended item
- **Reward:** value of user feedback



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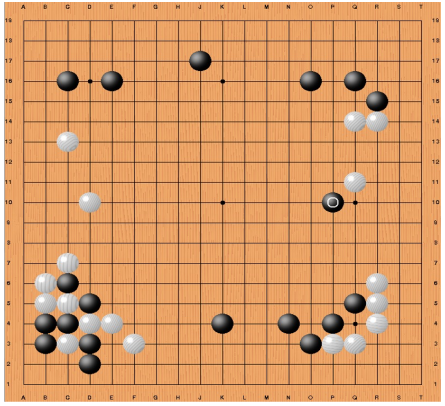
Operations Research (vehicle routing)

- **Agent:** vehicle routing system
- **Environment:** stochastic demand
- **State:** vehicle location, capacity and depot requests
- **Action:** vehicle route
- **Reward:** - travel costs

Game Playing (Computer Go)

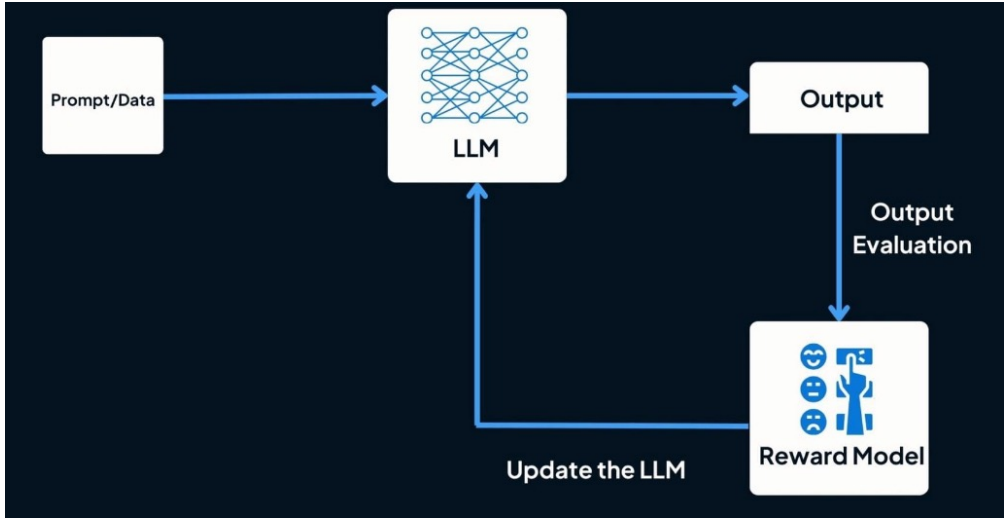
- **Agent:** player
- **Environment:** opponent
- **State:** board configuration
- **Action:** next stone location
- **Reward:** +1 win / -1 loose



- 2016: AlphaGo defeats Lee Sedol (4-1)
  - Game 2 move 37: AlphaGo plays unexpected move (odds 1/10,000)

Conversational Agents (RL from Human Feedback)

- **Agent:** system
- **Environment:** user
- **State:** history of past utterances
- **Action:** system utterance
- **Reward:** task completion, human feedback



Credit: https://www.twine.net/blog/what-is-reinforcement-learning-from-human-feedback-rlhf-and-how-does-it-work/

Computational Finance (Trading)

- **Agent:** trading software
- **Environment:** other traders
- **State:** price history
- **Action:** buy/sell/hold
- **Reward:** amount of profit

Example: how to purchase a large # of shares in a short period of time without affecting the price

Important Components in RL

RL agents may or may not estimate the following components:

- Model: $\Pr(s'|s,a)$, $\Pr(r|s,a)$
  - Environment dynamics and rewards
- Policy: $\pi(s)$
  - Agent action choices
- Value function: $V(s)$
  - Expected total rewards of the agent policy

Categorizing RL agents

Value based
- No policy (implicit)
- Value function

Policy based
- Policy
- No value function

Actor critic
- Policy
- Value function

Model based
- Transition and reward model

Model free
- No transition and no reward model (implicit)

Online RL
- Learn by interacting with environment

Offline RL
- No environment
- Learn only from saved data

Bellman's Equation

- Value Iteration:

$$V_n^*(s) \leftarrow \max_a E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) V_{n-1}^*(s')$$

- Bellman Equation (when $n \rightarrow \infty$):

$$V^*(s) = \max_a E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) V^*(s')$$

- State-action Bellman Equation:

$$Q^*(s,a) = E[r|s,a] + \gamma \sum_{s'} Pr(s'|s,a) \max_{a'} Q^*(s',a')$$

where $V^*(s) = max_a Q^*(s,a)$, $\pi^*(s) = argmax_a Q^*(s,a)$

Temporal Difference Control

- Approximate Q-function:

$$Q^*(s,a) = E[r|s,a] + \gamma \sum_{s'} \Pr(s'|s,a) \max_{a'} Q^*(s',a')$$

$$\approx r + \gamma \max_{a'} Q^*(s',a') \longleftarrow \text{one sample approximation}$$

- **Incremental update**

$$Q_n^*(s,a) \leftarrow Q_{n-1}^*(s,a) + \alpha_n \left(r + \gamma \max_{a'} Q_{n-1}^*(s',a') - Q_{n-1}^*(s,a) \right)$$

learning rate

Tabular Q-Learning

Qlearning()

Initialize s and Q^* arbitrarily

Repeat

Select and execute a

Observe s' and r

Update counts: $n(s,a) \leftarrow n(s,a) + 1$

Learning rate: $\alpha \leftarrow 1/n(s,a)$

$$Q^*(s,a) \leftarrow Q^*(s,a) + \alpha\left(r + \gamma \max_{a'} Q^*(s',a') - Q^*(s,a)\right)$$

$s \leftarrow s'$

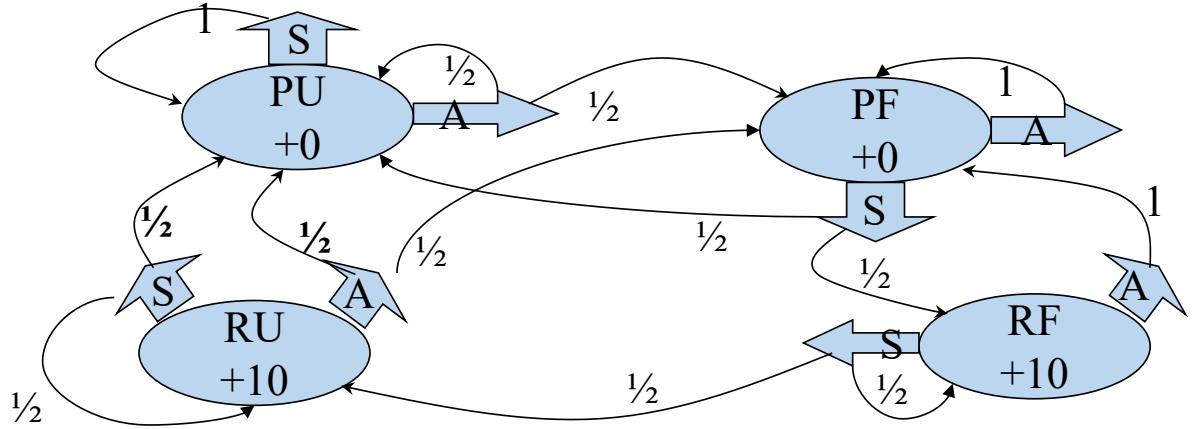
Until convergence of Q^*

Return Q^*

Q-learning Exercise

Current estimates:

$Q(RF, S) = 25$
$Q(RF, A) = 20$
$Q(RU, S) = 20$
$Q(RU, A) = 15$



Discount: $\gamma = 0.9$ Learning rate: $\alpha = 0.5$

Update $Q(RF, S)$ after executing S in RF and transitioning to RU:

$Q(RF,S) \leftarrow Q(RF,S) + \alpha(r + \gamma \max\{Q(RU,S), Q(RU,A)\} - Q(RF,S))$
$\leftarrow 25 + 0.5(10 + 0.9 \max\{20, 15\} - 25)$
$\leftarrow 26.5$

Convergence

- Q-learning converges to optimal Q-values if
  - Every state is visited infinitely often (due to exploration)
  - The action selection becomes greedy as time approaches infinity
  - The learning rate α is decreased fast enough, but not too fast (sufficient conditions for α):

    (1) $\sum_t \alpha_t \rightarrow \infty$ (2) $\sum_t (\alpha_t)^2 < \infty$

- NB: $\alpha_t(s,a) = 1/n_t(s,a)$ satisfies the above conditions

Common Exploration Methods

- ϵ-greedy:
  - With probability ϵ, execute random action
  - Otherwise execute best action $a^* = argmax_a\, Q(s,a)$

- Boltzmann exploration
  - Increasing temperature T increases stochasticity

$$\Pr(a) = \frac{e^{\frac{Q(s,a)}{T}}}{\sum_a e^{\frac{Q(s,a)}{T}}}$$