Lecture 12: Deep Neural Networks
CS486/686 Intro to Artificial Intelligence

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Outline

- Deep Neural Networks
  - Gradient Vanishing
    - Rectified linear units
  - Overfitting
    - Dropout

- Breakthroughs
  - Acoustic modeling in speech recognition
  - Image recognition

Deep Neural Networks

- Definition: neural network with many hidden layers

- Advantage: high expressivity
- Challenges:
  - How should we train a deep neural network?
  - How can we avoid overfitting?

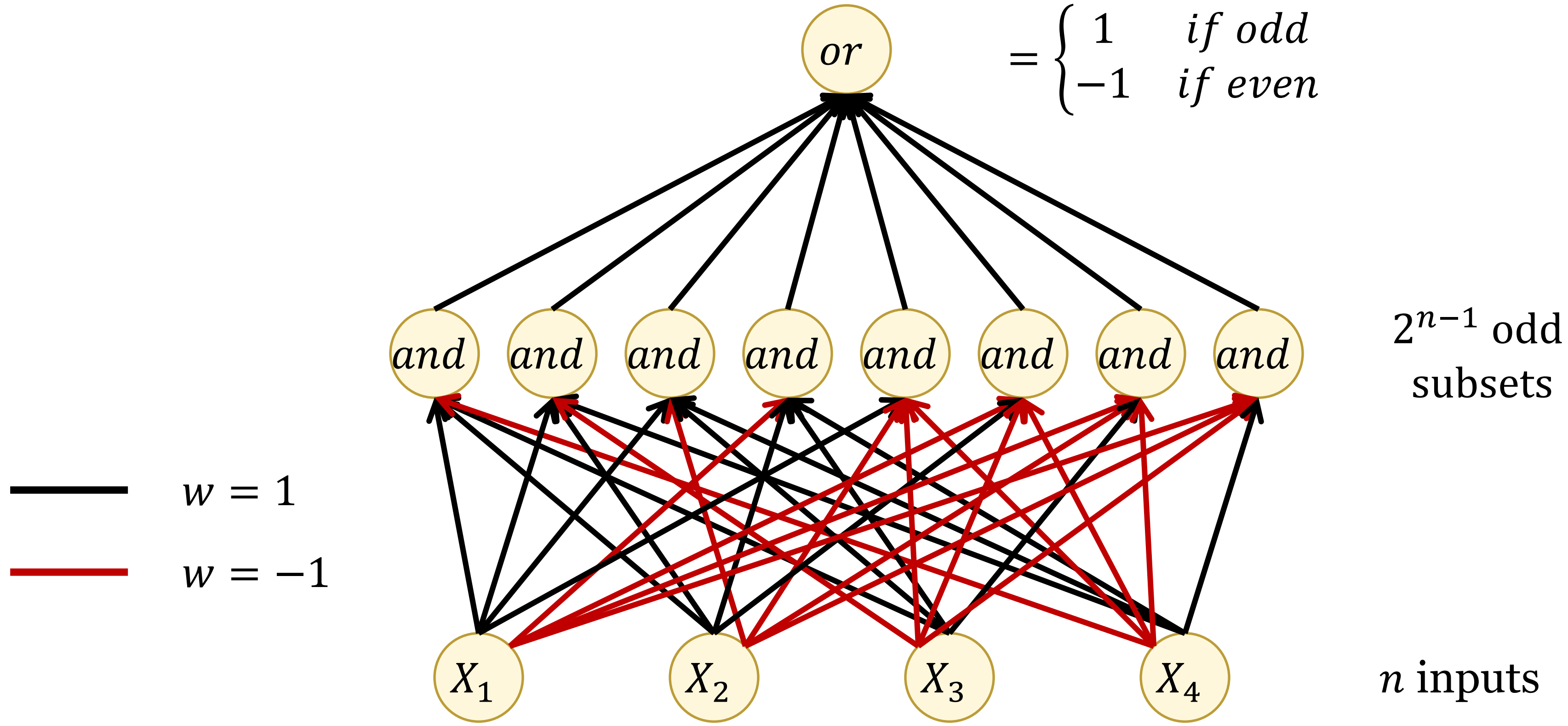
Expressiveness

- Neural networks with one hidden layer of sigmoid/hyperbolic units can approximate arbitrarily closely neural networks with several layers of sigmoid/hyperbolic units

- However as we increase the number of layers, the number of units needed may decrease exponentially (with the number of layers)

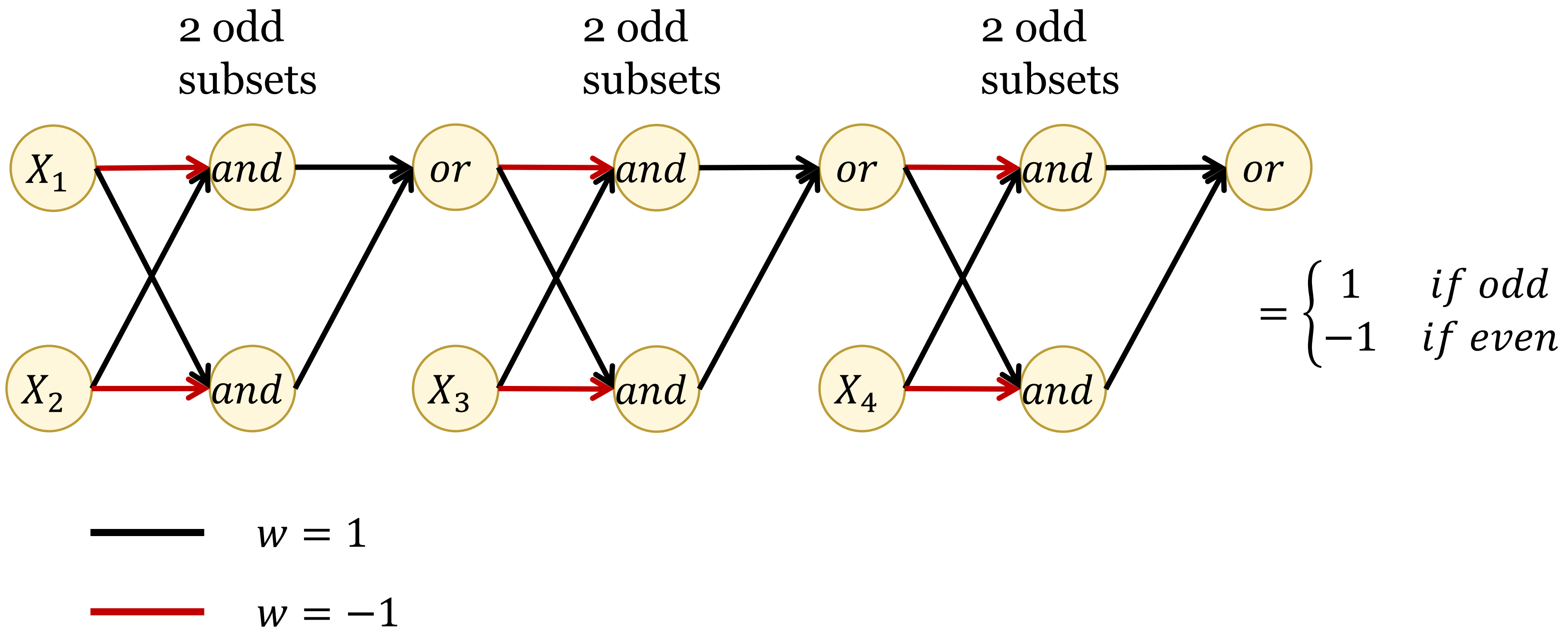
Example – Parity Function

- Single layer of hidden nodes



Example – Parity Function

- $2n - 2$ layers of hidden nodes



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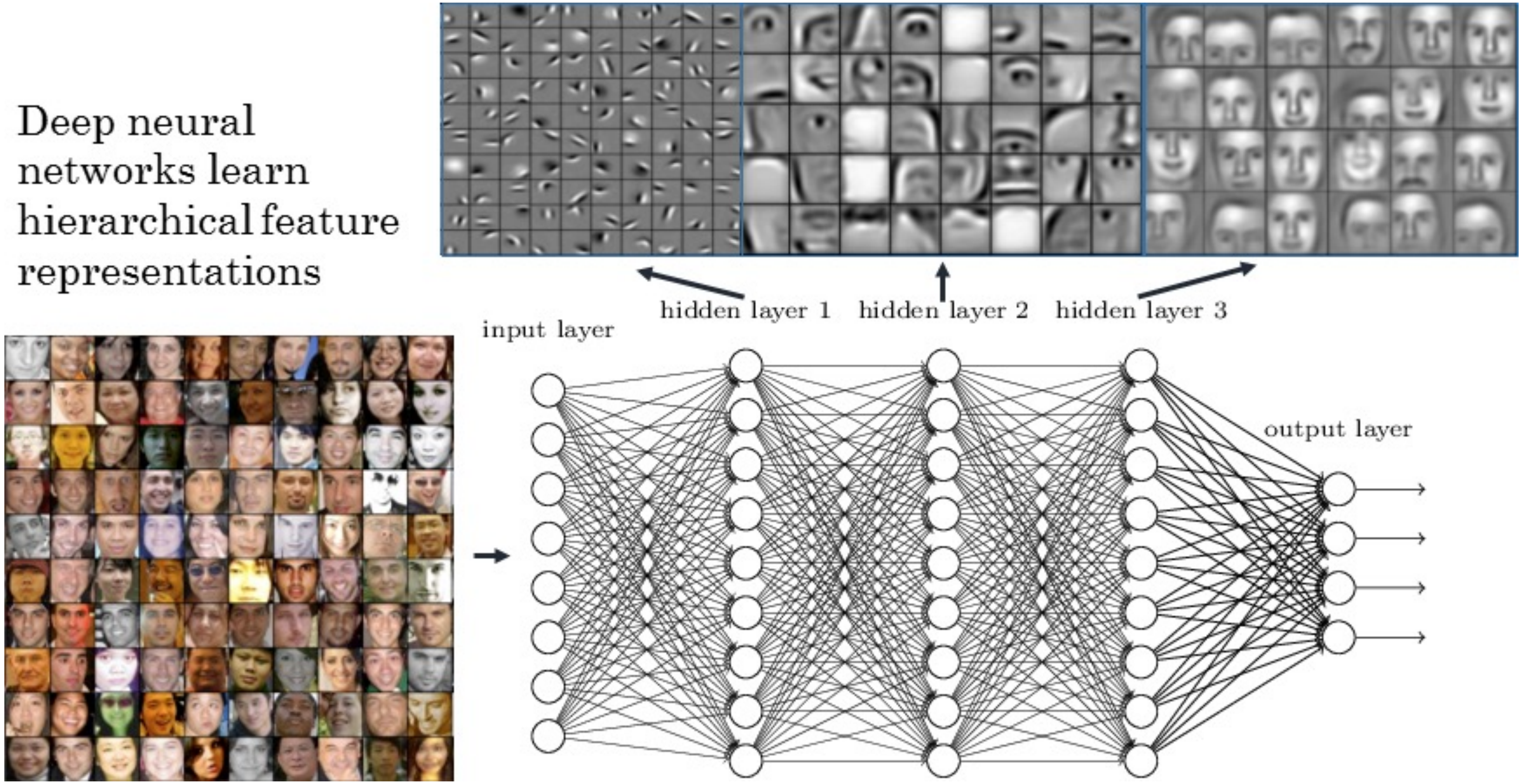
The power of depth (practice)

- Challenge: how to train deep NNs?



Speech

- 2006 (Hinton, al.): first effective algorithm for deep NN
  - layerwise training of Stacked Restricted Boltzmann Machines (SRBM)s

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  - Improved speech recognition at Google, Microsoft, IBM

Speech

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- 2009: Breakthrough in acoustic modeling
  - replace Gaussian Mixture Models by SRBMs
  - Improved speech recognition at Google, Microsoft, IBM
- 2013-2019: recurrent neural nets (LSTM)
  - Google error rate: 23% (2013) → 8% (2015)
  - Microsoft error rate: 5.9% (Oct 17, 2016) same as human performance

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Image Classification

- ImageNet Large Scale Visual Recognition Challenge

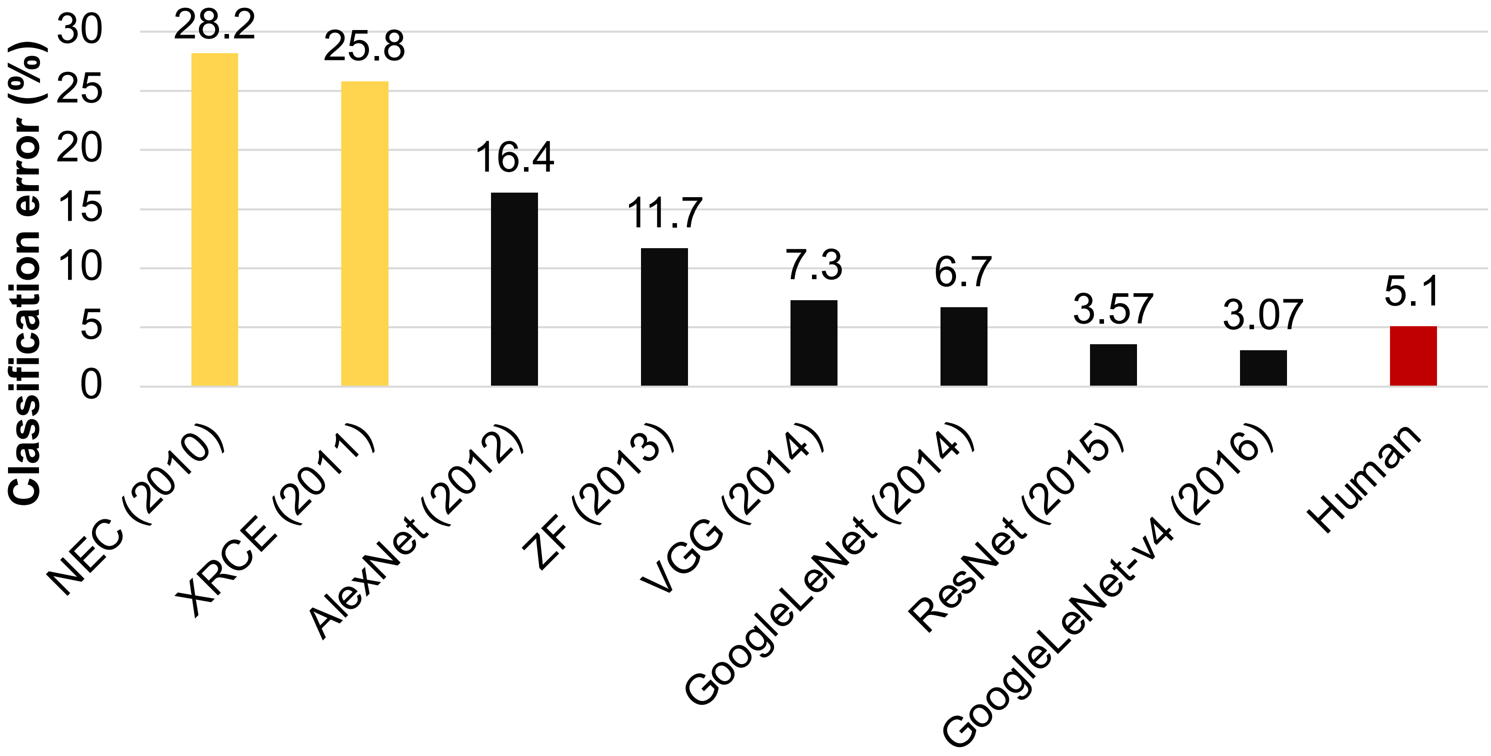

Image Classification

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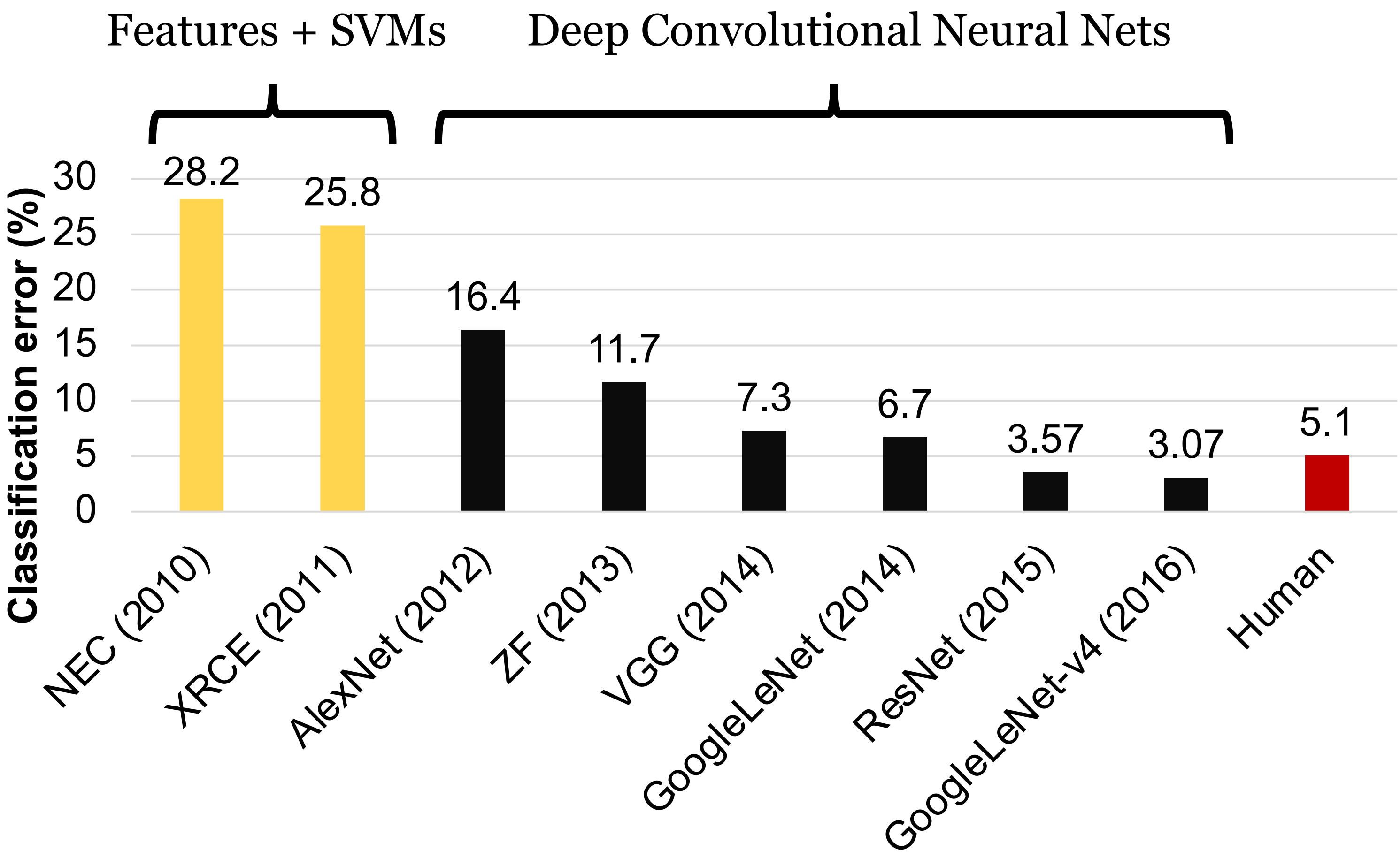

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Image Classification

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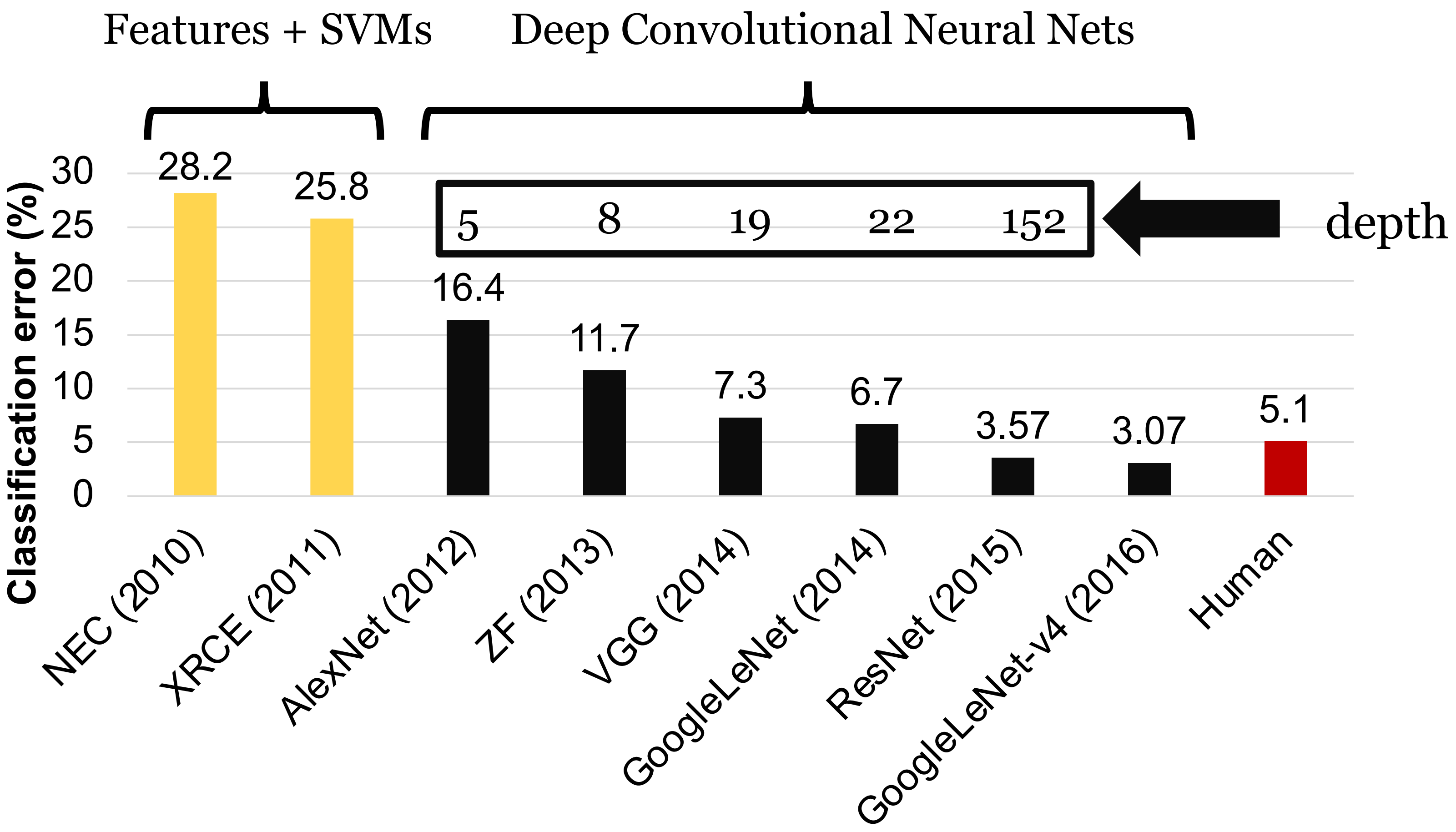
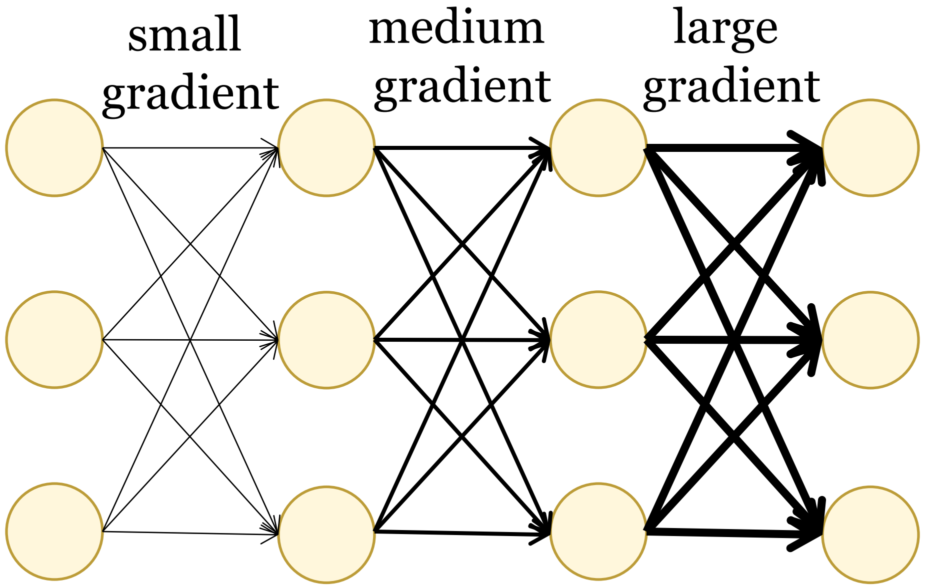


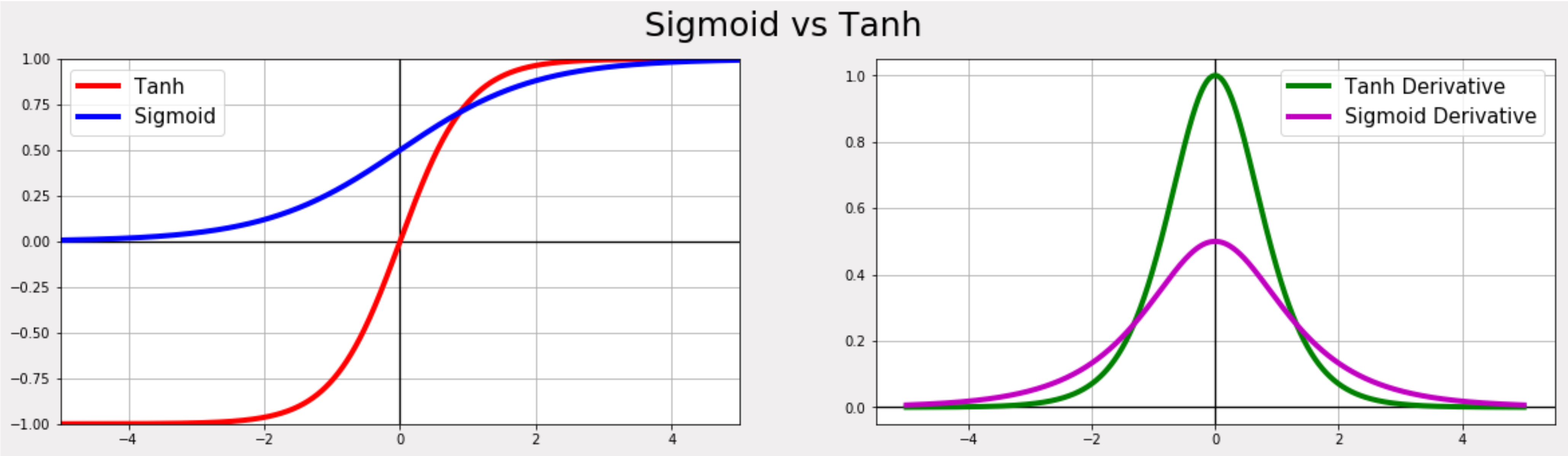
Vanishing Gradients

- Deep neural networks of sigmoid and hyperbolic units often suffer from vanishing gradients



Sigmoid and hyperbolic units

- Derivatives are always less than 1



From Aidan Wilson (https://a-i-dan.github.io/math_nn)

Simple Example

- $y = \sigma\left(w_4\, \sigma\left(w_3\, \sigma(w_2\, \sigma(w_1\, x))\right)\right)$

$$x \xrightarrow{w_1} h_1 \xrightarrow{w_2} h_2 \xrightarrow{w_3} h_3 \xrightarrow{w_4} y$$

- Common weight initialization in (-1,1)
- Sigmoid function and its derivative always less than 1
- This leads to vanishing gradients:

$$\frac{\partial y}{\partial w_4} = \sigma'(a_4)\sigma(a_3)$$
$$\frac{\partial y}{\partial w_3} = \sigma'(a_4)w_4\sigma'(a_3)\sigma(a_2)$$
$$\frac{\partial y}{\partial w_2} = \sigma'(a_4)w_4\sigma'(a_3)w_3\sigma'(a_2)\sigma(a_1)$$
$$\frac{\partial y}{\partial w_1} = \sigma'(a_4)w_4\sigma'(a_3)w_3\sigma'(a_2)w_2\sigma'(a_1)x$$

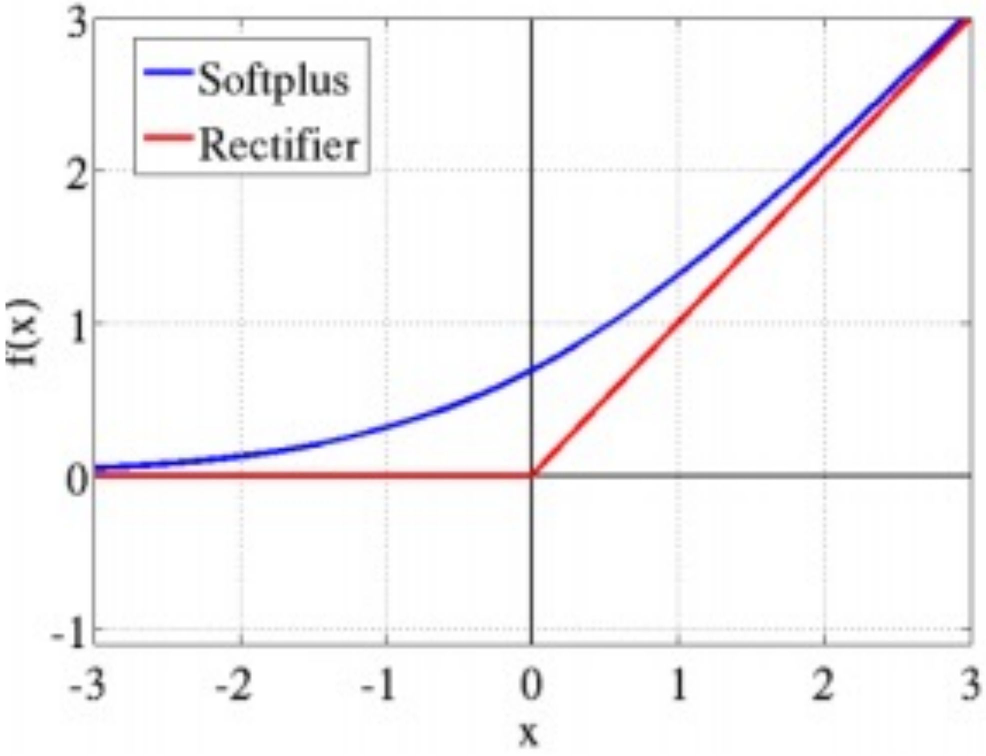
As products of factors less than 1 gets longer, gradient vanishes

Avoiding Vanishing Gradients

- Several popular solutions:
  - Pre-training
  - **Rectified linear units**
  - Skip connections
  - Batch normalization

Rectified Linear Units

From Abhinav Ralhan (https://medium.com/@abhinavr8/activation-functions-neural-networks-66220238e1ff)



- Rectifier (ReLU): $h(a) = \max(0, a)$
  - Gradient is 0 or 1
  - Sparse computation

- Soft version ("Softplus") : $h(a) = \log(1 + e^{a})$

- Warning: softplus does not prevent gradient vanishing (gradient < 1)

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Overfitting

- High expressivity increases the risk of overfitting
  - # of parameters is often larger than the amount of data

- Some solutions:
  - Regularization
  - **Dropout**
  - Data augmentation

Dropout

- Idea: randomly "drop" some units from the network when training

- Training: at each iteration of gradient descent
  - Each input unit is dropped with probability p_1 (e.g., 0.2)
  - Each hidden unit is dropped with probability p_2 (e.g., 0.5)

- Prediction (testing):
  - Multiply each input unit by $1 - p_1$
  - Multiply each hidden unit by $1 - p_2$

Dropout Illustration

Dropout Algorithm

Training: let $\odot$ denote elementwise multiplication

- Repeat
  - For each training example $(\boldsymbol{x}_n, y_n)$ do
    - Sample $\boldsymbol{z}_n^{(l)}$ from $Bernoulli(1-p_l)^{k_l}$ for $1 \leq l \leq L$
    - Neural network with dropout applied:

$$f_n(\boldsymbol{x}_n, \boldsymbol{z}_n; \boldsymbol{W}) = h_l\left(\boldsymbol{W}^{(L)}\left[\dots h_2\left(\boldsymbol{W}^{(2)}\left[h_1\left(\boldsymbol{W}^{(1)}\left[\overline{\boldsymbol{x}}_n \odot \boldsymbol{z}_n^{(1)}\right]\right) \odot \boldsymbol{z}_n^{(2)}\right]\right) \dots \odot \boldsymbol{z}_n^{(L)}\right]\right)$$

    - Loss: $Err(y_n, f_n(\boldsymbol{x}_n, \boldsymbol{z}_n; \boldsymbol{W}))$
    - Update: $w_{kj} \leftarrow w_{kj} - \eta \frac{\partial Err}{\partial w_{kj}}$
  - End for
- Until convergence

Prediction: $f(\boldsymbol{x}_n; \boldsymbol{W}) = h_l(\boldsymbol{W}^{(L)}[\dots h_2(\boldsymbol{W}^{(2)}[h_1(\boldsymbol{W}^{(1)}[\overline{\boldsymbol{x}}_n(1-p_1])(1-p_2)]) \dots (1-p_L)])$

Intuition

- Dropout can be viewed as an approximate form of ensemble learning

- In each training iteration, a different subnetwork is trained
- At test time, these subnetworks are “merged” by averaging their weights

Early Applications of Deep Neural Networks

- Speech Recognition
- Image recognition
- Machine translation
- Control

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Acoustic Modeling in Speech Recognition

TABLE III

A comparison of the Percentage Word Error Rates using DNN-HMMs and GMM-HMMs on five different large vocabulary tasks.

task	hours of training data	DNN-HMM	GMM-HMM with same data	GMM-HMM with more data
Switchboard (test set 1)	309	18.5	27.4	18.6 (2000 hrs)
Switchboard (test set 2)	309	16.1	23.6	17.1 (2000 hrs)
English Broadcast News	50	17.5	18.8	
Bing Voice Search (Sentence error rates)	24	30.4	36.2	
Google Voice Input	5,870	12.3		16.0 (>>5,870hrs)
Youtube	1,400	47.6	52.3	

Architecture of a DNN-HMM hybrid system

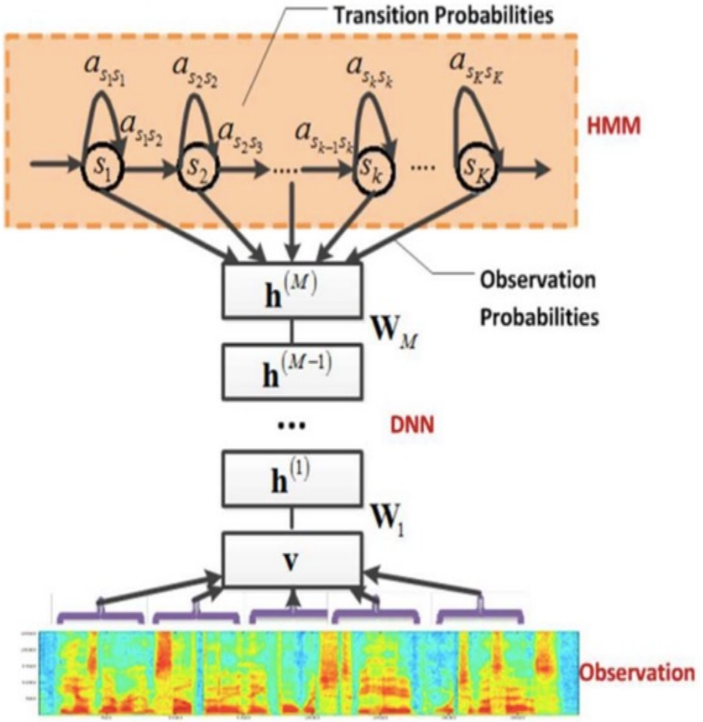

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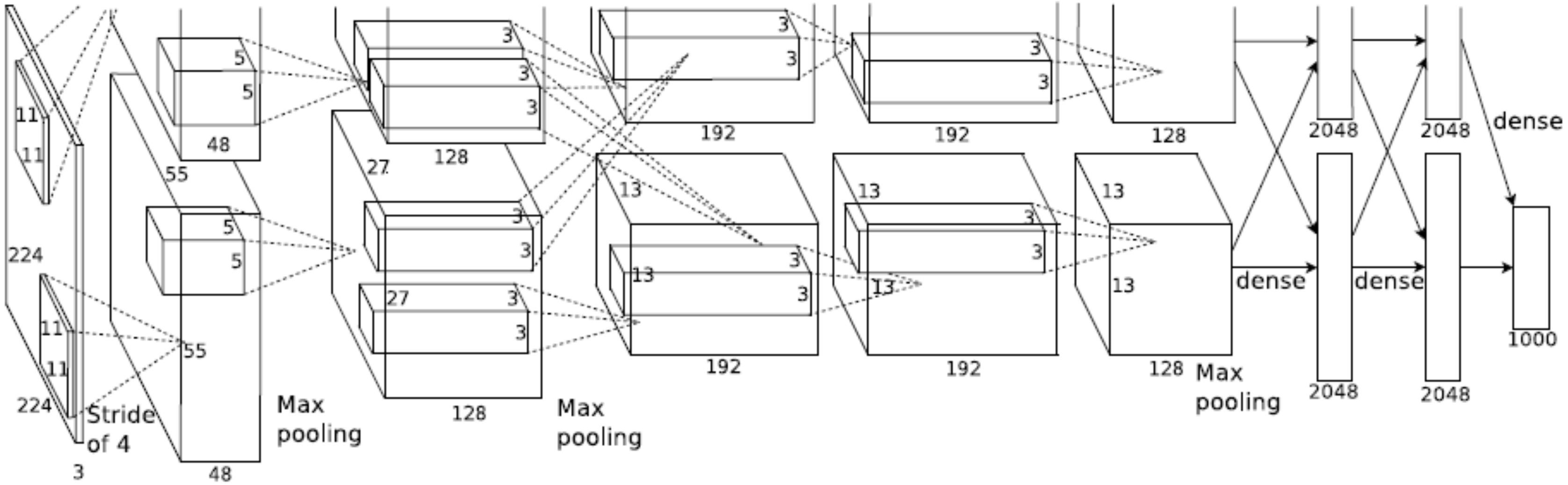
Image Recognition

- Convolutional Neural Network
  - With rectified linear units and dropout
  - Data augmentation for transformation invariance



ImageNet Breakthrough

- Results: ILSVRC-2012
- From Krizhevsky, Sutskever, Hinton

Model	Top-1 (val)	Top-5 (val)	Top-5 (test)
SIFT + FVs [7]	—	—	*26.2%*
1 CNN	40.7%	18.2%	—
5 CNNs	38.1%	16.4%	**16.4%**
1 CNN*	39.0%	16.6%	—
7 CNNs*	36.7%	15.4%	**15.3%**

Table 2: Comparison of error rates on ILSVRC-2012 validation and test sets. In *italics* are best results achieved by others. Models with an asterisk* were "pre-trained" to classify the entire ImageNet 2011 Fall release. See Section 6 for details.

ImageNet Breakthrough

- From Krizhevsky, Sutskever, Hinton

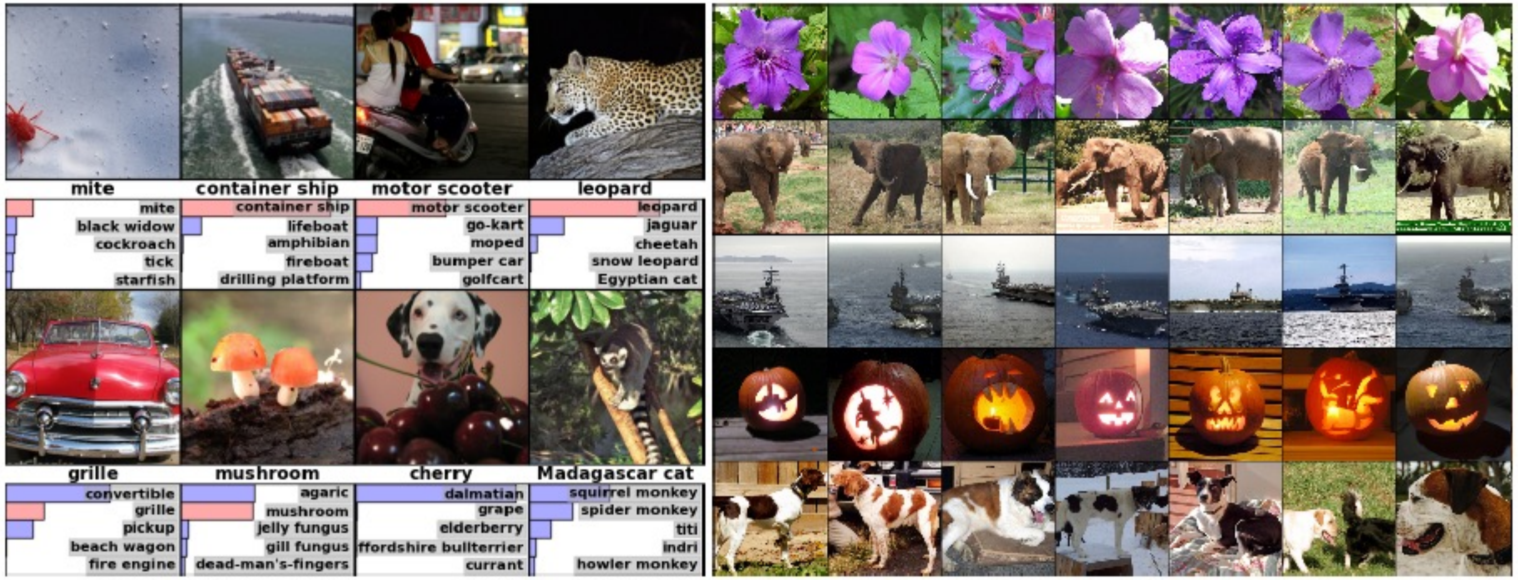


Figure 4: (**Left**) Eight ILSVRC-2010 test images and the five labels considered most probable by our model. The correct label is written under each image, and the probability assigned to the correct label is also shown with a red bar (if it happens to be in the top 5). (**Right**) Five ILSVRC-2010 test images in the first column. The remaining columns show the six training images that produce feature vectors in the last hidden layer with the smallest Euclidean distance from the feature vector for the test image.