Reinforcement Learning

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CS 486/686

University of Waterloo

Outline

- Russell & Norvig Sect 21.1-21.3
- What is reinforcement learning
- Temporal-Difference learning
- Q-learning

Machine Learning

- Supervised Learning
  - Teacher tells learner what to remember
- Reinforcement Learning
  - Environment provides hints to learner
- Unsupervised Learning
  - Learner discovers on its own

What is RL?

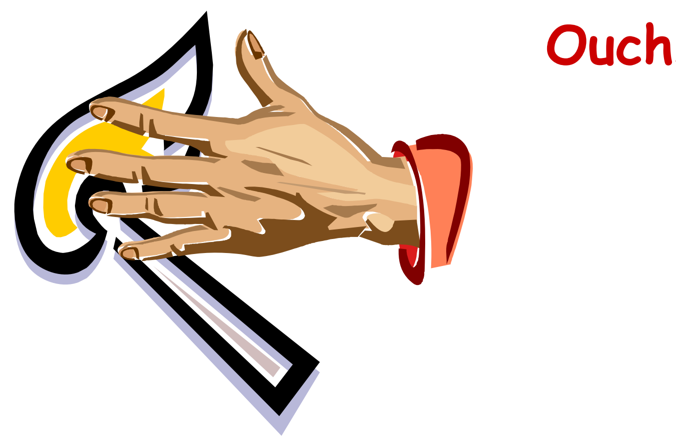
- Reinforcement learning is learning what to do so as to maximize a numerical reward signal
  - Learner is not told what actions to take, but must discover them by trying them out and seeing what the reward is

What is RL

- Reinforcement learning differs from supervised learning

Supervised learning

Reinforcement learning



Animal Psychology

- Negative reinforcements:
  - Pain and hunger
- Positive reinforcements:
  - Pleasure and food
- Reinforcements used to train animals

- Let's do the same with computers!

RL Examples

- Game playing (backgammon, solitaire)
- Operations research (pricing, vehicule routing)
- Elevator scheduling
- Helicopter control

- http://neuromancer.eecs.umich.edu/cgi-bin/twiki/view/Main/SuccessesOfRL

Reinforcement Learning

- Definition:
  - Markov decision process with unknown transition and reward models

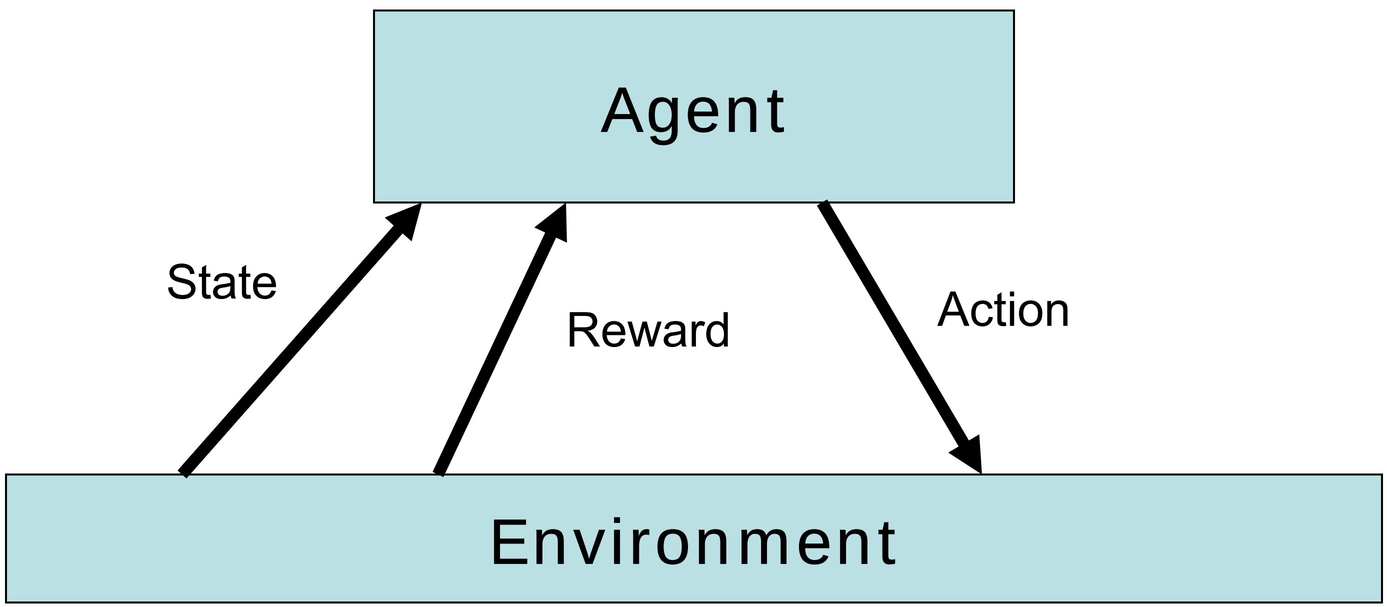
- Set of states S
- Set of actions A
  - Actions may be stochastic
- Set of reinforcement signals (rewards)
  - Rewards may be delayed

Policy optimization

- Markov Decision Process:
  - Find optimal policy given transition and reward model
  - Execute policy found

- Reinforcement learning:
  - Learn an optimal policy while interacting with the environment

Reinforcement Learning Problem

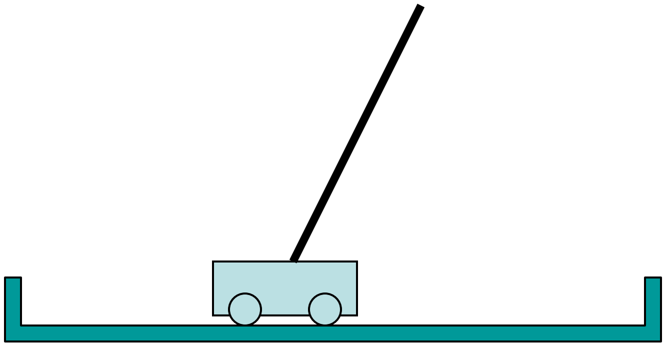


$$s0 \xrightarrow[r0]{a0} s1 \xrightarrow[r1]{a1} s2 \xrightarrow[r2]{a2} \cdots$$

Goal: Learn to choose actions that maximize $r_0+\gamma\, r_1+\gamma^2 r_2+\ldots$, where $0 \leq \gamma < 1$

Example: Inverted Pendulum

- State: $x(t)$, $x'(t)$, $\theta(t)$, $\theta'(t)$
- Action: Force F
- Reward: 1 for any step where pole balanced



Problem: Find $\delta: S \rightarrow A$ that maximizes rewards

Rl Characterisitics

- Reinforcements: rewards
- Temporal credit assignment: when a reward is received, which action should be credited?
- Exploration/exploitation tradeoff: as agent learns, should it exploit its current knowledge to maximize rewards or explore to refine its knowledge?
- Lifelong learning: reinforcement learning

Types of RL

- Passive vs Active learning
  - Passive learning: the agent executes a fixed policy and tries to evaluate it
  - Active learning: the agent updates its policy as it learns

- Model based vs model free
  - Model-based: learn transition and reward model and use it to determine optimal policy
  - Model free: derive optimal policy without learning the model

Passive Learning

- Transition and reward model known:
  - Evaluate δ:
  - $V^{\delta}(s) = R(s) + \gamma \Sigma_{s'} Pr(s'|s,\delta(s)) V^{\delta}(s')$

- Transition and reward model unknown:
  - Estimate policy value as agent executes policy: $V^{\delta}(s) = E_{\delta}[\Sigma_t \gamma^t R(s_t)]$
  - Model based vs model free

Passive learning

3	r	r	r	+1
2	u		u	-1
1	u	l	l	l
	1	2	3	4

$\gamma = 1$

r_i = -0.04 for non-terminal states

Do not know the transition probabilities

(1,1)→ (1,2)→ (1,3)→ (1,2)→ (1,3)→ (2,3)→ (3,3)→ $(4,3)_{+1}$
(1,1)→ (1,2)→ (1,3)→ (2,3)→ (3,3)→ (3,2)→ (3,3)→ $(4,3)_{+1}$
(1,1)→ (2,1)→ (3,1)→ (3,2)→ $(4,2)_{-1}$

What is the value V(s) of being in state s?

Passive ADP

- Adaptive dynamic programming (ADP)
  - Model-based
  - Learn transition probabilities and rewards from observations
  - Then update the values of the states

ADP Example

$\gamma = 1$

3	r	r	r	+1
2	u		u	-1
1	u	l	l	l
	1	2	3	4

r_i = -0.04 for non-terminal states

$$V^{\delta}(s) = R(s) + \gamma\ \Sigma_{s'}\ Pr(s'|s,\delta(s))\ V^{\delta}(s')$$

(1,1)→ (1,2)→ **(1,3)**→ (1,2)→ **(1,3)**→ (2,3)→ (3,3)→ $(4,3)_{+1}$
(1,1)→ (1,2)→ **(1,3)**→ (2,3)→ (3,3)→ (3,2)→ (3,3)→ $(4,3)_{+1}$
(1,1)→ (2,1)→ (3,1)→ (3,2)→ $(4,2)_{-1}$

$$\left.\begin{array}{l} P((2,3)|(1,3),r) = 2/3 \\ P((1,2)|(1,3),r) = 1/3 \end{array}\right\}$$ Use this information in

We need to learn all the transition probabilities!

Passive TD

- Temporal difference (TD)
  - Model free
- At each time step
  - Observe: s,a,s',r
  - Update $V^\delta(s)$ after each move
  - $V^\delta(s) = V^\delta(s) + \underbrace{\alpha}_{\text{Learning rate}} \underbrace{(R(s) + \gamma V^\delta(s') - V^\delta(s))}_{\text{Temporal difference}}$

TD Convergence

Thm: If α is appropriately decreased with number of times a state is visited then $V^{\delta}(s)$ converges to correct value

- α must satisfy:
  - $\Sigma_t \alpha_t \rightarrow \infty$
  - $\Sigma_t (\alpha_t)^2 < \infty$
- Often $\alpha(s) = 1/n(s)$
  - $n(s)$ = # of times s is visited

Active Learning

- Ultimately, we are interested in improving δ
- Transition and reward model known:
  - $V^*(s) = \max_a R(s) + \gamma \Sigma_{s'} Pr(s'|s,a) V^*(s')$
- Transition and reward model unknown:
  - Improve policy as agent executes policy
  - Model based vs model free

Q-learning (aka active temporal difference)

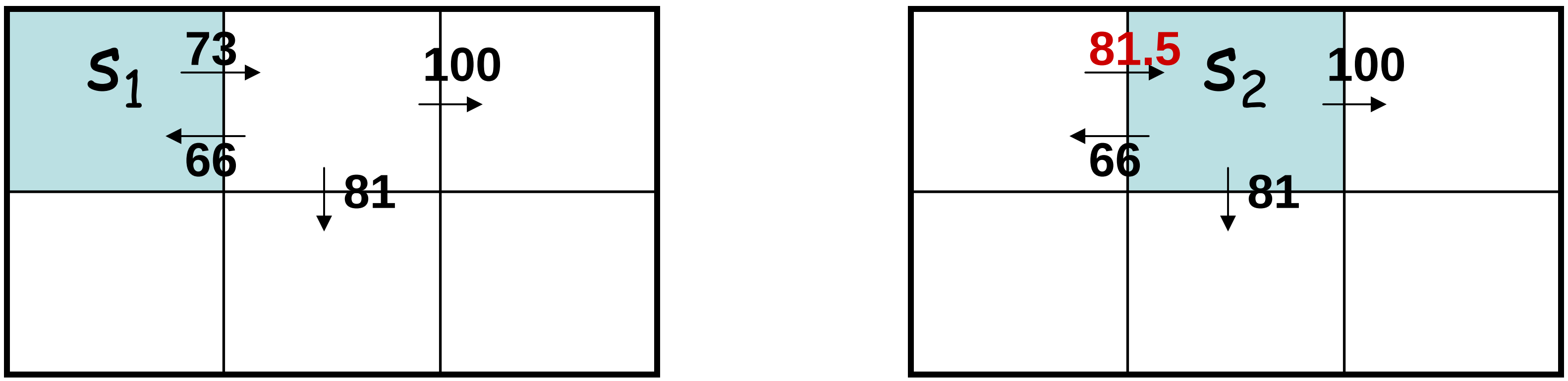
- Q-function: $Q: S \times A \rightarrow \Re$
  - Value of state-action pair
  - Policy $\delta(s) = \mathrm{argmax}_a\, Q(s,a)$ is the optimal policy
- Bellman's equation:

$$\boxed{Q^*(s,a) = R(s) + \gamma\, \Sigma_{s'}\, \Pr(s'|s,a)\, \max_{a'} Q^*(s',a')}$$

Q-learning

- For each state s and action a initialize $Q(s,a)$ (0 or random)
- Observe current state
- Loop
  - Select action a and execute it
  - Receive immediate reward r
  - Observe new state s'
  - Update $Q(a,s)$
    - $Q(s,a) = Q(s,a) + \alpha(r(s)+\gamma \max_{a'} Q(s',a') - Q(s,a))$
  - $s=s'$

Q-learning example



r=0 for non-terminal states

γ=0.9

α=0.5

$$
\begin{aligned}
Q(s_1,\text{right}) &= Q(s_1,\text{right}) + \alpha\,(r(s_1) + \gamma \max_{a'} Q(s_2,a') - Q(s_1,\text{right})) \\
&= 73 + 0.5\,(0 + 0.9 \max[66,81,100] - 73) \\
&= 73 + 0.5\,(17) \\
&= 81.5
\end{aligned}
$$

Q-learning

- For each state s and action a initialize Q(s,a) (0 or random)
- Observe current state
- Loop
  - Select action a and execute it
  - Receive immediate reward r
  - Observe new state s'
  - Update Q(a,s)
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  - s=s'

Exploration vs Exploitation

- If an agent always chooses the action with the highest value then it is exploiting
  - The learned model is not the real model
  - Leads to suboptimal results
- By taking random actions (pure exploration) an agent may learn the model
  - But what is the use of learning a complete model if parts of it are never used?
- Need a balance between exploitation and exporation

Common exploration methods

- ε-greedy:
  - With probability ε execute random action
  - Otherwise execute best action a*
    $a^* = \text{argmax}_a\, Q(s,a)$

- Boltzmann exploration

$$P(a) = \frac{e^{Q(s,a)/T}}{\Sigma_a e^{Q(s,a)/T}}$$

Exploration and Q-learning

- Q-learning converges to optimal Q-values if
  - Every state is visited infinitely often (due to exploration)
  - The action selection becomes greedy as time approaches infinity
  - The learning rate a is decreased fast enough but not too fast

A Triumph for Reinforcement Learning: TD-Gammon

- Backgammon player: TD learning with a neural network representation of the value function:

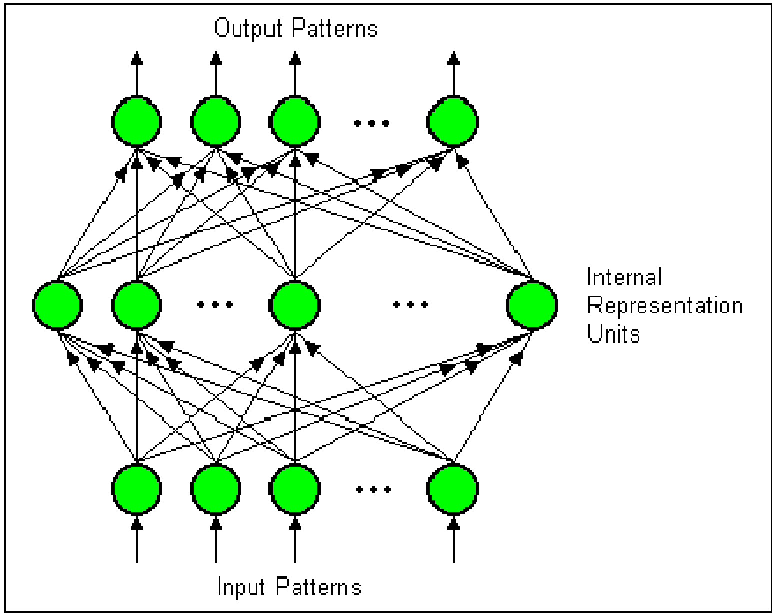

Figure 1. An illustration of the multilayer perception architecture used in TD-Gammon's neural network. This architecture is also used in the popular backpropagation learning procedure. Figure reproduced from [9].

Next Class

- Machine learning
- Decision trees
- Russell and Norvig: chapter 18