Lecture 14

June 16, 2005
CS 486/686

Outline

- Multi-agent systems
- Game theory
- Russell and Norvig: Sect 17.6

Course Topics

- Search
  - Uninformed and heuristic search
  - CSP's and optimization
  - Game playing
- Reasoning under uncertainty
  - Probability theory, utility theory and decision theory
  - Bayesian networks and decision networks
  - Multi-agent systems
- Learning
  - Decision trees, neural networks, ensemble learning, reinforcement learning
- Specialized areas
  - Natural language processing, computational vision and robotics

---

Multi-agent systems

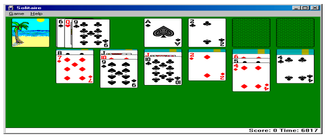
- So far…
  - Single agent optimizing some objectives in a possibly uncertain environment
  - But, what if there are several agents?
- Multi-agent systems
  - Two (or more) agents can influence the world
  - How should an agent act given that it shares "control" with other agents?

Multi-agent Systems

- Search techniques for deterministic games with alternating play
  - Minimax algorithm
  - Alpha-beta pruning
- Today:
  - Extend decision theory to multi-agent systems
  - View other agents as sources of uncertainty
  - Framework: **Game theory**

What is game theory?

- Game theory is a formal way to analyze **interactions** among a **group** of **rational** agents who behave **strategically**
  - Group: Must have more than 1 decision maker
    - Otherwise you have a decision problem, not a game



Solitaire is not a game!

What is game theory?

- Game theory is a formal way to analyze **interactions** among a **group** of **rational** agents who behave **strategically**
  - Interaction: What one agent does directly affects at least one other agent in the group
  - Rational: An agent chooses its best action
  - Strategic: Agents take into account how other agents influence the game

Games

- Examples:
  - Chess, soccer, poker, etc.
  - Elections
  - Auctions, Trades
  - Taxation system
  - Negotiation
  - Packet routing protocols,
  - Driving laws

Two aspects

- Agent design
  - Given a game, what is a rational strategy?
  - Ex: playing chess, driving, voting, filling up an income tax report, etc.
- Mechanism design
  - Given that agents behave rationally, what should the rules of the game be?
  - Ex: designing driving laws, an election, a taxation system, an auction, etc.

Strategic Games (aka normal form)

- Formally: $<I,\{S_i\},\{U_i\}>$
- Set of **agents** $I=\{1,2,\ldots,n\}$
- Each agent i can choose a **strategy** $s_i \in S_i$
- Outcome of the game is defined by a **strategy profile** $(s_1,\ldots,s_n) \in S$
- Agents have **preferences** over the outcomes
  - utility functions: $U_i(s_1,\ldots,s_n) \in \Re$

Example: Election

- Agents: electors
- Strategies: possible votes for different candidates
- Outcome: set of all votes determines a winner (elected candidate)
- Utility fn: preferences for each candidate

Simple Games

- Assumptions:
  - Single decision
  - Deterministic game
  - Fully observable game
  - Simultaneous play
- Possible to relax those assumptions… but this is beyond the scope of this course

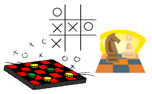
Example: Even or Odd

Agent 2

		One	**Two**
Agent 1	**One**	2,-2	-3,3
	Two	-3,3	4,-4

Zero-sum game.
$\Sigma_{i=1}^{n} u_i(o)=0$

I={1,2}
S_i={One,Two}
An outcome is (One, Two)
U_1((One,Two))=-3 and U_2((One,Two))=3



Examples of strategic games

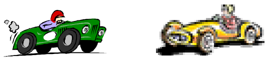
Baseball or Soccer

	B	**S**
B	2,1	0,0
S	0,0	1,2

Chicken

	T	**C**
T	-1,-1	10,0
C	0,10	5,5

Coordination Game



Anti-Coordination Game

Example: Prisoner's Dilemma

	Confess	Don't Confess
Confess	-5,-5	0,-10
Don't Confess	-10,0	-1,-1

Playing a game

- We now know how to describe a game
- Next step – Playing the game!
- Recall, agents are **rational**
  - Let p_i be agent i's beliefs about what its opponent will do
  - Agent i is rational if it chooses to play strategy s_i* where

$$s_i^* = \text{argmax}_{s_i} \Sigma_{s_{-i}} u_i(s_i',s_{-i})p_i(s_{-i})$$

Notation: $s_{-i} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$

Dominated Strategies

- **Definition**: A strategy s_i is *strictly dominated* if

$$\exists s_i', \forall s_{-i}, u_i(s_i,s_{-i}) < u_i(s_i',s_{-i})$$

- A rational agent will never play a strictly dominated strategy!
  - This allows us to solve some games!

Example: Prisoner's Dilemma

	Confess	Don't Confess
Confess	-5,-5	0,-10
Don't Confess	~~-10,0~~	~~-1,-1~~

	Confess	Don't Confess
Confess	-5,-5	~~0,-10~~

	Confess
Confess	-5,-5

Equilibrium Outcome

Strict Dominance does not capture the whole picture

	A	B	C
A	0,4	4,0	5,3
B	4,0	0,4	5,3
C	3,5	3,5	6,6

What strict dominance eliminations can we do?

None…
So what should the players of this game do?

Nash Equilibrium

- Sometimes an agent's best-response depends on the strategies other agents are playing
- A strategy profile, s*, is a **Nash equilibrium** if no agent has incentive to deviate from its strategy *given that others do not deviate*:

$$\forall i \; u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*) \; \forall s_i'$$

Nash Equilibrium

- Equivalently, s^* is a N.E. iff
  $\forall i \quad s_i^* = \text{argmax}_{s_i} u_i(s_i, s_{-i}^*)$

	A	B	C
A	0,4	4,0	5,3
B	4,0	0,4	5,3
C	3,5	3,5	6,6

(C,C) is a N.E. because

$$u_1(C,C) = \max \begin{bmatrix} u_1(A,C) \\ u_1(B,C) \\ u_1(C,C) \end{bmatrix}$$

AND

$$u_2(C,C) = \max \begin{bmatrix} u_2(C,A) \\ u_2(C,B) \\ u_2(C,C) \end{bmatrix}$$

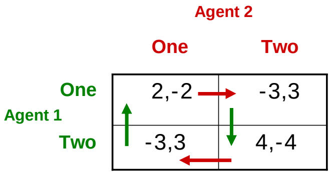
Another example

	B	S
B	2,1	0,0
S	0,0	1,2

2 Nash Equilibria

Coordination Game

Yet another example



		Agent 2	
		One	Two
Agent 1	One	2,-2	-3,3
	Two	-3,3	4,-4

There is no **PURE** strategy Nash Equilibrium for this game

(Mixed) Nash Equilibria

- Mixed strategy σ_i:
  - $\sigma_i \in \Sigma_i$ defines a probability distribution over S_i
- Strategy profile: $\sigma = (\sigma_1, \ldots, \sigma_n)$
- Expected utility: $u_i(\sigma) = \Sigma_{s \in S} (\Pi_j \; \sigma(s_j)) u_i(s)$
- Nash Equilibrium: σ^* is a (mixed) Nash equilibrium if

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i', \sigma_{-i}^*) \; \forall \sigma_i'$$

Yet another example

		B: One	B: Two
A	One	2,-2	-3,3
A	Two	-3,3	4,-4

p = Pr(one)
q = Pr(one)

How do we determine p and q?

$$U_A(p,q) = 2pq - 3p(1-q) - 3(1-p)q + 4(1-p)(1-q)$$

$$U_B(p,q) = -2pq + 3p(1-q) + 3(1-p)q - 4(1-p)(1-q)$$

$$\frac{\partial}{\partial p} U_A(p,q) = 12q - 7 \Rightarrow q = \frac{7}{12}$$

$$\frac{\partial}{\partial q} U_B(p,q) = -12p + 7 \Rightarrow p = \frac{7}{12}$$

Exercise

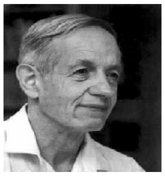
	B	S
B	2,1	0,0
S	0,0	1,2

This game has 3 Nash Equilibrium (2 pure strategy NE and 1 mixed strategy NE). Find them.

Mixed Nash Equilibrium

- **Theorem** (Nash 50):

  Every game in which the strategy sets, $S_1,\ldots, S_n$ have a finite number of elements has a mixed strategy equilibrium.



John Nash
Nobel Prize in Economics (1994)

Other Useful Theorems

- **Thm:** In an n-player pure strategy game $G=(S_1,\ldots,S_n; u_1,\ldots,u_n)$, if iterated elimination of strictly dominated strategies eliminates all but the strategies $(S_1^*,\ldots,S_n^*)$ then these strategies are the unique NE of the game

- **Thm:** Any NE will survive iterated elimination of strictly dominated strategies.

Nash Equilibrium

- Interpretations:
  - Focal points, self-enforcing agreements, stable social convention, consequence of rational inference..
- Criticisms
  - They may not be unique
    - Ways of overcoming this: Refinements of equilibrium concept, Mediation, Learning
  - They may be hard to find
  - People don't always behave based on what equilibria would predict (ultimatum games and notions of fairness,…)

Bayesian Games

- What should player A do?

		Player B: L	Player B: R
Player A	U	3,?	-2,?
Player A	D	0,?	6,?

Question: When does such a situation arise?

Bayesian Games

- Hockey lover get 2 units for watching hockey and 1 unit for watching curling
- Curling lover gets 2 units for watching curling and 1 unit for watching hockey
- Pat is a hockey lover
- Pat thinks that Chris is probably a hockey lover, but is not sure

		Chris	
		H	C
Pat	H	2,2	0,0
	C	0,0	1,1

With 2/3 chance

		Chris	
		H	C
Pat	H	2,1	0,0
	C	0,0	1,2

With 1/3 chance

Bayesian Games

- In a Bayesian game each player has a **type**
- All players know their own type, but have only a probability distribution over their opponents' types
- Game G
  - Set of action spaces: $A_1,\ldots,A_n$
  - Set of type spaces: $T_1,\ldots,T_n$
  - Set of beliefs: $P_1,\ldots,P_n$
  - Set of payoff functions: $u_1,\ldots,u_n$

  - $P_i(t_{-i}|t_i)$ is the prob distribution of the types for the other players, given player i has type t_i
  - $u_i(a_1,\ldots,a_n;t_i)$ is the utility (payoff) to agent i if player j chooses action a_j and agent i has type $t_i \in T_i$

Knowledge Assumptions (Who knows what)

- All players know A_i's, T_i's, P_i's and u_i's
- The i'th player knows t_i but not $t_1,t_2,\ldots t_{i-1}, t_{i+1},\ldots,t_n$
- All players know that all players know the above
- And they know that they know that they know…… (common knowledge)
- **Def:** A **strategy** $s_i(t_i)$ in a Bayesian game is a mapping from T_i to A_i (i.e. it specifies what action should be taken for each type)

Back to our game

- A_1={H, C} A_2={H,C}
- T_1={hl, cl} T_2={hl, cl}
- P_1
  - $P_1(t_2=hl|t_1=hl)=2/3$, $P_1(t_2=cl|t_1=hl)=1/3$, $P_1(t_2=hl|h_1=cl)=2/3$, $P_1(t_2=cl|t_1=cl)=1/3$
- P_2
  - $P_2(t_1=hl|t_2=hl)=1$, $P_2(t_1=cl|t_2=hl)=0$, $P_2(t_1=hl|t_2=cl)=1$, $P_2(t_1=cl|t_2=cl)=0$
- U_1
  - $u_1(H,H,hl)=2$, $u_1(H,H,cl)=1$, $u_1(H,C,hl)=0,\ldots$
- U_2
  - $u_2(H,H,hl)=2$, $u_2(H,H,cl)=1$, $u_2(H,C,cl)=0,\ldots$

Bayesian Nash Equilibrium

- A set of strategies $(s_1^*,\ldots,s_n^*)$ are a Pure Bayesian Nash Equilibrium if and only if for each player i, and for all possible types $t_i \in T_i$

$$s_i^*(t_i) = \mathrm{argmax}_{a_i \in A_i} \Sigma_{t_{-i}} u_i(a_i,s_{-i}^*(t_{-i}))$$

No player, for any of their type, wants to change their strategy

---

Next Class

- Reinforcement Learning
- Russell and Norvig: Chapter 21