Machine Learning
[RN2] Sec 18.1-18.4
[RN3] Sec 18.1-18.4

CS 486/686

University of Waterloo

Lecture 15: June 23, 2015

Outline

- Inductive learning
- Decision trees

What is Machine Learning?

- Definition:
  - A computer program is said to **learn** from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.

[T Mitchell, 1997]

Examples

- Backgammon (reinforcement learning):
  - T: playing backgammon
  - P: percent of games won against an opponent
  - E: playing practice games against itself
- Handwriting recognition (supervised learning):
  - T: recognize handwritten words within images
  - P: percent of words correctly recognized
  - E: database of handwritten words with given classifications
- Customer profiling (unsupervised learning):
  - T: cluster customers based on transaction patterns
  - P: homogeneity of clusters
  - E: database of customer transactions

Representation

- Representation of the learned information is important
  - Determines how the learning algorithm will work
- Common representations:
  - Linear weighted polynomials
  - Propositional logic
  - First order logic
  - Bayes nets
  - …

Inductive learning (aka concept learning)

- Induction:
  - Given a training set of examples of the form (x,f(x))
    - x is the input, f(x) is the output
  - Return a function h that approximates f
    - h is called the hypothesis

Classification

- Training set:

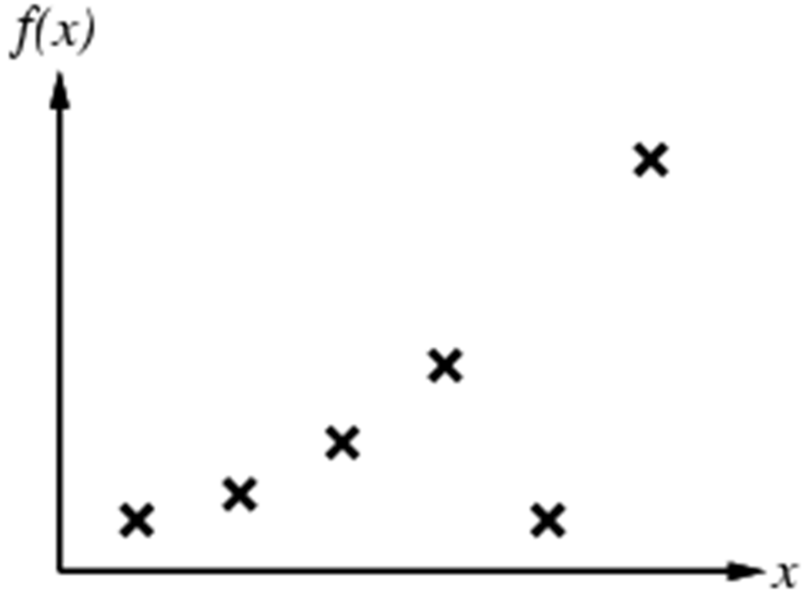
Sky	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Normal	Strong	Warm	Same	Yes
Sunny	High	Strong	Warm	Same	Yes
Sunny	High	Strong	Warm	Change	No
Sunny	High	Strong	Cool	Change	Yes

x $f(x)$

- Possible hypotheses:
  - h_1: S=sunny → ES=yes
  - h_2: Wa=cool or F=same → enjoySport

Regression

- Find function **h** that fits f at instances x



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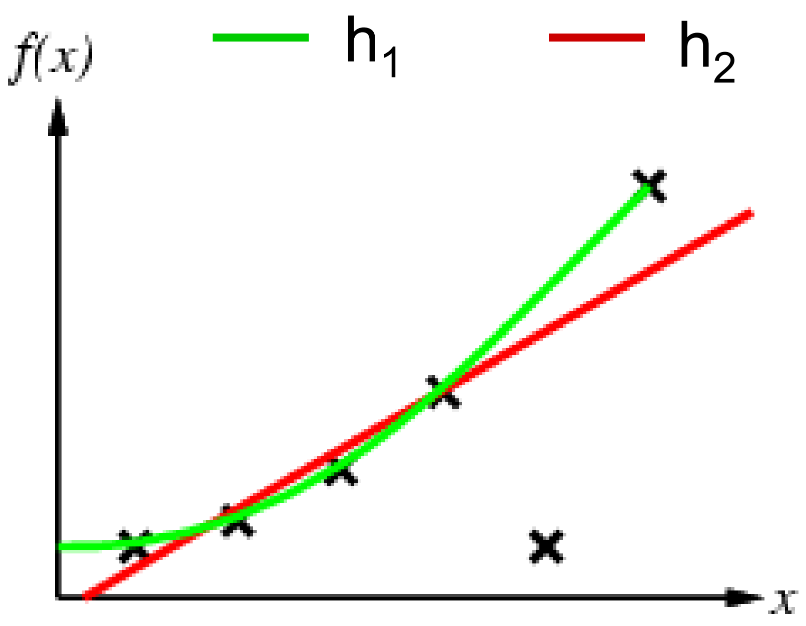


Hypothesis Space

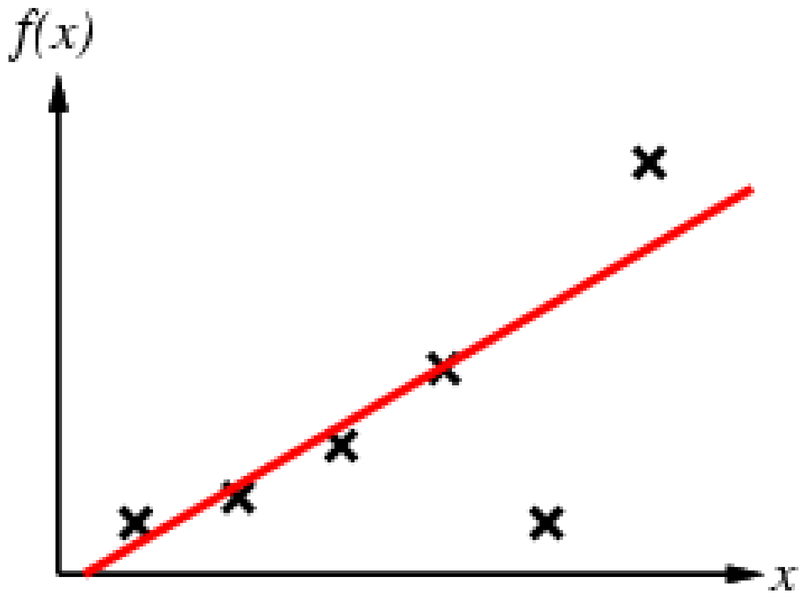
- Hypothesis space H
  - Set of all hypotheses h that the learner may consider
  - Learning is a search through hypothesis space
- Objective:
  - Find hypothesis that agrees with training examples
  - But what about unseen examples?

Generalization

- A good hypothesis will generalize well (i.e. predict unseen examples correctly)
- Usually…
  - Any hypothesis h found to approximate the target function f well over a sufficiently large set of training examples will also approximate the target function well over any unobserved examples

Inductive learning

- Construct/adjust *h* to agree with *f* on training set
- (*h* is consistent if it agrees with *f* on all examples)
- E.g., curve fitting:



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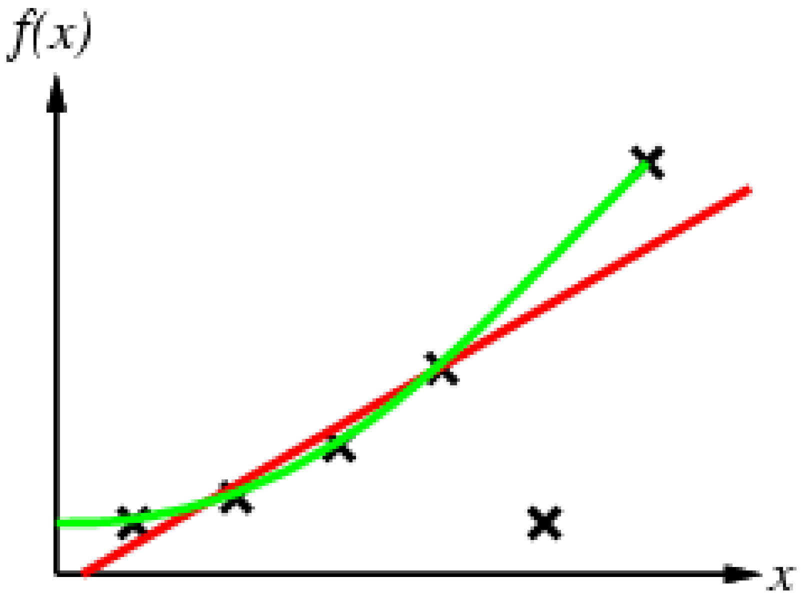

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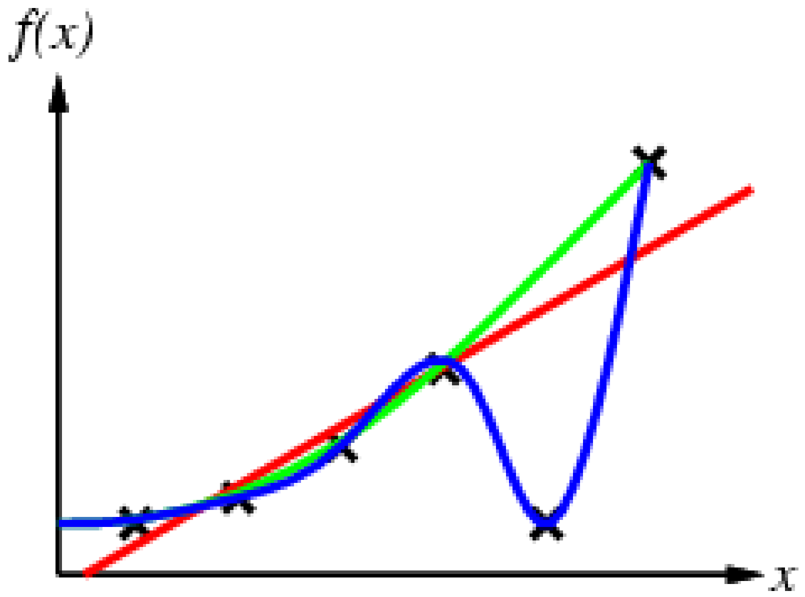

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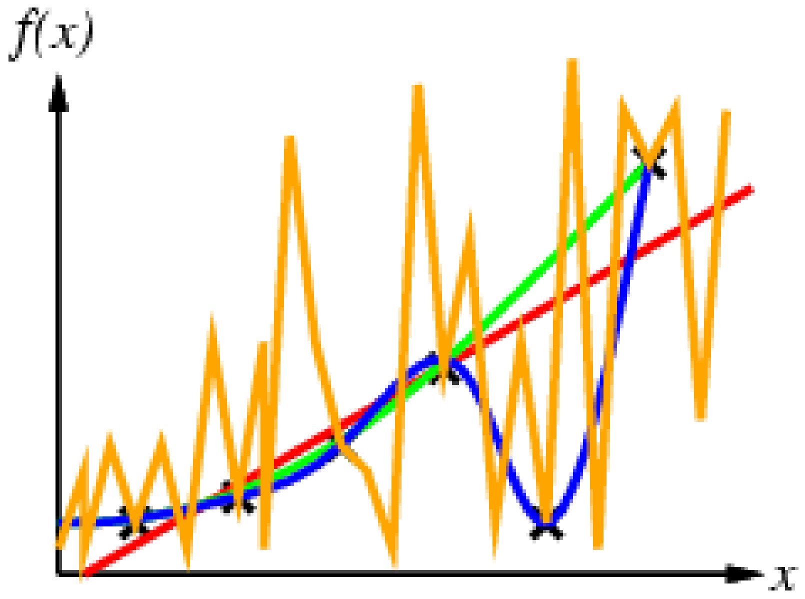

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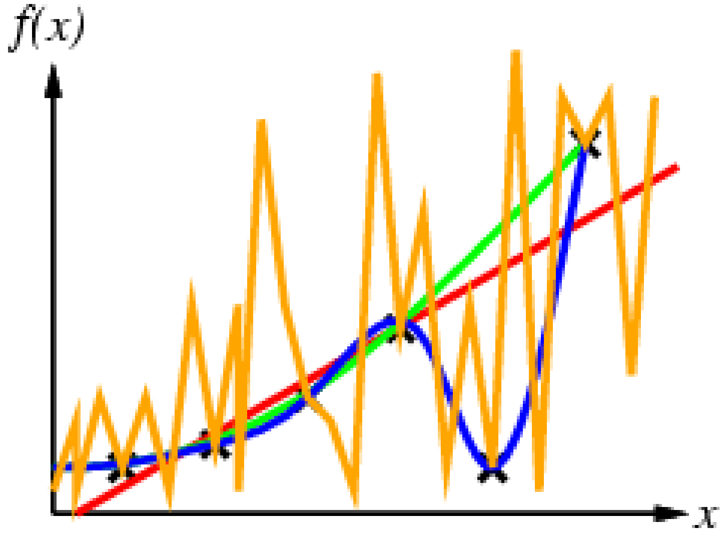


- Ockham's razor: prefer the simplest hypothesis consistent with data

Inductive learning

- Finding a consistent hypothesis depends on the hypothesis space
  - For example, it is not possible to learn exactly $f(x)=ax+b+x\sin(x)$ when H=space of polynomials of finite degree

- A learning problem is realizable if the hypothesis space contains the true function, otherwise it is unrealizable
  - Difficult to determine whether a learning problem is realizable since the true function is not known

Inductive learning

- It is possible to use a very large hypothesis space
  - For example, H=class of all Turing machines

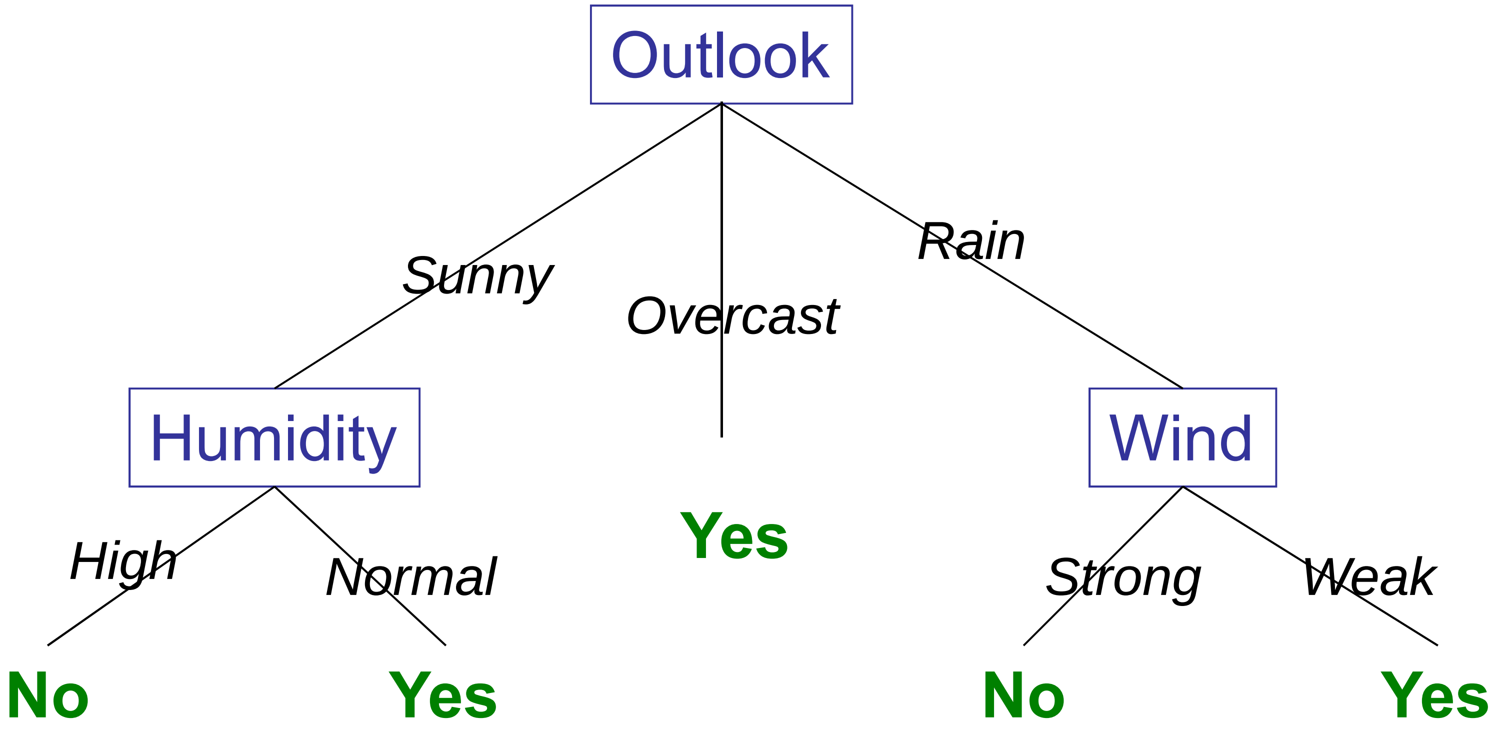
- But there is a tradeoff between expressiveness of a hypothesis class and complexity of finding simple, consistent hypothesis within the space
  - Fitting straight lines is easy, fitting high degree polynomials is hard, fitting Turing machines is very hard!

Decision trees

- Decision tree classification
  - Nodes: labeled with attributes
  - Edges: labeled with attribute values
  - Leaves: labeled with classes

- Classify an instance by starting at the root, testing the attribute specified by the root, then moving down the branch corresponding to the value of the attribute
  - Continue until you reach a leaf
  - Return the class

Decision tree (playing tennis)

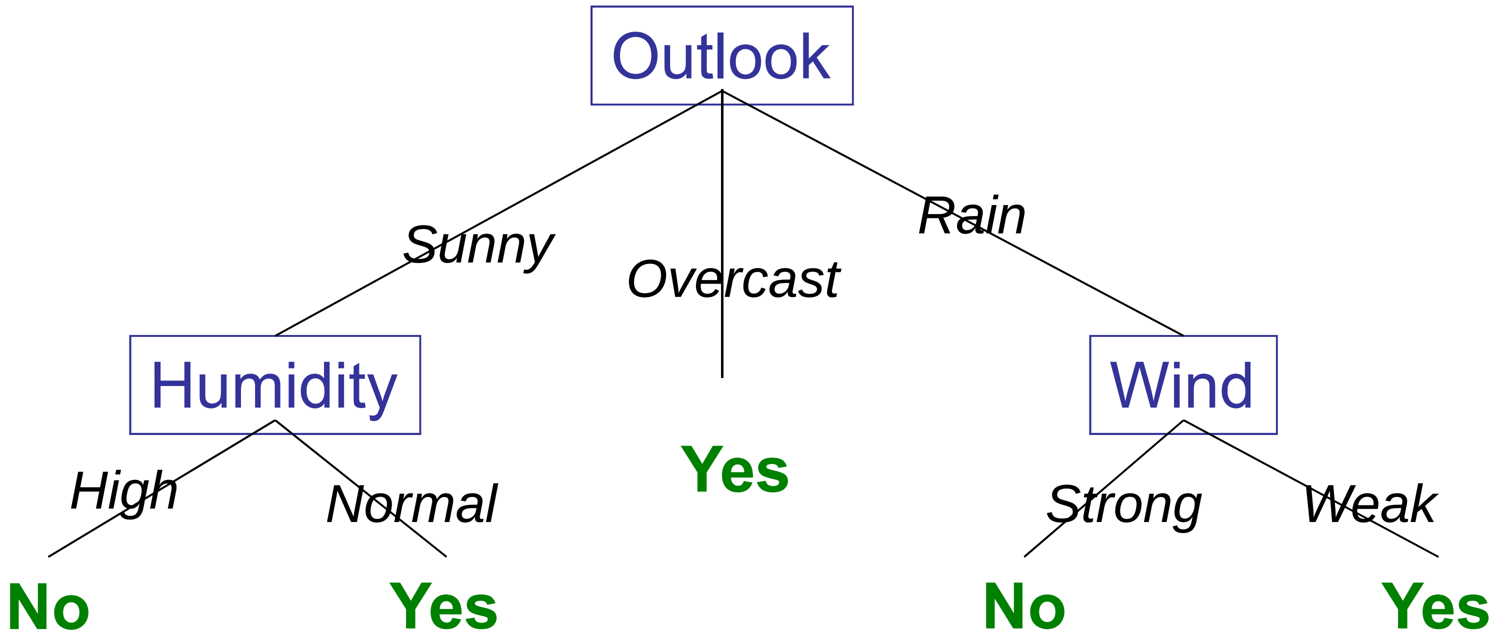


An instance
<Outlook=Sunny, Temp=Hot, Humidity=High, Wind=Strong>

Classification: No

Decision tree representation

- Decision trees can represent disjunctions of conjunctions of constraints on attribute values



(Outlook=Sunny $\wedge$ Humidity=Normal)
$\vee$ (Outlook=Overcast)
$\vee$ (Outlook=Rain $\wedge$ Wind=Weak)

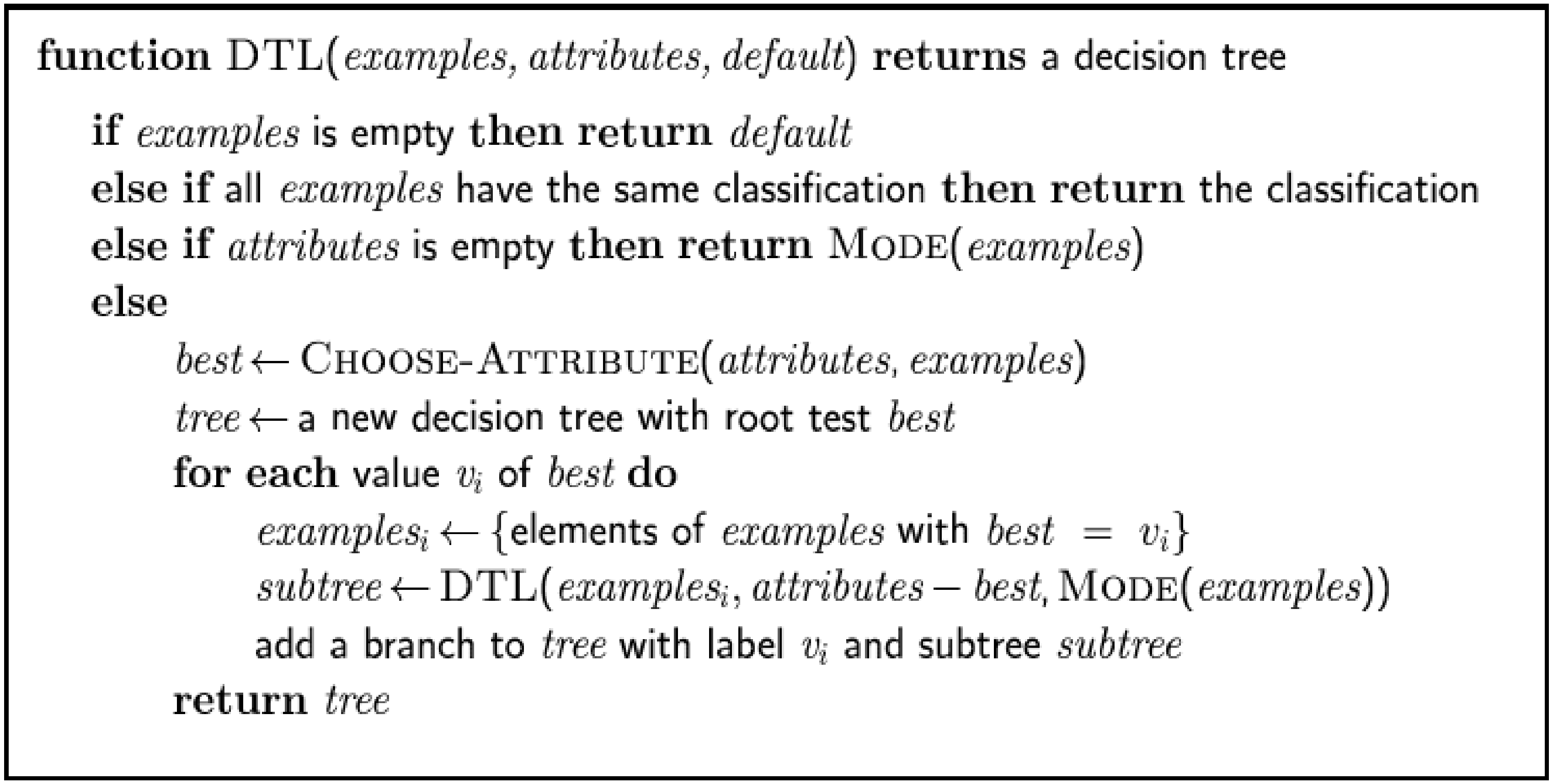
Decision tree representation

- Decision trees are fully expressive within the class of propositional languages
  - Any Boolean function can be written as a decision tree
    - Trivially by allowing each row in a truth table correspond to a path in the tree
    - Can often use small trees
    - Some functions require exponentially large trees (majority function, parity function)
  - However, there is no representation that is efficient for all functions

Inducing a decision tree

- Aim: find a small tree consistent with the training examples
- Idea: (recursively) choose "most significant" attribute as root of (sub)tree

Decision Tree Learning



```
function DTL($examples$, $attributes$, $default$) returns a decision tree
    if $examples$ is empty then return $default$
    else if all $examples$ have the same classification then return the classification
    else if $attributes$ is empty then return MODE($examples$)
    else
        $best \leftarrow$ CHOOSE-ATTRIBUTE($attributes$, $examples$)
        $tree \leftarrow$ a new decision tree with root test $best$
        for each value $v_i$ of $best$ do
            $examples_i \leftarrow$ {elements of $examples$ with $best = v_i$}
            $subtree \leftarrow$ DTL($examples_i$, $attributes - best$, MODE($examples$))
            add a branch to $tree$ with label $v_i$ and subtree $subtree$
        return $tree$
```

Choosing attribute tests

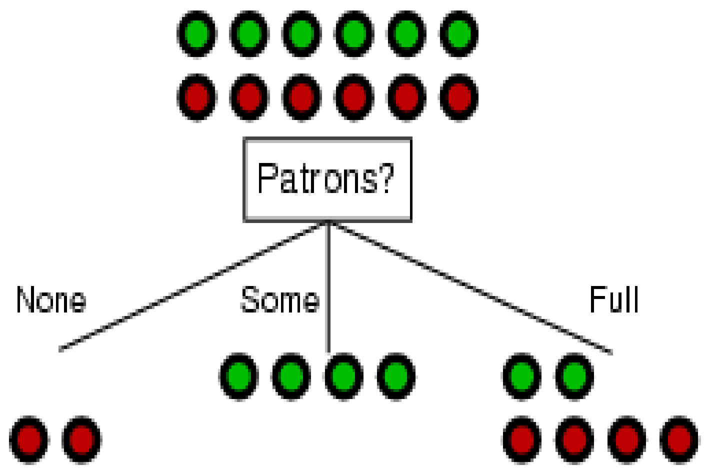
- The central choice is deciding which attribute to test at each node
- We want to choose an attribute that is most useful for classifying examples

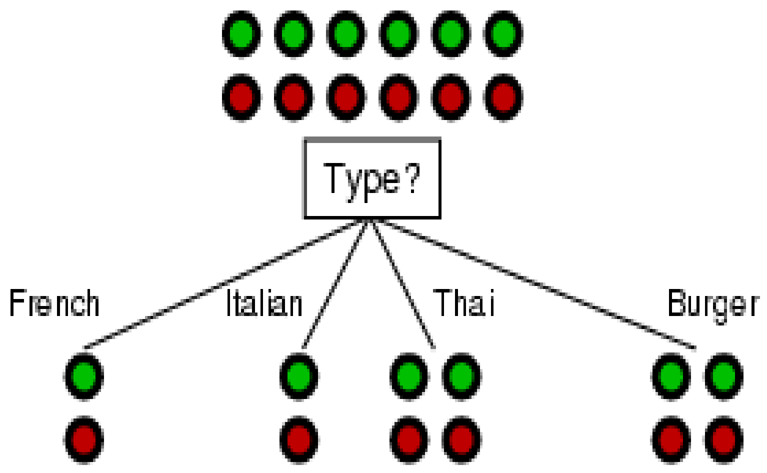
Example -- Restaurant

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	\$\$\$	F	T	French	0–10	T
X_2	T	F	F	T	Full	\$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	\$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	\$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	\$\$\$	F	T	French	>60	F
X_6	F	T	F	T	Some	\$\$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	\$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	\$\$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	\$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	\$\$\$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	\$	F	F	Burger	30–60	T

Choosing an attribute

- Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"





- *Patrons?* is a better choice

Using information theory

- To implement **Choose-Attribute** in the DTL algorithm
- Measure uncertainty (Entropy):

  $I(P(v_1), \dots , P(v_n)) = \Sigma_{i=1} -P(v_i) \log_2 P(v_i)$

- For a training set containing *p* positive examples and *n* negative examples:

$$I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) = -\frac{p}{p+n}\log_2 \frac{p}{p+n} - \frac{n}{p+n}\log_2 \frac{n}{p+n}$$

Information gain

- *A* chosen attribute *A* divides the training set *E* into subsets $E_1, \dots, E_v$ according to their values for *A*, where *A* has v distinct values.

$$remainder(A) = \sum_{i=1}^{v} \frac{p_i + n_i}{p+n} I\left(\frac{p_i}{p_i+n_i}, \frac{n_i}{p_i+n_i}\right)$$

- Information Gain (IG) or reduction in uncertainty from the attribute test:

$$IG(A) = I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - remainder(A)$$

- Choose the attribute with the largest IG

Information gain

For the training set, $p = n = 6$, *I(6/12, 6/12) = 1* bit

Consider the attributes *Patrons* and *Type* (and others too):

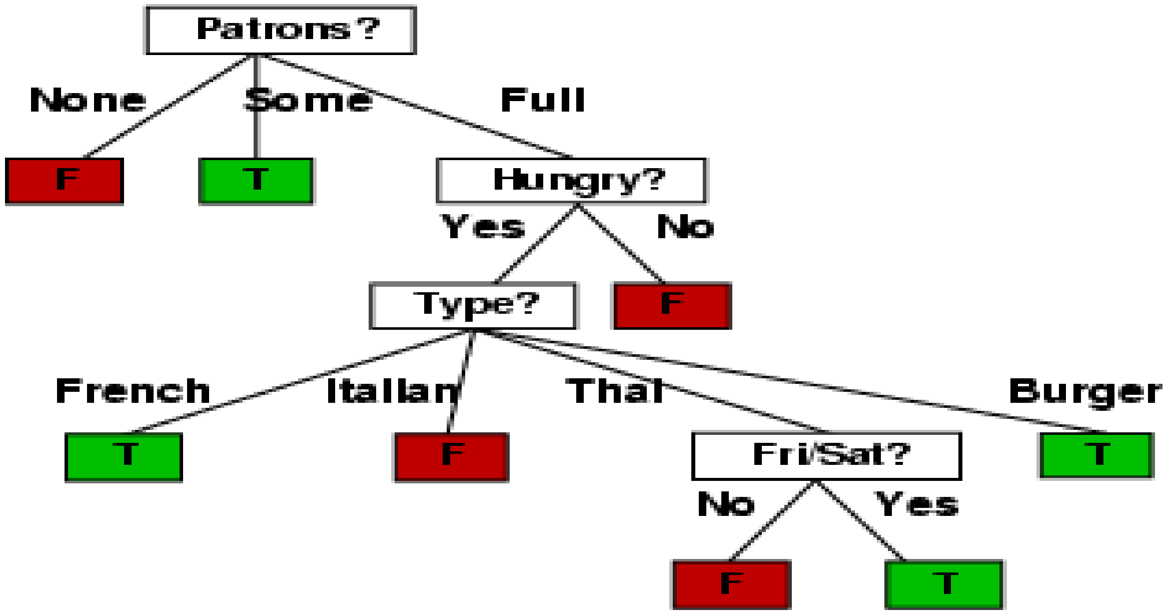
$$IG(Patrons) = 1 - [\frac{2}{12} I(0,1) + \frac{4}{12} I(1,0) + \frac{6}{12} I(\frac{2}{6}, \frac{4}{6})] = \ .541 \ \ \text{bits}$$

$$IG(Type) = 1 - [\frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4})] = 0 \text{ bits}$$

Patrons has the highest IG of all attributes and so is chosen by the DTL algorithm as the root

Example

- Decision tree learned from the 12 examples:



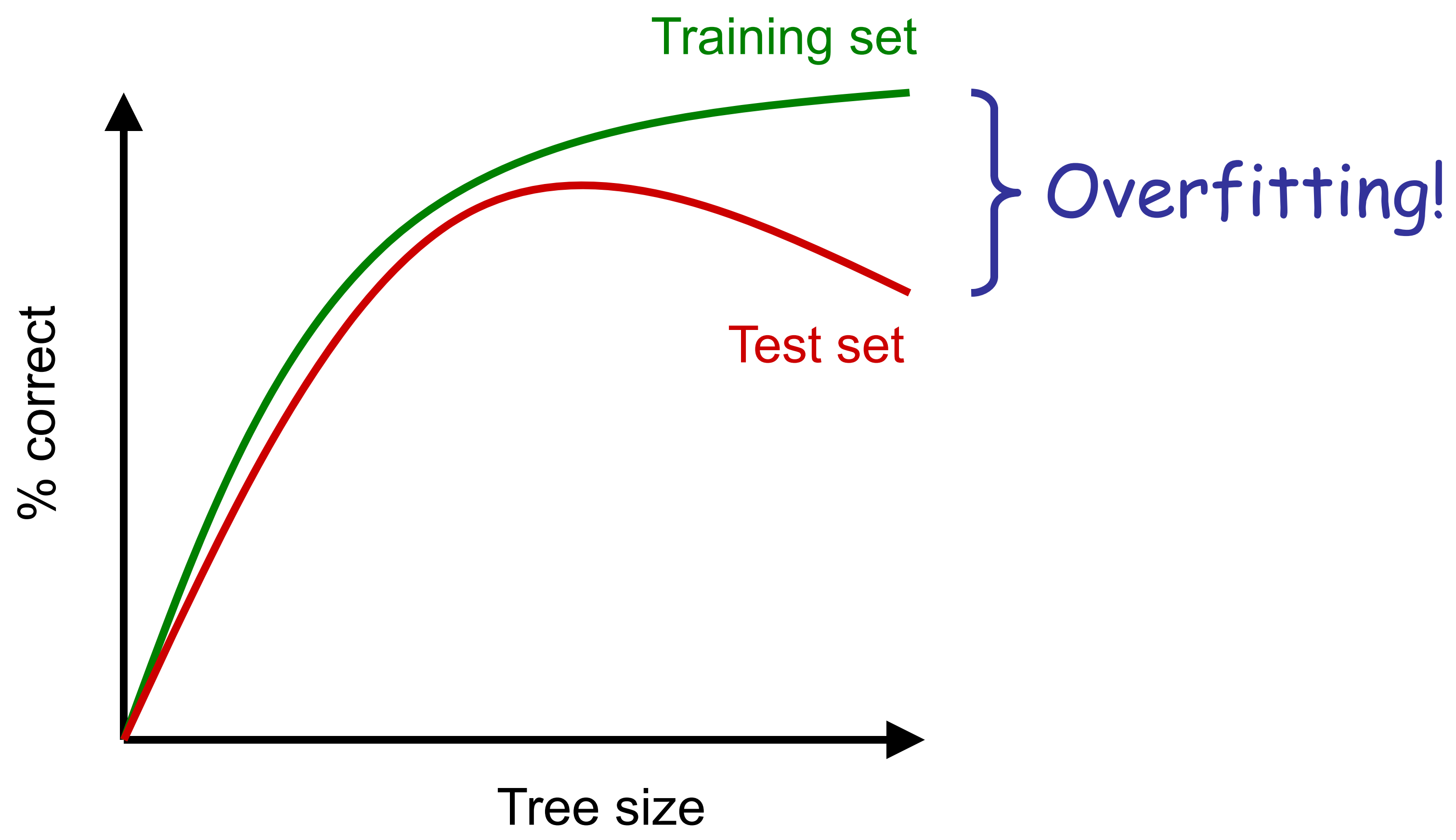
- Substantially simpler than "true" tree---a more complex hypothesis isn't justified by small amount of data

Performance of a learning algorithm

- A learning algorithm is good if it produces a hypothesis that does a good job of predicting classifications of unseen examples

- Verify performance with a **test set**
  1. Collect a large set of examples
  2. Divide into 2 disjoint sets: training set and test set
  3. Learn hypothesis h with training set
  4. Measure percentage of correctly classified examples by h in the test set
  5. Repeat 2-4 for different randomly selected training sets of varying sizes

Learning curves



Overfitting

- Decision-tree grows until all training examples are perfectly classified
- But what if…
  - Data is noisy
  - Training set is too small to give a representative sample of the target function
- May lead to **Overfitting!**
  - Common problem with most learning algo

Overfitting

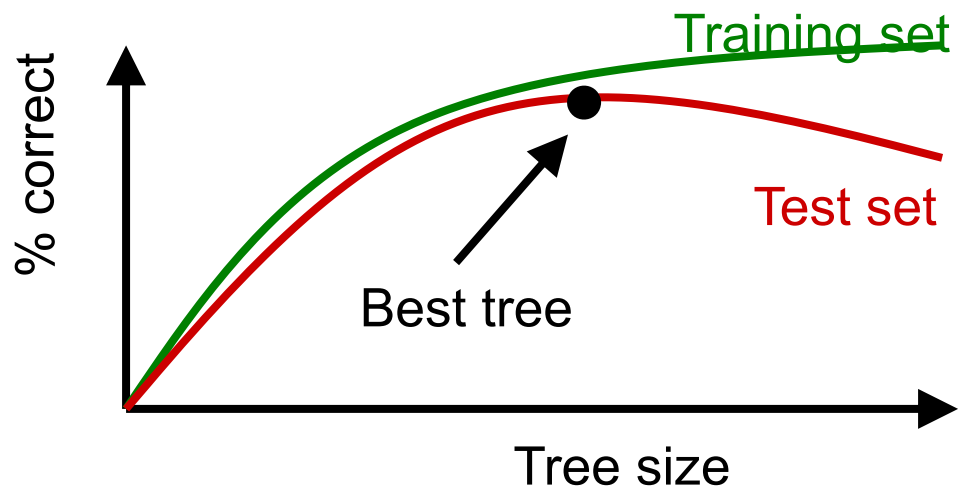
- **Definition**: Given a hypothesis space H, a hypothesis h $\in$ H is said to overfit the training data if there exists some alternative hypothesis h' $\in$ H such that h has smaller error than h' over the training examples but h' has smaller error than h over the entire distribution of instances

- Overfitting has been found to decrease accuracy of decision trees by 10-25%

Avoiding overfitting

Two popular techniques:

1. Prune statistically irrelevant nodes
   - Measure irrelevance with χ^2 test
2. Stop growing tree when test set performance starts decreasing
   - Use cross-validation



Cross-validation

- Split data in two parts, one for training, one for testing the accuracy of a hypothesis

- K-fold cross validation means you run k experiments, each time putting aside 1/k of the data to test on