Local Search

CS 486/686
University of Waterloo
May 11, 2006

Outline

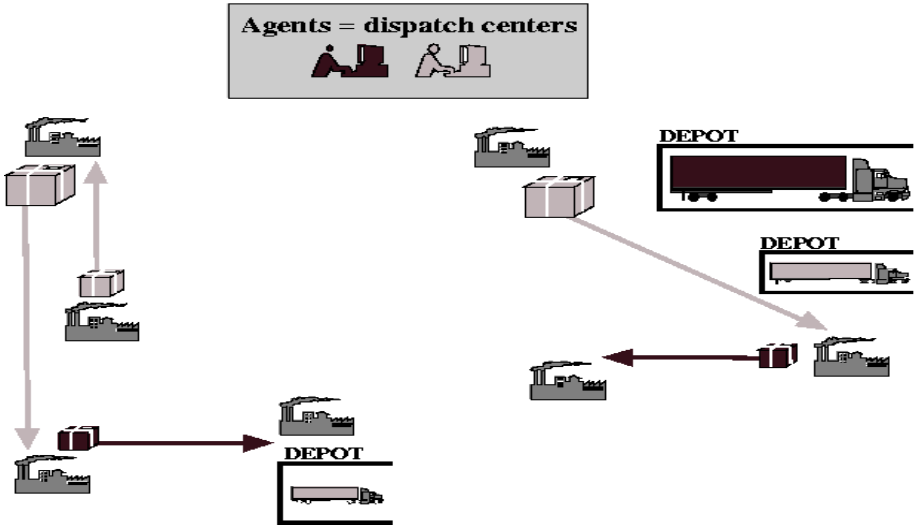
- Iterative improvement algorithms
- Hill climbing search
- Simulated annealing
- Genetic algorithms

Introduction

- So far we have studied algorithms which systematically explore search spaces
  - Keep one or more paths in memory
  - When the goal is found, the solution consists of a path to the goal

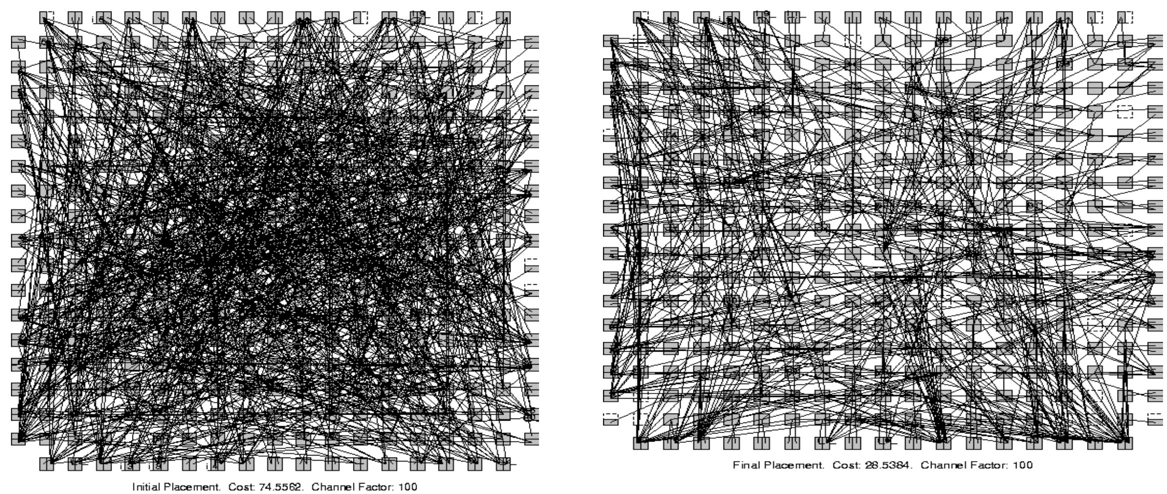
- For many problems the path is unimportant

Examples

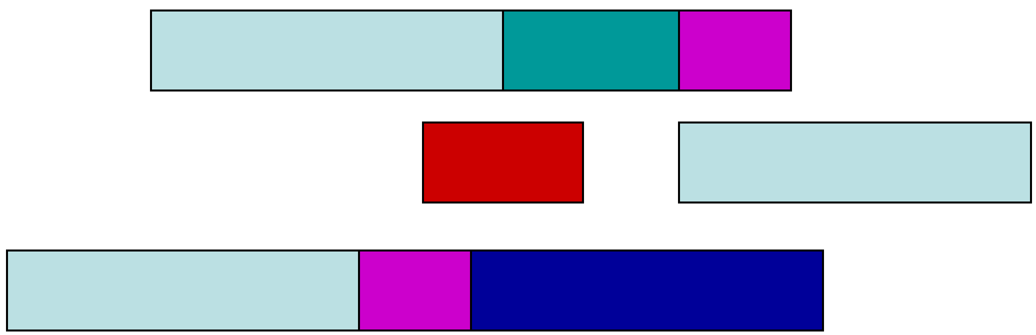


Vehicle routing

Channel Routing



Examples



Job shop scheduling

A v ~B v C

~A v C v D

B v D v ~E

~C v ~D v ~E

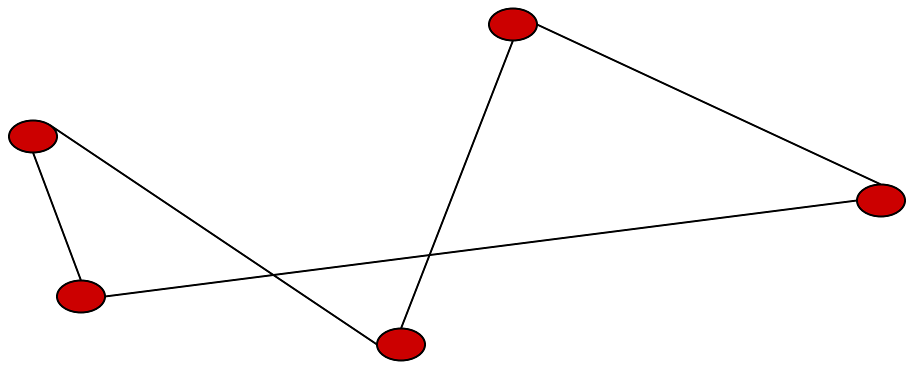
…

Boolean Satisfiability

Introduction

- Informal characterization
  - Combinatorial structure being optimized
  - There is a cost function to be optimized
    - At least we want to find a **good** solution
  - Searching all possible states is infeasible
  - No known algorithm for finding the solution efficiently
  - Some notion of similar states having similar costs

Example - TSP



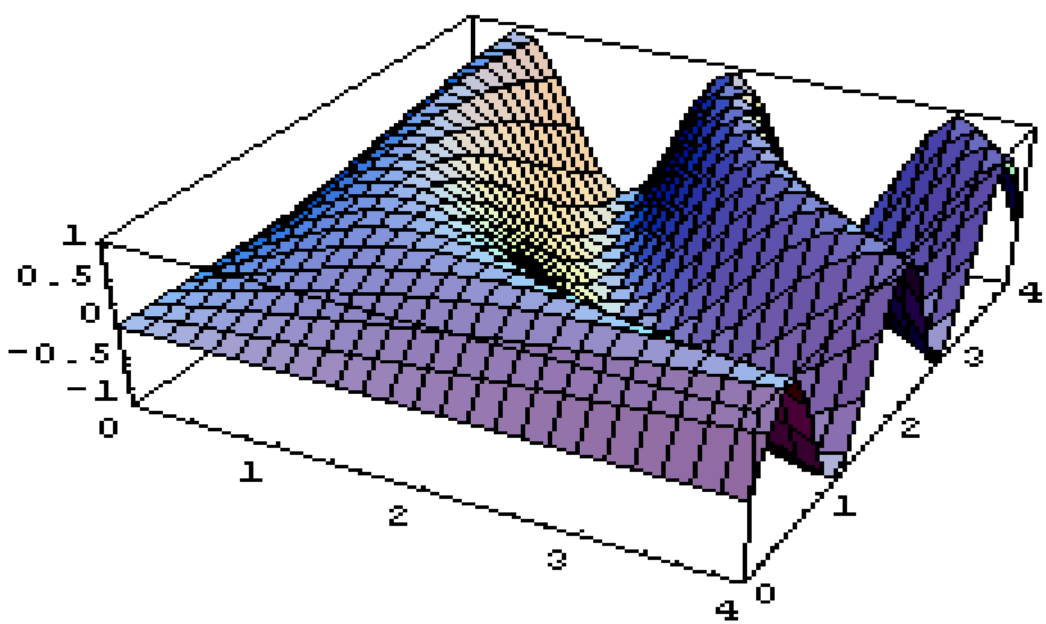
- Goal is to minimize the length of the route
- **Constructive method:**
  - Start from scratch and build up a solution
- **Iterative improvement method:**
  - Start with a solution and try to improve it

Constructive method

- For the optimal solution we could use A*!
  - But we do not really need to know how we got to the solution – we just want the solution
  - Can be very expensive to run

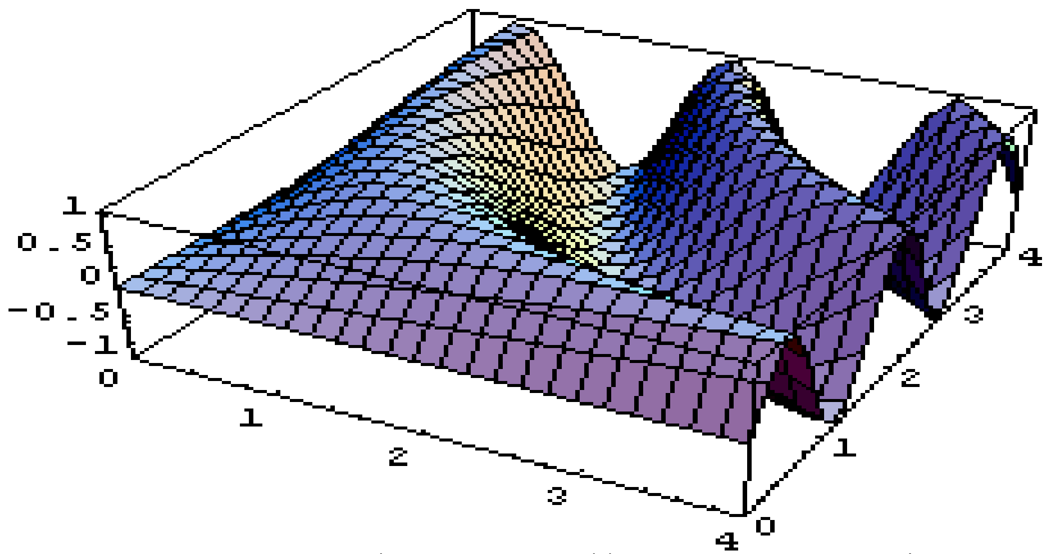
Iterative improvement methods

- Idea: Imagine all possible solutions laid out on a landscape
  - We want to find the highest (or lowest) point



Iterative improvement methods

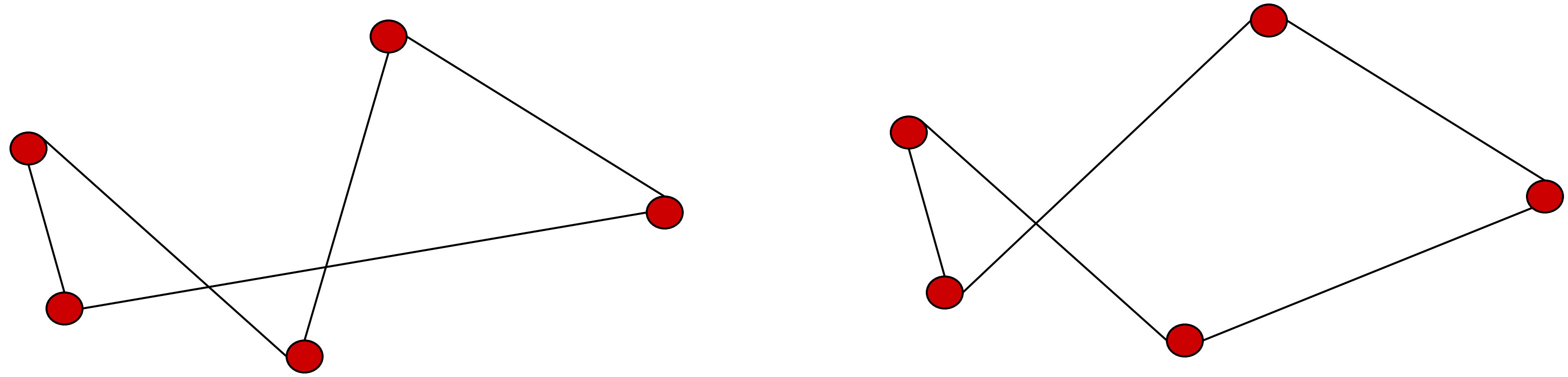
1. Start at some random point on the landscape
2. Generate all possible points to move to
3. Choose a point of improvement and move to it
4. If you are stuck then restart



Iterative improvement methods

- What does it mean to "generate points to move to"
  - Sometimes called generating the **moveset**
- Depends on the application

TSP



2-swap

Hill-climbing

1. Start at some initial configuration S
2. Let V=Eval(S)
3. Let N=Move_Set(S)
4. For each $X_i \in N$
   - Let V_{max}=max_i Eval(X_i) and X_{max}=$argmax_i$ Eval(X_i)
5. If $V_{max} \leq V$, return S
6. Let S=X_{max} and V=V_{max}. Go to 3

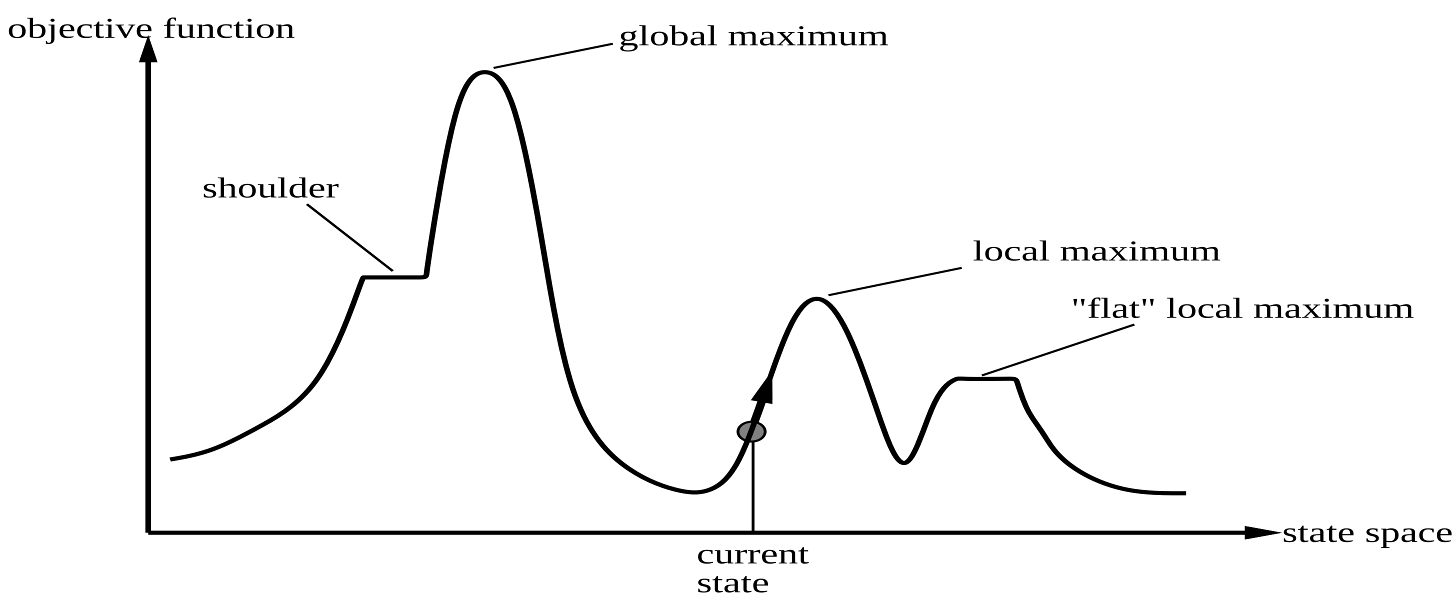
"Like trying to find the peak of Mt Everest in the fog", Russell and Norvig

Hill Climbing

- Always take a step in the direction that improves the current solution value the most
  - Greedy
- Good things about hill climbing
  - Easy to program!
  - Requires no memory of where we have been!
  - It is important to have a "good" set of moves
    - Not too many, not too few

Hill Climbing

- Issues with hill climbing
  - It can get stuck!
  - Local maximum (local minimum)
  - Plateaus



Improving on hill climbing

- Plateaus
  - Allow for sideways moves, but be careful since may move sideways forever!
- Local Maximum
  - Random restarts: "If at first you do not succeed, try, try again"
  - Random restarts works well in practice
- Randomized hill climbing
  - Like hill climbing except you choose a **random state from the move set**, and then move to it if it is better than current state. Continue until you are bored

Hill climbing example: GSAT

Av~BvC	1
~AvCvD	1
BvDv~E	0
~Cv~Dv~E	1
~Av~CvE	1

Configuration A=1, B=0, C=1, D=0, E=1

Goal is to maximize the number of satisfied clauses: Eval(config)=# satisfied clauses

GSAT Move_Set: Flip any 1 variable

WALKSAT (Randomized GSAT)

Pick a random unsatisfied clause;

Consider flipping each variable in the clause

- If any improve Eval, then accept the best
- If none improve Eval, then with prob p pick the move that is least bad; prob (1-p) pick a random one

Simulated Annealing

- Hill climbing algorithms which never make downhill moves are incomplete
  - Can get stuck at local maxima (minima)
- A random walk is complete but very inefficient

New Idea:

Allow the algorithm to make some "bad" moves in order to escape local maxima.

Simulated annealing

1. Let S be the initial configuration and V=Eval(S)
2. Let i be a random move from the moveset and let S_i be the next configuration, V_i=Eval(S_i)
3. If V<V_i then S=S_i and V=V_i
4. Else with probability p, S=S_i and V=V_i
5. Goto 2 until you are bored

Simulated annealing

- How should we choose the probability of accepting a "bad" move?
  - Idea 1: p=0.1 (or some other fixed value)?
  - Idea 2: Probability that decreases with time?
  - Idea 3: Probability that decreases with time and as $V-V_i$ increases?

Selecting moves in simulated annealing

- If new value V_i is better than old value V then definitely move to new solution
- If new value V_i is worse than old value V then move to new solution with probability

$$\boxed{\mathrm{Exp}(-(V-V_i)/T)}$$

Boltzmann distribution: T>0 is a parameter called temperature. It starts high and decreases over time towards 0

If T is close to 0 then the probability of making a bad move is almost 0

Properties of simulated annealing

- If T is decreased slowly enough then simulated annealing is guaranteed (in theory) to reach best solution
  - Annealing schedule is critical
- When T is high: Exploratory phase (random walk)
- When T is low: Exploitation phase (randomized hill climbing)

Genetic Algorithms

- Problems are encoded into a representation which allows certain operations to occur
  - Usually use a bit string
  - The representation is key – needs to be thought out carefully
- An encoded candidate solution is an individual
- Each individual has a fitness which is a numerical value associated with its quality of solution
- A population is a set of individuals
- Populations change over generations by applying strategies to them

Typical genetic algorithm

- Initialize: Population P consists of N random individuals (bit strings)
- Evaluate: for each $x \in P$, compute fitness(x)
- Loop
  - For i=1 to N do
    - Choose 2 parents each with probability proportional to fitness scores
    - **Crossover** the 2 parents to produce a new bit string (child)
    - With some small probability **mutate** child
    - Add child to the population
- Until some child is fit enough or you get bored
- Return the best child in the population according to fitness function

Crossover

- Consists of combining parts of individuals to create new individuals
- Choose a random crossover point
  - Cut the individuals there and swap the pieces

101|0101 011|1110

Cross over

011|0101 101|1110

Implementation: use a crossover mask m

Given two parents a and b the offspring are

$(a \wedge m) \vee (b \wedge \sim m)$ and $(a \wedge \sim m) \vee (b \wedge m)$

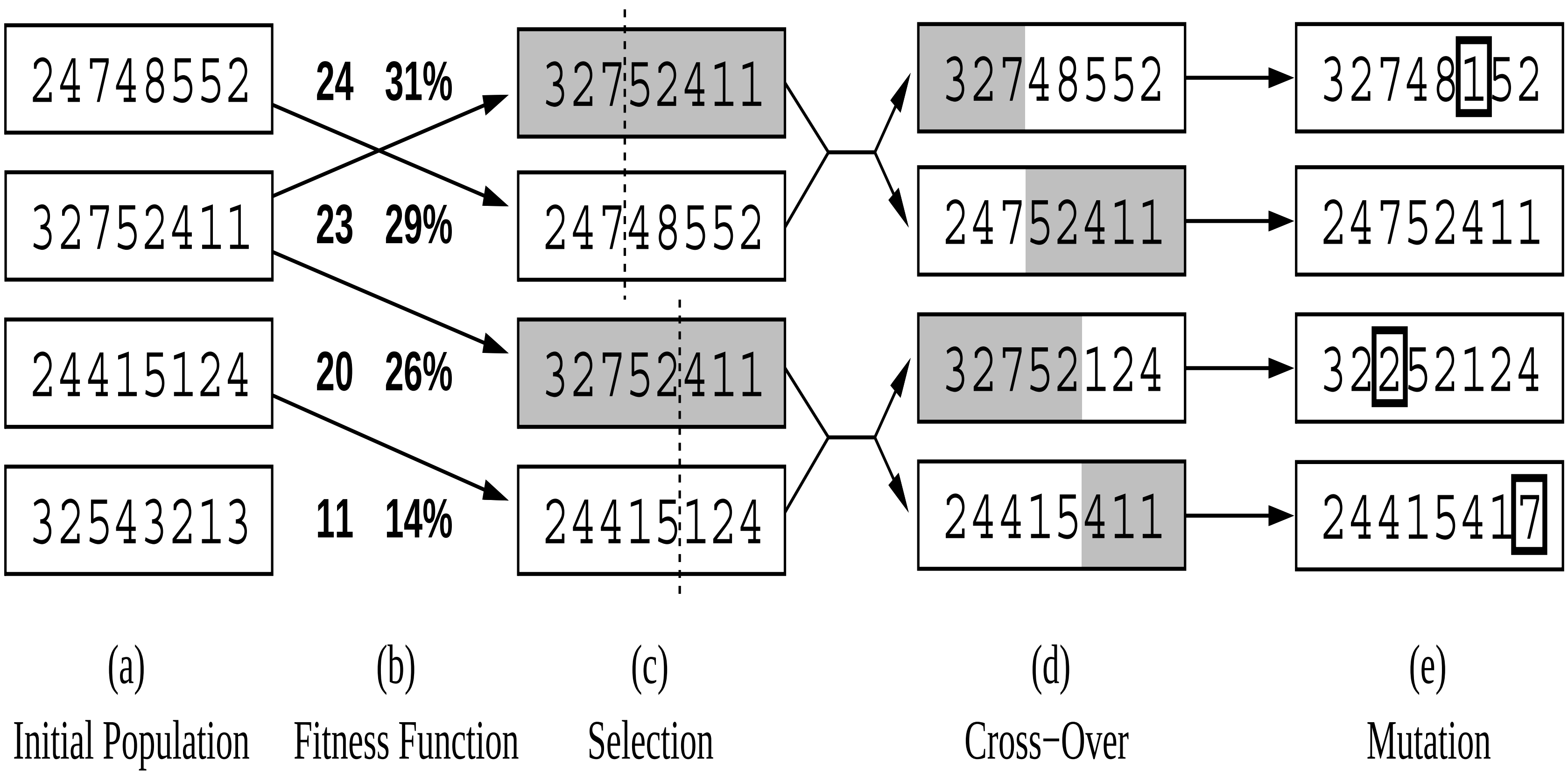
Mutation

- Mutation allows us to generate desirable features that are not present in the original population

- Typically mutation just means flipping a bit in the string

100111 mutates to 100101

Genetic Algorithms



Genetic algorithms and search

- Why are genetic algorithms a type of search?

Genetic algorithms and search

- Why are genetic algorithms a type of search?
  - States: possible solutions
  - Operators: mutation, crossover, selection
  - Parallel search: since several solutions are maintained in parallel
  - Hill-climbing on the fitness function
  - Mutation and crossover allow us to get out of local optima

Discussion of local search

- Useful for optimization problems!
- Often the second best way to solve a problem
  - If you can, use A* or linear programming or…
  - But local search is easy to program ☺
- Hill climbing always moves in the (locally) best direction
  - Can get stuck, but random restarts can be really effective
- Simulated annealing allows moves downhill

Next class

- Constraint satisfaction (CSPs)
  - Russell and Norving, Chapter 5 (mainly sections 5.1-5.3)