Markov Decision Processes
[RN2] Sec 17.1, 17.2, 17.4, 17.5
[RN3] Sec 17.1, 17.2, 17.4

CS 486/686

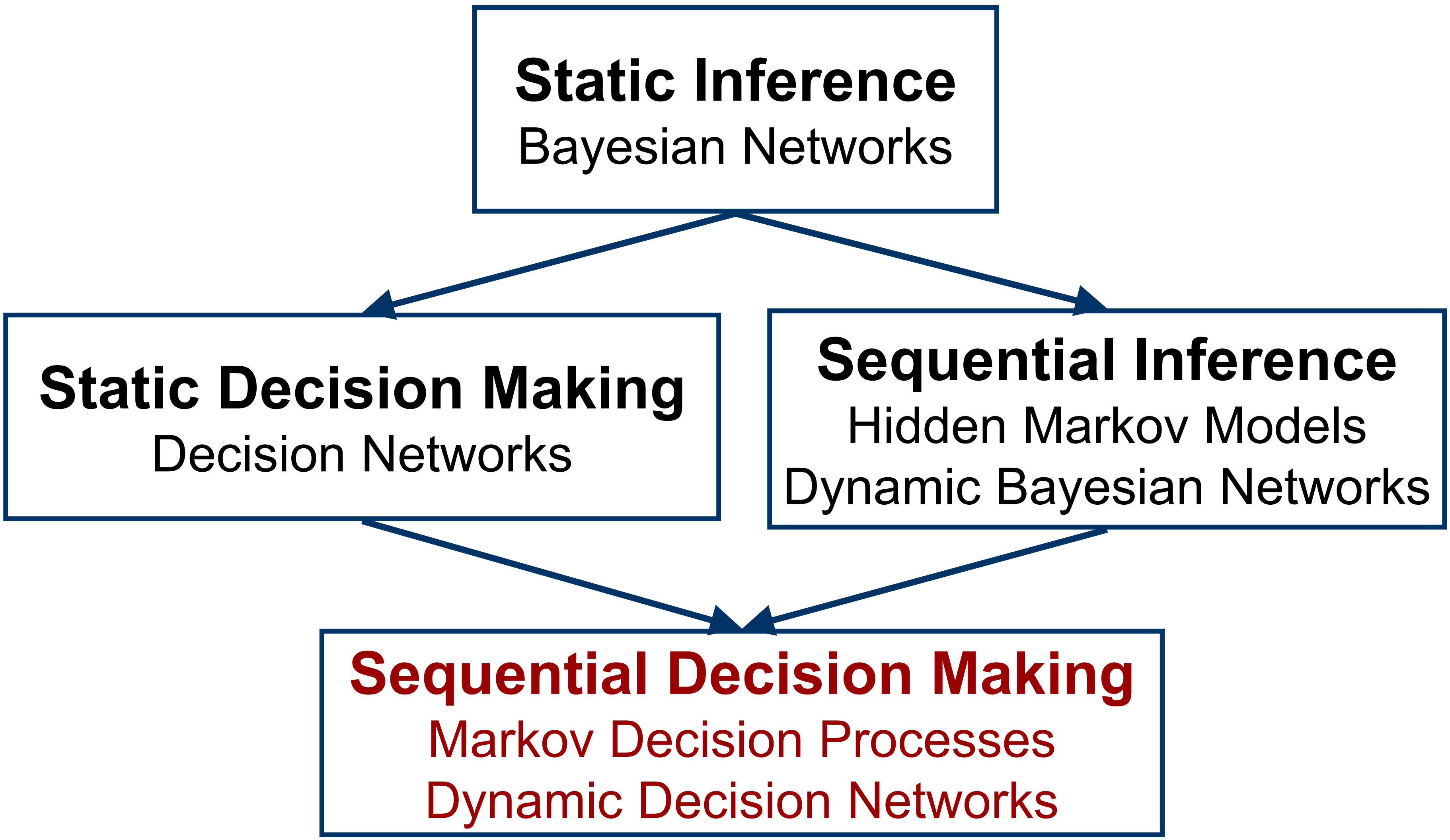
University of Waterloo

Lecture 20: March 20, 2014

Outline

- Markov Decision Processes
- Dynamic Decision Networks

Sequential Decision Making

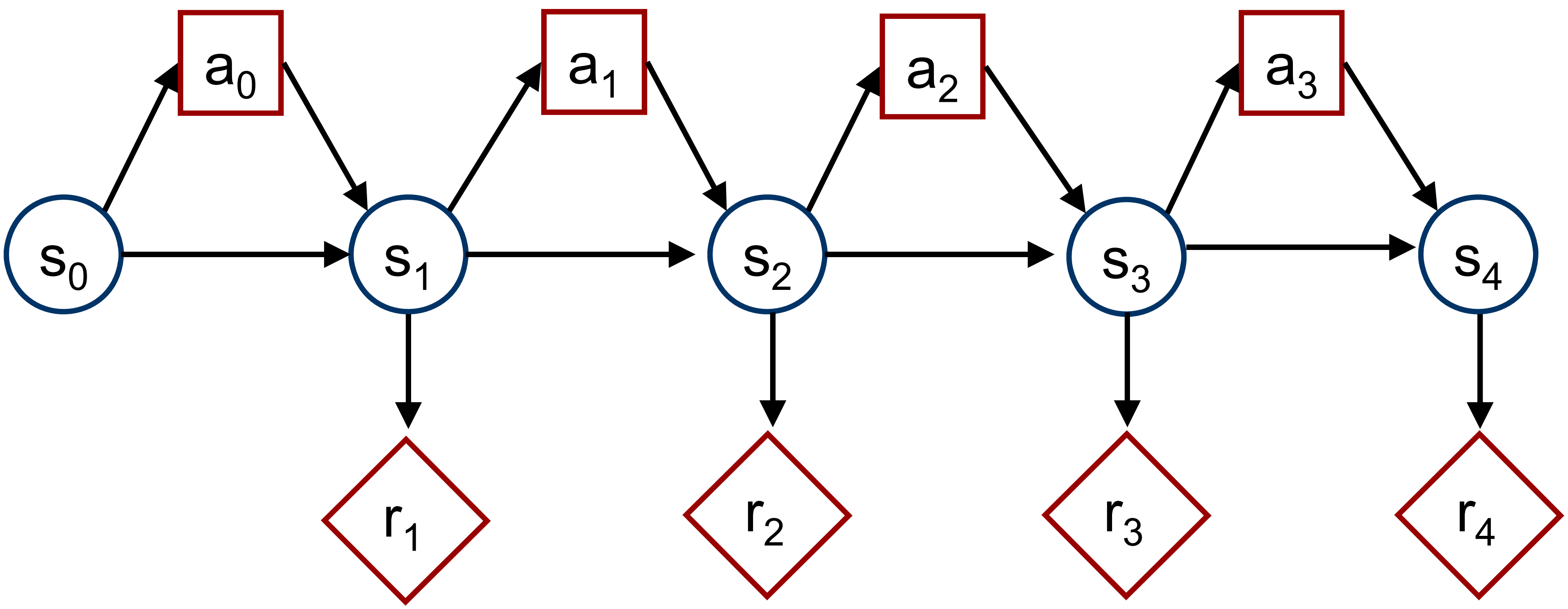


Sequential Decision Making

- Wide range of applications
  - Robotics (e.g., control)
  - Investments (e.g., portfolio management)
  - Computational linguistics (e.g., dialogue management)
  - Operations research (e.g., inventory management, resource allocation, call admission control)
  - Assistive technologies (e.g., patient monitoring and support)

Markov Decision Process

- Intuition: Markov Process with…
  - Decision nodes
  - Utility nodes



Stationary Preferences

- Hum… but why many utility nodes?

- $U(s_0,s_1,s_2,\ldots)$
  - Infinite process → infinite utility function

- Solution:
  - Assume stationary and additive preferences
  - $U(s_0,s_1,s_2,\ldots) = \Sigma_t R(s_t)$

Discounted/Average Rewards

- If process infinite, isn't $\Sigma_t R(s_t)$ infinite?

- Solution 1: discounted rewards
  - Discount factor: $0 \leq \gamma \leq 1$
  - Finite utility: $\Sigma_t \gamma^t R(s_t)$ is a geometric sum
  - γ is like an inflation rate of $1/\gamma - 1$
  - Intuition: prefer utility sooner than later

- Solution 2: average rewards
  - More complicated computationally
  - Beyond the scope of this course

Markov Decision Process

- Definition
  - Set of states: S
  - Set of actions (i.e., decisions): A
  - Transition model: $Pr(s_t|a_{t-1},s_{t-1})$
  - Reward model (i.e., utility): $R(s_t)$
  - Discount factor: $0 \leq \gamma \leq 1$
  - Horizon (i.e., # of time steps): h

- Goal: find optimal policy

Inventory Management

- Markov Decision Process
  - States: inventory levels
  - Actions: {doNothing, orderWidgets}
  - Transition model: stochastic demand
  - Reward model: Sales – Costs - Storage
  - Discount factor: 0.999
  - Horizon: ∞
- Tradeoff: increasing supplies decreases odds of missed sales but increases storage costs

Policy

- Choice of action at each time step
- Formally:
  - Mapping from states to actions
  - i.e., $\delta(s_t) = a_t$
  - Assumption: fully observable states
    - Allows a_t to be chosen only based on current state s_t. Why?

Policy Optimization

- Policy evaluation:
  - Compute expected utility
  - $EU(\delta) = \Sigma_{t=0}^{h} \gamma^t \Pr(s_t|\delta) R(s_t)$
- Optimal policy:
  - Policy with highest expected utility
  - $EU(\delta) \leq EU(\delta^*)$ for all δ

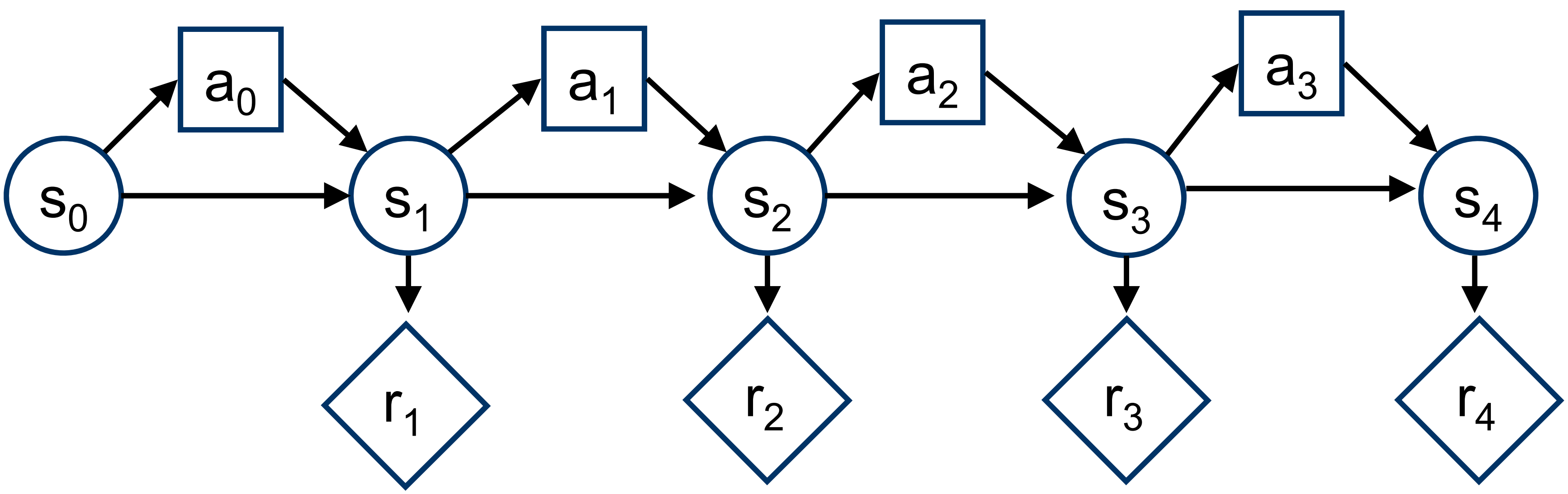
Policy Optimization

- Three algorithms to optimize policy:
  - Value iteration
  - Policy iteration
  - Linear Programming

- Value iteration:
  - Equivalent to variable elimination

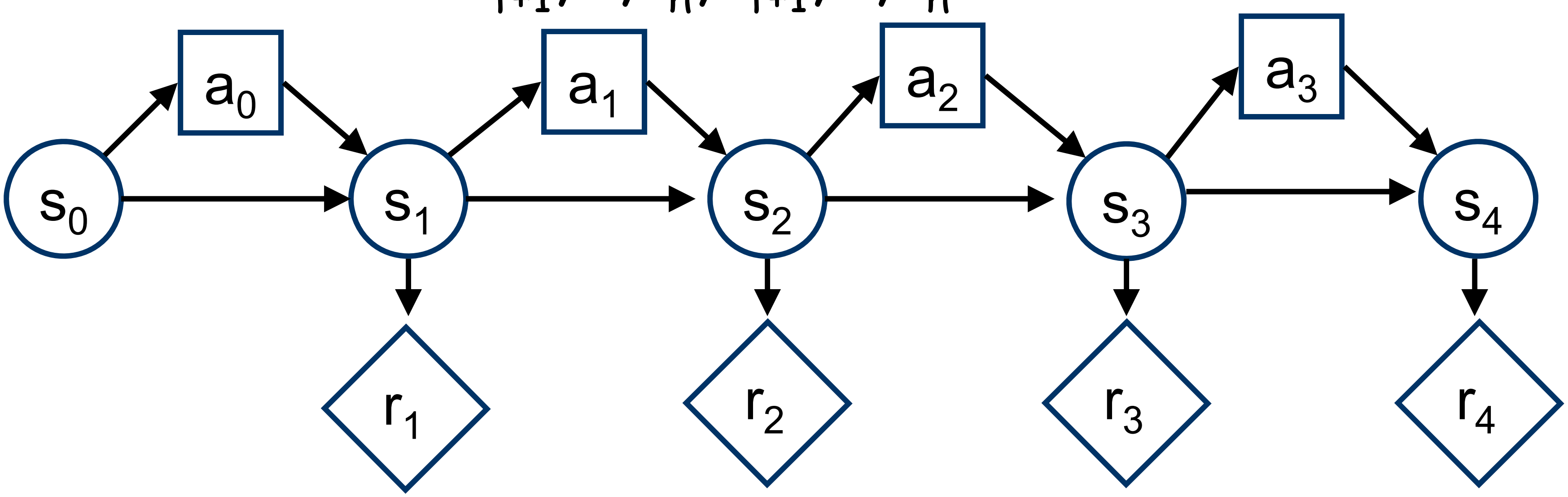
Value Iteration

- Nothing more than variable elimination
- Performs dynamic programming
- Optimize decisions in reverse order



Value Iteration

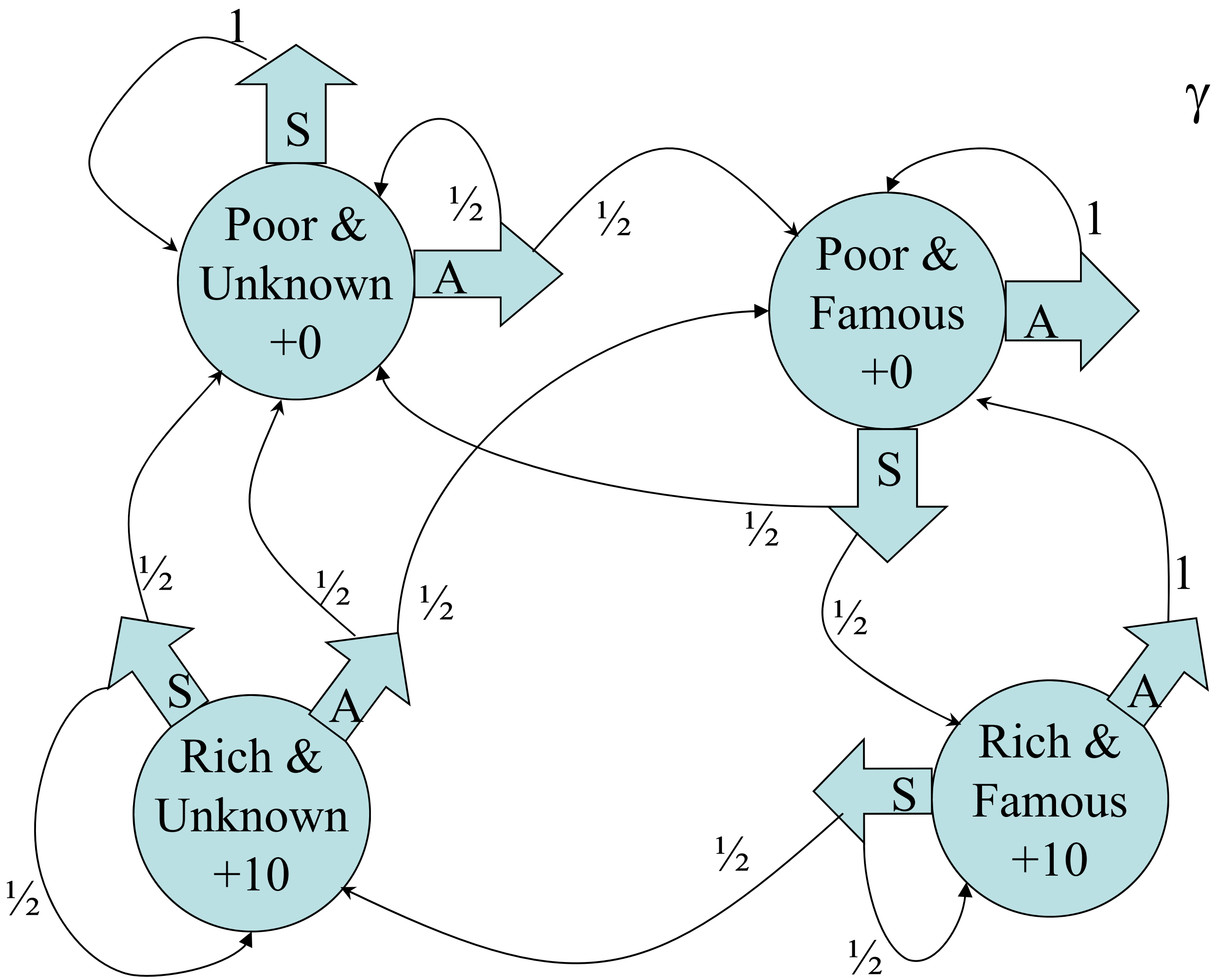
- At each t, starting from t=h down to 0:
  - Optimize a_t: $EU(a_t|s_t)$?
  - Factors: $Pr(s_{i+1}|a_i,s_i)$, $R(s_i)$, for $0 \leq i \leq h$
  - Restrict s_t
  - Eliminate $s_{t+1},\ldots,s_h,a_{t+1},\ldots,a_h$



Value Iteration

- Value when no time left:
  - $V(s_h) = R(s_h)$
- Value with one time step left:
  - $V(s_{h-1}) = \max_{a_{h-1}} R(s_{h-1}) + \gamma \sum_{s_h} \Pr(s_h|s_{h-1},a_{h-1}) V(s_h)$
- Value with two time steps left:
  - $V(s_{h-2}) = \max_{a_{h-2}} R(s_{h-2}) + \gamma \sum_{s_{h-1}} \Pr(s_{h-1}|s_{h-2},a_{h-2}) V(s_{h-1})$
- ...
- Bellman's equation:
  - $V(s_t) = \max_{a_t} R(s_t) + \gamma \sum_{s_{t+1}} \Pr(s_{t+1}|s_t,a_t) V(s_{t+1})$
  - $a_t^* = \mathrm{argmax}_{a_t} R(s_t) + \gamma \sum_{s_{t+1}} \Pr(s_{t+1}|s_t,a_t) V(s_{t+1})$

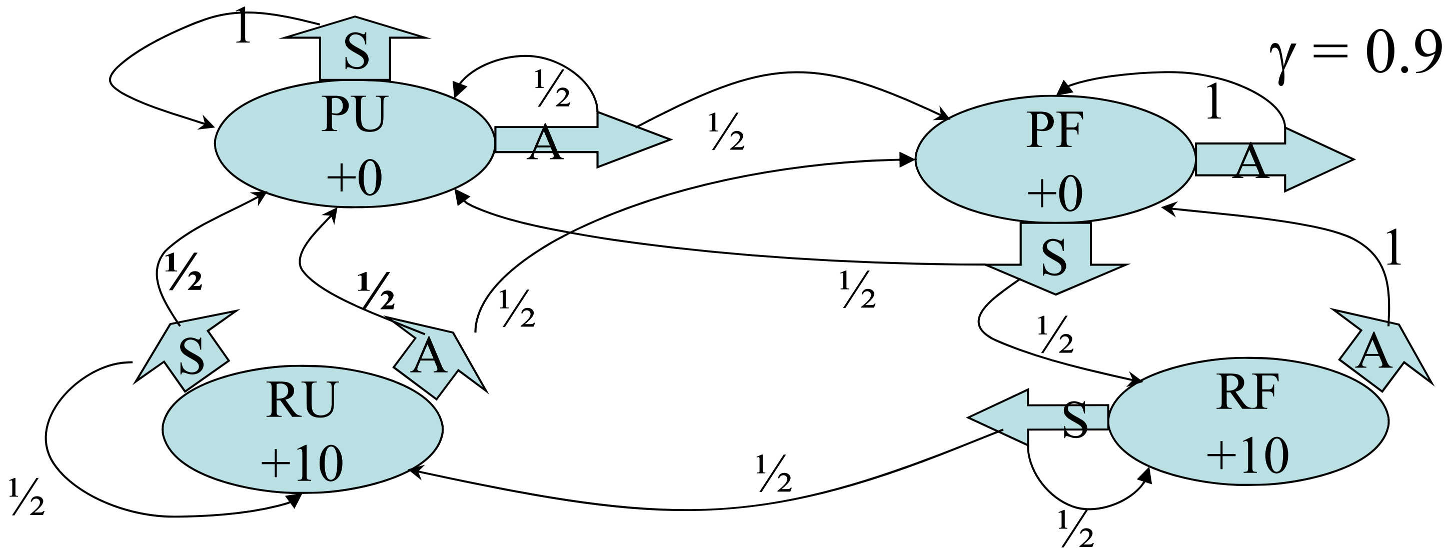
A Markov Decision Process



$\gamma = 0.9$

You own a company

In every state you must choose between Saving money or Advertising



t	V(PU)	V(PF)	V(RU)	V(RF)
h	0	0	10	10
h-1	0	4.5	14.5	19
h-2	2.03	8.55	16.53	25.08
h-3	4.76	12.20	18.35	28.72
h-4	7.63	15.07	20.40	31.18
h-5	10.21	17.46	22.61	33.21

Finite Horizon

- When h is finite,
- Non-stationary optimal policy
- Best action different at each time step
- Intuition: best action varies with the amount of time left

Infinite Horizon

- When h is infinite,
- Stationary optimal policy
- Same best action at each time step
- Intuition: same (infinite) amount of time left at each time step, hence same best action

- Problem: value iteration does an infinite number of iterations…

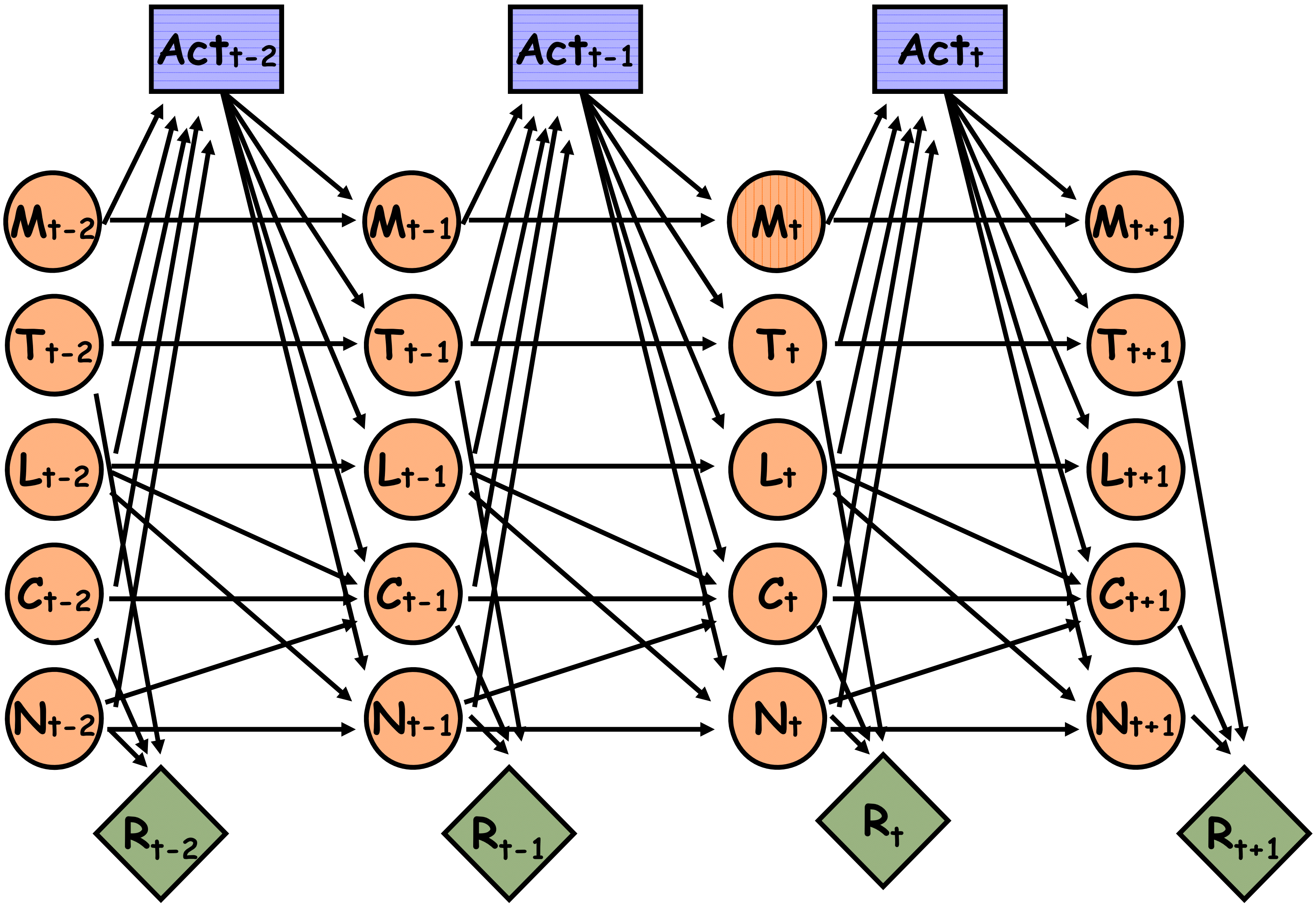
Infinite Horizon

- Assuming a discount factor γ, after k time steps, rewards are scaled down by γ^k
- For large enough k, rewards become insignificant since $\gamma^k \rightarrow 0$
- Solution:
  - pick large enough k
  - run value iteration for k steps
  - Execute policy found at the k^{th} iteration

Computational Complexity

- Space and time: $O(k|A||S|^2)$ ☺
  - Here k is the number of iterations
- But what if |A| and |S| are defined by several random variables and consequently exponential?
- Solution: exploit conditional independence
  - Dynamic decision network

Dynamic Decision Network

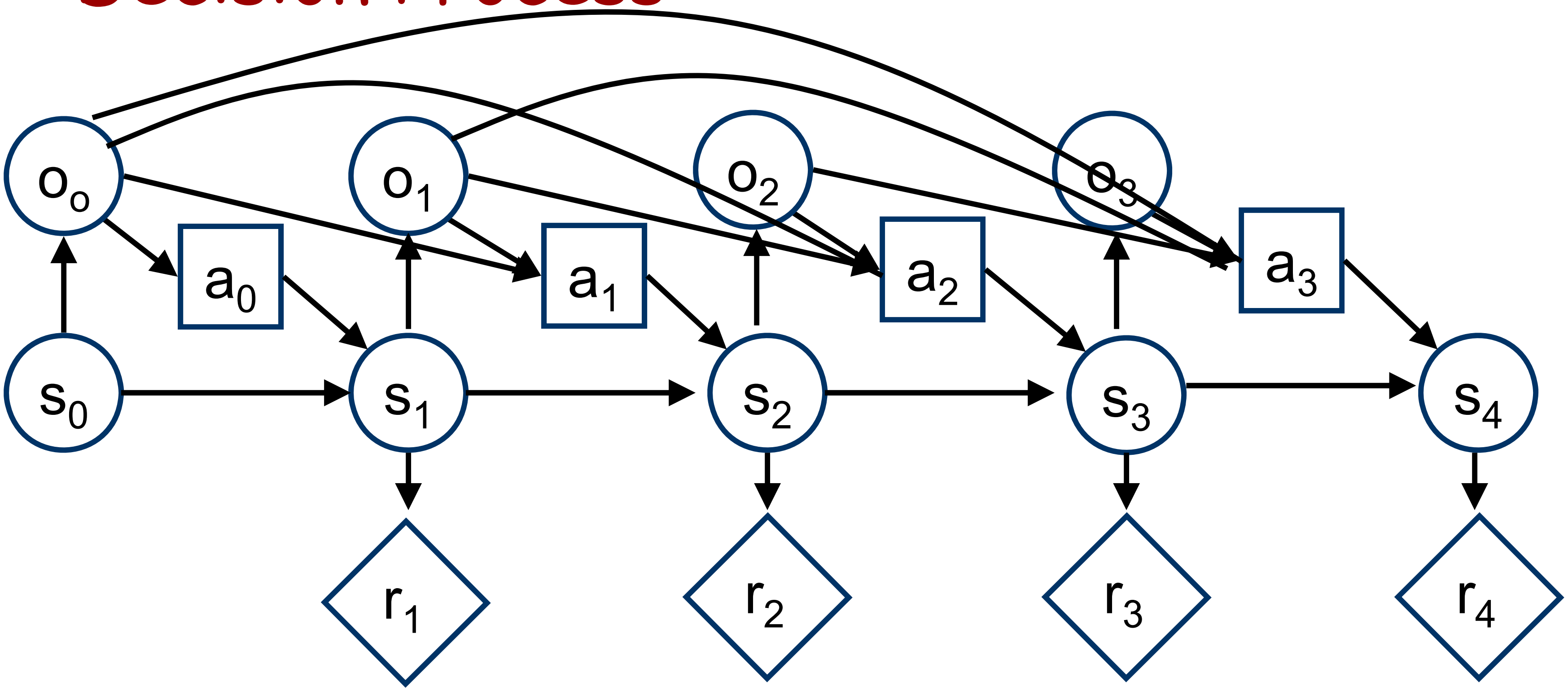


Dynamic Decision Network

- Similarly to dynamic Bayes nets:
  - Compact representation ☺
  - Exponential time for decision making ☹

Partial Observability

- What if states are not fully observable?
- Solution: Partially Observable Markov Decision Process



Partially Observable Markov Decision Process (POMDP)

- Definition
  - Set of states: S
  - Set of actions (i.e., decisions): A
  - Set of observations: O
  - Transition model: $Pr(s_t|a_{t-1},s_{t-1})$
  - Observation model: $Pr(o_t|s_t)$
  - Reward model (i.e., utility): $R(s_t)$
  - Discount factor: $0 \leq \gamma \leq 1$
  - Horizon (i.e., # of time steps): h

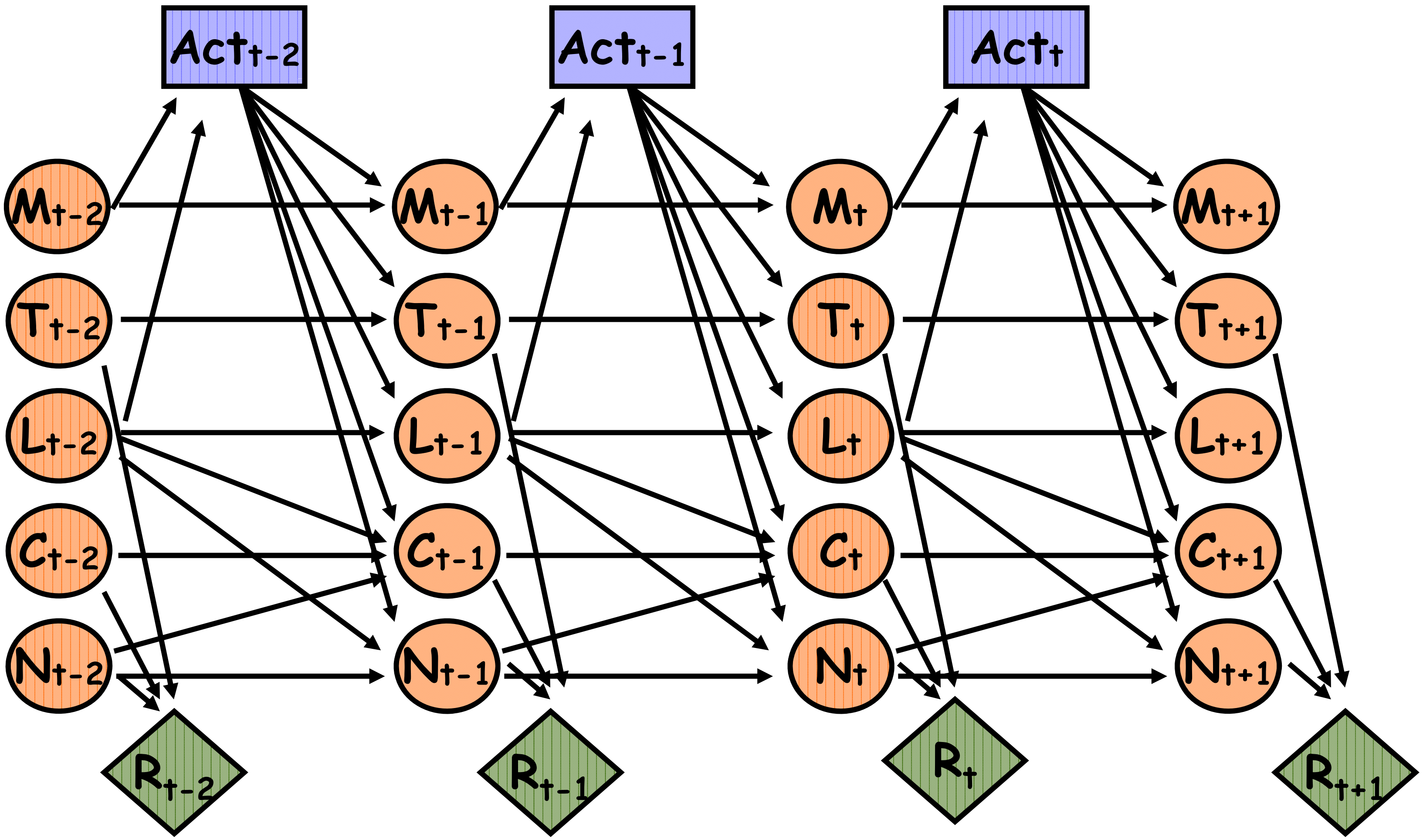
- Policy: mapping from past obs. to actions

POMDP

- Problem: action choice generally depends on all previous observations…
- Two solutions:
  - Consider only policies that depend on a finite history of observations
  - Find stationary sufficient statistics encoding relevant past observations

Partially Observable DDN

- Actions do not depend on all state variables



Policy Optimization

- Policy optimization:
  - Value iteration (variable elimination)
  - Policy iteration

- POMDP and PODDN complexity:
  - Exponential in $|O|$ and k when action choice depends on all previous observations ☹
  - In practice, good policies based on subset of past observations can still be found

COACH project

- Automated prompting system to help elderly persons wash their hands
- IATSL: Alex Mihailidis, Pascal Poupart, Jennifer Boger, Jesse Hoey, Geoff Fernie and Craig Boutilier

Aging Population

- Dementia
  - Deterioration of intellectual faculties
  - Confusion
  - Memory losses (e.g., Alzheimer’s disease)

- Consequences:
  - Loss of autonomy
  - Continual and expensive care required

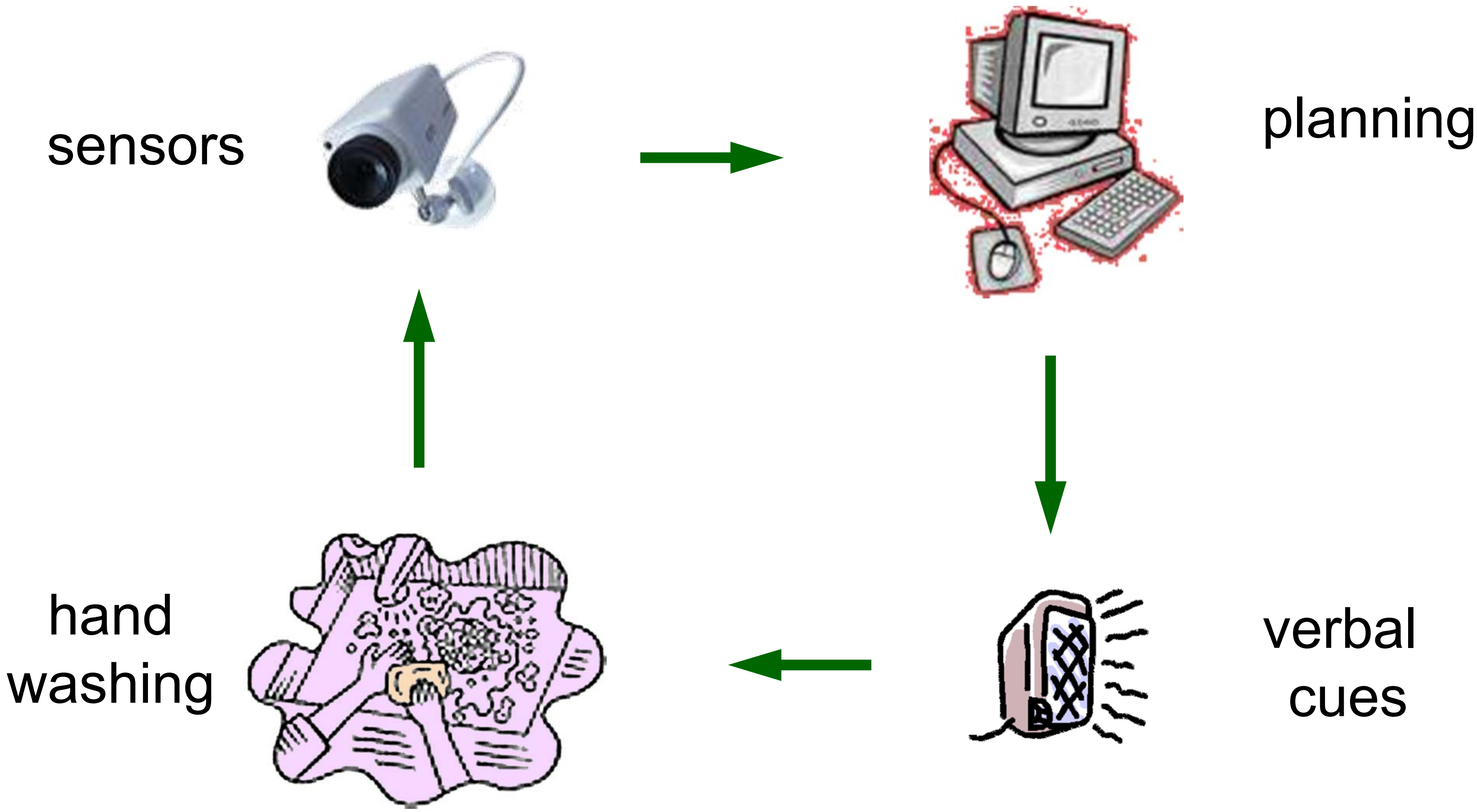
Intelligent Assistive Technology

- Let's facilitate aging in place

- Intelligent assistive technology
  - Non-obtrusive, yet pervasive
  - Adaptable

- Benefits:
  - Greater autonomy
  - Feeling of independence

System Overview



Prompting Strategy

- Sequential decision problem
  - Sequence of prompts
- Noisy sensors & imprecise actuators
  - Noisy image processing, uncertain prompt effects
- Partially unknown environment
  - Unknown user habits, preferences and abilities
- Tradeoff between complex concurrent goals
  - Rapid task completion vs greater autonomy
- Approach: Partially Observable Markov Decision Processes (POMDPs)

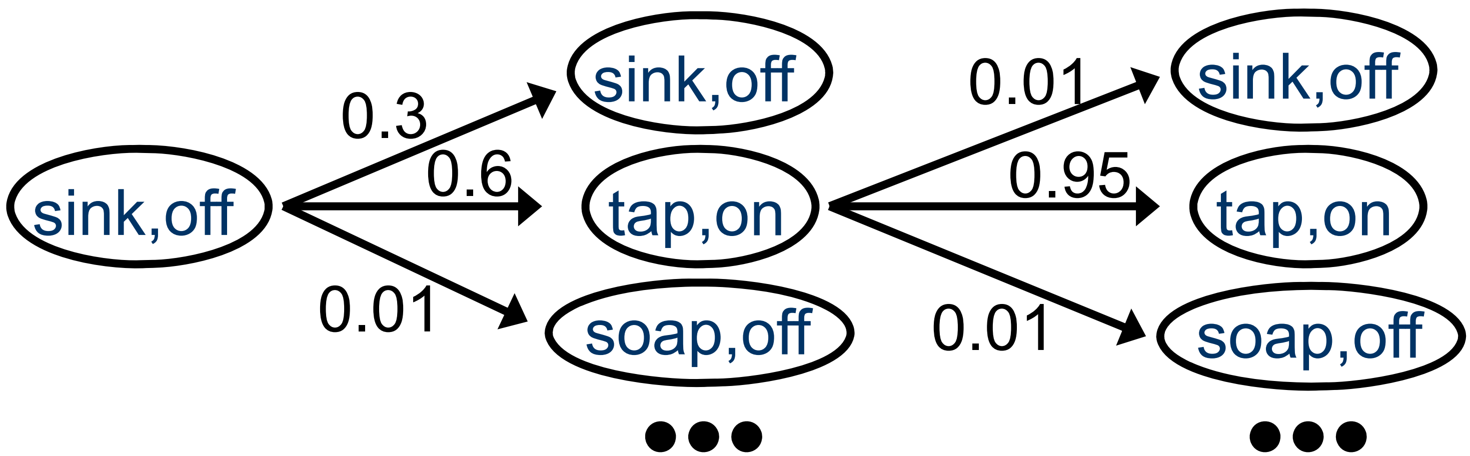
POMDP components

- State set **S** = dom(HL) x dom(WF) x dom(D) x …
  - Hand Location $\in$ {tap,water,soap,towel,sink,away,…}
  - Water Flow $\in$ {on, off},
  - Dementia $\in$ {high, low}, etc.
- Observation set **O** = dom(C) x dom(FS)
  - Camera $\in$ {handsAtTap, handsAtTowel, …}
  - Faucet sensor $\in$ {waterOn, waterOff}
- Action set **A**
  - DoNothing, CallCaregiver, Prompt $\in$ {turnOnWater, rinseHands, useSoap, …}

POMDP components

- Transition function Pr(s'|s,a)

Observation function Pr(o|s)



- Reward function **R(s,a)**
  – Task completed → +100
  – Call caregiver → -30
  – Each prompt → -1, -2 or -3