

# Lecture 14

June 15, 2006

CS 486/686

# Outline

- Multi-agent systems
- Game theory
- Russell and Norvig: Sect 17.6

# Course Topics

- Search
  - Uninformed and heuristic search
  - CSP's and optimization
  - Game playing
- Reasoning under uncertainty
  - Probability theory, utility theory and decision theory
  - Bayesian networks and decision networks
  - **Multi-agent systems**
- Learning
  - Decision trees, neural networks, ensemble learning, reinforcement learning
- Specialized areas
  - Natural language processing, and robotics

# Multi-agent systems

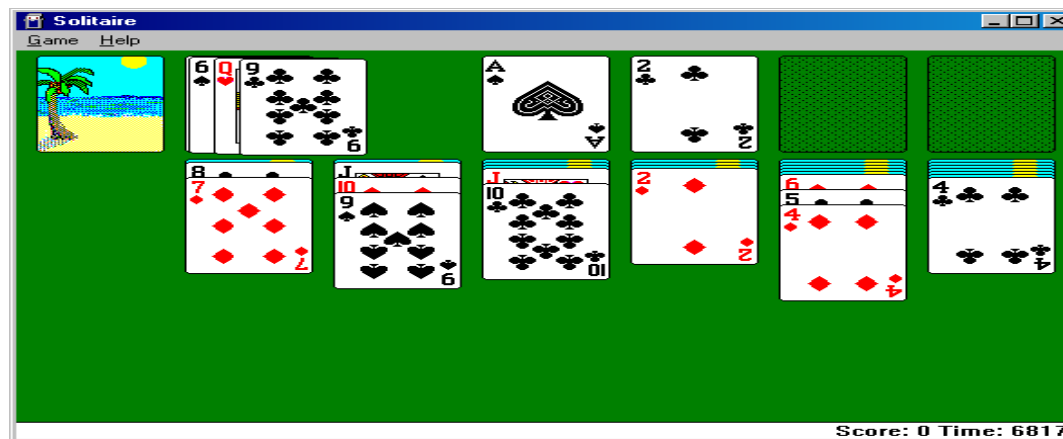
- So far...
  - Single agent optimizing some objectives in a possibly uncertain environment
  - But, what if there are several agents?
- Multi-agent systems
  - Two (or more) agents can influence the world
  - How should an agent act given that it shares "control" with other agents?

# Multi-agent Systems

- Search techniques for deterministic games with alternating play
  - Minimax algorithm
  - Alpha-beta pruning
- Today:
  - Extend decision theory to multi-agent systems
  - View other agents as sources of uncertainty
  - Framework: **Game theory**

# What is game theory?

- Game theory is a formal way to analyze **interactions** among a **group** of **rational** agents who behave **strategically**
  - **Group**: Must have more than 1 decision maker
    - Otherwise you have a decision problem, not a game



Solitaire  
is not a  
game!

# What is game theory?

- Game theory is a formal way to analyze **interactions** among a **group** of **rational** agents who behave **strategically**
  - **Interaction**: What one agent does directly affects at least one other agent in the group
  - **Rational**: An agent chooses its best action
  - **Strategic**: Agents take into account how other agents influence the game

# Games

- Examples:
  - Chess, soccer, poker, etc.
  - Elections
  - Auctions, Trades
  - Taxation system
  - Negotiation
  - Packet routing protocols,
  - Driving laws



# Two aspects

- **Agent design**
  - Given a game, what is a rational strategy?
  - Ex: playing chess, driving, voting, filling up an income tax report, etc.
- **Mechanism design**
  - Given that agents behave rationally, what should the rules of the game be?
  - Ex: designing driving laws, an election, a taxation system, an auction, etc.

# Strategic Games (aka normal form)

- Formally:  $\langle I, \{S_i\}, \{U_i\} \rangle$
- Set of **agents**  $I = \{1, 2, \dots, n\}$
- Each agent  $i$  can choose a **strategy**  $s_i \in S_i$
- Outcome of the game is defined by a **strategy profile**  $(s_1, \dots, s_n) \in S$
- Agents have **preferences** over the outcomes
  - utility functions:  $U_i(s_1, \dots, s_n) \in \mathbb{R}$

# Example: Election

- **Agents:** electors
- **Strategies:** possible votes for different candidates
- **Outcome:** set of all votes determines a winner (elected candidate)
- **Utility fn:** preferences for each candidate

# Simple Games

- Assumptions:
  - Single decision
  - Deterministic game
  - Fully observable game
  - Simultaneous play
- Possible to relax those assumptions... but this is beyond the scope of this course

# Example: Even or Odd

Agent 2

One Two

Agent 1

One

Two

|     |      |      |
|-----|------|------|
|     | One  | Two  |
| One | 2,-2 | -3,3 |
| Two | -3,3 | 4,-4 |

Zero-sum  
game.

$$\sum_{i=1}^n u_i(o)=0$$

$I=\{1,2\}$

$S_i=\{\text{One}, \text{Two}\}$

An outcome is (One, Two)

$U_1((\text{One}, \text{Two}))=-3$  and  $U_2((\text{One}, \text{Two}))=3$



# Examples of strategic games

## Baseball or Soccer

|   | B   | S   |
|---|-----|-----|
| B | 2,1 | 0,0 |
| S | 0,0 | 1,2 |



## Chicken

|   | T     | C    |
|---|-------|------|
| T | -1,-1 | 10,0 |
| C | 0,10  | 5,5  |



## Coordination Game

## Anti-Coordination Game

# Example: Prisoner's Dilemma



Confess

Don't Confess

Confess

-5,-5

0,-10

Don't  
Confess

-10,0

-1,-1

# Playing a game

- We now know how to describe a game
- Next step - **Playing the game!**
- Recall, agents are **rational**
  - Let  $p_i$  be agent  $i$ 's beliefs about what its opponent will do
  - Agent  $i$  is rational if it chooses to play strategy  $s_i^*$  where

$$s_i^* = \operatorname{argmax}_{s_i} \sum_{s_{-i}} u_i(s_i, s_{-i}) p_i(s_{-i})$$

**Notation:**  $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$



# Dominated Strategies

- **Definition:** A strategy  $s_i$  is *strictly dominated* if

$$\exists s_i', \forall s_{-i}, u_i(s_i, s_{-i}) < u_i(s_i', s_{-i})$$

- A rational agent will never play a strictly dominated strategy!
  - This allows us to solve some games!

# Example: Prisoner's Dilemma

|               | Confess | Don't Confess |
|---------------|---------|---------------|
| Confess       | -5,-5   | 0,-10         |
| Don't Confess | -10,0   | -1,-1         |

|         | Confess | Don't Confess    |
|---------|---------|------------------|
| Confess | -5,-5   | <del>0,-10</del> |

|         | Confess |
|---------|---------|
| Confess | -5,-5   |

Equilibrium Outcome

# Strict Dominance does not capture the whole picture

|   | A   | B   | C   |
|---|-----|-----|-----|
| A | 0,4 | 4,0 | 5,3 |
| B | 4,0 | 0,4 | 5,3 |
| C | 3,5 | 3,5 | 6,6 |

What strict dominance eliminations can we do?

None...

So what should the players of this game do?

# Nash Equilibrium

- Sometimes an agent's best-response depends on the strategies other agents are playing
- A strategy profile,  $s^*$ , is a **Nash equilibrium** if no agent has incentive to deviate from its strategy *given that others do not deviate*:

$$\forall i \ u_i(s_i^*, s_{-i}^*) \geq u_i(s'_i, s_{-i}^*) \ \forall s'_i$$

# Nash Equilibrium

- Equivalently,  $s^*$  is a N.E. iff  
 $\forall i \quad s_i^* = \operatorname{argmax}_{s_i} u_i(s_i, s_{-i}^*)$

|   | A   | B   | C   |
|---|-----|-----|-----|
| A | 0,4 | 4,0 | 5,3 |
| B | 4,0 | 0,4 | 5,3 |
| C | 3,5 | 3,5 | 6,6 |

**(C,C) is a N.E. because**

$$u_1(C, C) = \max \begin{bmatrix} u_1(A, C) \\ u_1(B, C) \\ u_1(C, C) \end{bmatrix}$$

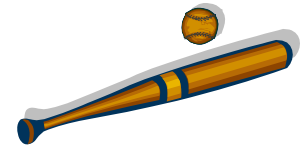
**AND**

$$u_2(C, C) = \max \begin{bmatrix} u_2(C, A) \\ u_2(C, B) \\ u_2(C, C) \end{bmatrix}$$

# Another example

|   | B   | S   |
|---|-----|-----|
| B | 2,1 | 0,0 |
| S | 0,0 | 1,2 |




A 2x2 payoff matrix for a coordination game. The columns are labeled B and S, and the rows are labeled B and S. The payoffs are (2,1) for (B,B), (0,0) for (B,S), (0,0) for (S,B), and (1,2) for (S,S). The (B,B) and (S,S) cells are circled in green. A green arrow points up to the (B,B) cell, and a green arrow points down from the (S,S) cell. A red arrow points left from the (B,S) cell, and a red arrow points right to the (S,B) cell.



**2 Nash Equilibria**

**Coordination Game**

# Yet another example

|         |     | Agent 2  |  |
|---------|-----|--|--|
|         |     | One  | Two  |
| Agent 1 | One | 2, -2  | -3, 3  |
|         | Two |  -3, 3  |  4, -4 |

There is no PURE strategy Nash Equilibrium for this game

# (Mixed) Nash Equilibria

- **Mixed strategy  $\sigma_i$ :**
  - $\sigma_i \in \Sigma_i$  defines a probability distribution over  $S_i$
- **Strategy profile:**  $\sigma = (\sigma_1, \dots, \sigma_n)$
- **Expected utility:**  $u_i(\sigma) = \sum_{s \in S} (\prod_j \sigma(s_j)) u_i(s)$
- **Nash Equilibrium:**  $\sigma^*$  is a (mixed) Nash equilibrium if
$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i', \sigma_{-i}^*) \quad \forall \sigma_i'$$



# Yet another example

|   |     | B    |      |
|---|-----|------|------|
|   |     | One  | Two  |
| A | One | 2,-2 | -3,3 |
|   | Two | -3,3 | 4,-4 |

$p = \text{Pr}(\text{one})$   
 $q = \text{Pr}(\text{two})$

How do we determine  $p$  and  $q$ ?

$$U_A(p, q) = 2pq - 3p(1-q) - 3(1-p)q + 4(1-p)(1-q)$$

$$U_B(p, q) = -2pq + 3p(1-q) + 3(1-p)q - 4(1-p)(1-q)$$

$$\frac{\partial}{\partial p} U_A(p, q) = 12q - 7 \Rightarrow q = \frac{7}{12}$$

$$\frac{\partial}{\partial q} U_B(p, q) = -12p + 7 \Rightarrow p = \frac{7}{12}$$

# Exercise

|   | B   | S   |
|---|-----|-----|
| B | 2,1 | 0,0 |
| S | 0,0 | 1,2 |

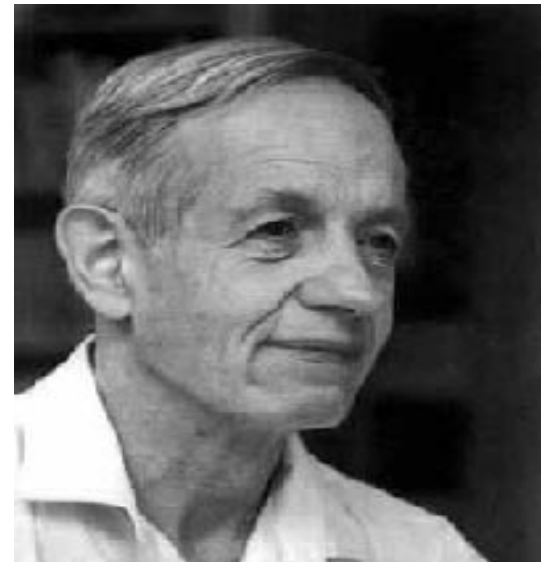
This game has 3 Nash Equilibria (2 pure strategy NE and 1 mixed strategy NE). Find them.

# Mixed Nash Equilibrium

- **Theorem (Nash 50):**

Every game in which the strategy sets  $S_1, \dots, S_n$  have a finite number of elements has a mixed strategy equilibrium.

**John Nash**  
**Nobel Prize in Economics (1994)**



# Other Useful Theorems

- **Thm:** In an  $n$ -player pure strategy game  $G=(S_1, \dots, S_n; u_1, \dots, u_n)$ , if iterated elimination of strictly dominated strategies eliminates all but the strategies  $(S_1^*, \dots, S_n^*)$  then these strategies are the unique NE of the game
- **Thm:** Any NE will survive iterated elimination of strictly dominated strategies.

# Nash Equilibrium

- Interpretations:
  - Focal points, self-enforcing agreements, stable social convention, consequence of rational inference..
- Criticisms
  - They may not be unique
    - Ways of overcoming this: Refinements of equilibrium concept, Mediation, Learning
  - They may be hard to find
  - People don't always behave based on what equilibria would predict (ultimatum games and notions of fairness,...)

# Bayesian Games

- What should player A do?

|          |   | Player B |      |
|----------|---|----------|------|
|          |   | L        | R    |
| Player A | U | 3,?      | -2,? |
|          | D | 0,?      | 6,?  |

Question: When does such a situation arise?

# Bayesian Games

- Hockey lover gets 2 units for watching hockey and 1 unit for watching curling
- Curling lover gets 2 units for watching curling and 1 unit for watching hockey
- Pat is a hockey lover
- Pat thinks that Chris is probably a hockey lover, but is not sure

|     |   | Chris |     |
|-----|---|-------|-----|
|     |   | H     | C   |
| Pat | H | 2,2   | 0,0 |
|     | C | 0,0   | 1,1 |

With 2/3 chance

|     |   | Chris |     |
|-----|---|-------|-----|
|     |   | H     | C   |
| Pat | H | 2,1   | 0,0 |
|     | C | 0,0   | 1,2 |

With 1/3 chance

# Bayesian Games

- In a Bayesian game each player has a **type**
- All players know their own type, but have only a probability distribution over their opponents' types
- **Game  $G$** 
  - Set of action spaces:  $A_1, \dots, A_n$
  - Set of type spaces:  $T_1, \dots, T_n$
  - Set of beliefs:  $P_1, \dots, P_n$
  - Set of payoff functions:  $u_1, \dots, u_n$
  - $P_i(t_{-i} | t_i)$  is the prob distribution of the types for the other players, given player  $i$  has type  $t_i$
  - $u_i(a_1, \dots, a_n; t_i)$  is the utility (payoff) to agent  $i$  if player  $j$  chooses action  $a_j$  and agent  $i$  has type  $t_i \in T_i$



# Knowledge Assumptions (Who knows what)

- All players know  $A_i$ 's,  $T_i$ 's,  $P_i$ 's and  $u_i$ 's
- The  $i$ 'th player knows  $t_i$   
but not  $t_1, t_2, \dots, t_{i-1}, t_{i+1}, \dots, t_n$
- All players know that all players know the above
- And they know that they know that they know..... (common knowledge)
- **Def:** A **strategy**  $s_i(t_i)$  in a Bayesian game is a mapping from  $T_i$  to  $A_i$  (i.e. it specifies what action should be taken for each type)

# Back to our game

- $A_1 = \{H, C\}$   $A_2 = \{H, C\}$
- $T_1 = \{hl, cl\}$   $T_2 = \{hl, cl\}$
- $P_1$ 
  - $P_1(t_2=hl|t_1=hl)=2/3$ ,  $P_1(t_2=cl|t_1=hl)=1/3$ ,  $P_1(t_2=hl|h_1=cl)=2/3$ ,  
 $P_1(t_2=cl|h_1=cl)=1/3$
- $P_2$ 
  - $P_2(t_1=hl|t_2=hl)=1$ ,  $P_2(t_1=cl|t_2=hl)=0$ ,  $P_2(t_1=hl|t_2=cl)=1$ ,  
 $P_2(t_1=cl|t_2=cl)=0$
- $U_1$ 
  - $u_1(H,H,hl)=2$ ,  $u_1(H,H,cl)=1$ ,  $u_1(H,C,hl)=0$ ,...
- $U_2$ 
  - $u_2(H,H,hl)=2$ ,  $u_2(H,H,cl)=1$ ,  $u_2(H,C,cl)=0$ ,...

# Bayesian Nash Equilibrium

- A set of strategies  $(s_1^*, \dots, s_n^*)$  are a Pure Bayesian Nash Equilibrium if and only if for each player  $i$ , and for all possible types  $t_i \in T_i$

$$s_i^*(t_i) = \operatorname{argmax}_{a_i \in A_i} \sum_{t_{-i}} u_i(a_i, s_{-i}^*(t_{-i}))$$

No player, for any of their type, wants to change their strategy

# Next Class

- Reinforcement Learning
- Russell and Norvig: Chapter 21